A COMBINED EXPERIMENTAL AND SPH APPROACH TO SLOSHING AND SHIP ROLL MOTIONS

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ABSTRACT

Passive anti-roll tanks have been used for a long time in ships to damp their roll motion. The coupled roll motion response of a single degree of freedom (SDOF) system to which a passive anti-roll tank has been attached is considered in the present paper. The performance of the anti-roll tank has been studied both experimentally and numerically, with weakly compressible SPH. The sloshing flows inside the tank comprise the onset of breaking waves. In order to characterise the wave breaking effects on the response curves, tests have been performed with liquids of different viscosity, the increasing viscosity preventing the onset of breaking waves. The capabilities of SPH to treat this coupling problem are assessed and the results show that SPH is able to capture a part of the physics involved in the addressed phenomena but further work remains still to be done.

Keywords: *Smoothed Particle Hydrodynamics, SPH, antiroll tanks, TLD, Tuned Liquid Damper, wave breaking, roll, SDOF, single degree of freedom systems.*

1. INTRODUCTION

Roll motion is one of the most important responses of a ship in waves. Antiroll tanks have been used for a long time in ships to damp their roll motion. A free surface roll-damping tank was first introduced by Philip Watts at the end of the 19th century (Watts, 1883). Framh in 1911 presented a U-tube form roll damper that became popular at the end of the second world war. The mechanical equivalent to an antiroll tank is a damped vibration absorber, (Anderson et al., 2002, Ikeda and Nakagawa, 1997). These absorbers, in the field of civil engineering, are usually rectangular tanks filled with water and are named tuned liquid dampers (TLD). The TLD systems rely on the sloshing waves that appear at the free surface of the fluid to produce a counter force and/or torque thus dampening the original forcing motion if the correct frequency ratio is met. In the existing literature two approaches can be found to characterise the behaviour of a TLD to external excitations. The first one consists of imposing a periodic motion on the TLD by using a shaking table or a forced roll motion device and measuring the response in terms of lateral force or moment (Tait *et al*., 2005; Souto-Iglesias et al. 2006). The other approach, more complex, and the one the present paper deals with, is to consider the motion response of the coupled system tank-structure, subjected to external excitation in terms of force, moment or even induced motion to the tank interfaced with an elastic structure. With this second approach the damping characteristics, inertia and restoring terms are also relevant in the dynamic analysis. Real motions of the structure are the outcome of this process, that can be compared with design limit states (Delorme *et al*, 2006; Attari and Rofooei, 2008).

In contrast to the civil engineering case, where rectilinear motions are usually the main concern, the dampening effect on roll given by an anti-roll tank is obtained through the moment generated by the tank with angular motion. Van den Bosch and Vugts (1966) coupled the roll motion equation with experimentally determined fluid moments for an oscillating free surface tank and Francescutto and Contento (1999) exploited a mechanical equivalence to provide a 2-DOF analytical model which is nonlinear in respect to roll, and linear in respect to the DOF associated to the fluid sloshing. SDOF models for roll are often used to simulate the roll behaviour in beam seas (Bulian and Francescutto, 2004) and an example of analytical nonlinear SDOF descriptions of roll motion coupled with direct CFD calculations for the free surface tank can be found in Armenio et al, (1996). Regarding experimental work on the coupling problem, Rognebakke and Faltinsen (2003) analyzed the coupled problem in the sway case of a box excited by waves, in comparison to experimental data. Also, a few studies have been reported using the shaking table for horizontal excitations (Sun and Fujino, 1994), but not much in relation to angular motions (Pirner and Urushadze, 2007).

In this paper, a SDOF (single degree of freedom) structure with a partially filled tank is considered, with roll motion modelled by means of an "exact" (from the dynamics point of view) 1-DOF approach. This means that the damper acts as an angular damper whilst most of the previously described works correspond to horizontal excitation. The moment created by the fluid with respect to the rolling axis is simulated and results for the roll angle are compared. The motion is excited by the moment created by a transversally (in a tank fixed reference system) moving mass with imposed motion. Experiments have been performed using fluids of different viscosity in order to assess the influence of breaking. A similar approach was taken by Pirner and Urushadze (2007), because water was not a suitable liquid to be used in footbridges which was the problem they were interested in.

The numerical simulations have been performed using SPH. SPH has been applied with success to shallow depth sloshing problems with periodic oscillation in sway (Landrini et al. 2003) and roll (Souto et.al., 2006) motions. It had also been applied to a coupled motion problem (Delorme *et al,* 2006), showing promising results that had been compared with those obtained with a multimodal approach. Nevertheless, in Delorme *et al* (2006), comparisons with experimental results were not possible because they were not available at that time. In the present paper, an experimental SDOF model has been modeled and it will serve as a benchmark data supplier for the comparisons with the SPH computations. The problem represents a significant challenge specially for SPH due to the extent in time of the real phenomena to be simulated.

It is also important to underline that the present experimental / numerical approach allows to remove the difficulties usually encountered in a correct modelling of the actual ship roll motion. Indeed, when numerical simulations are compared with experimental tests carried out on ship models excited by waves, it is almost never completely clear which is the real source of discrepancy between experimental results and numerical prediction, i.e. whether the reason is to be sought in the modelling of sea-ship interaction or in the modelling of sea-tank interaction. In the present tests, being the dynamics of the mechanical system practically known "exactly" (at least at a reasonable level of accuracy, with some question mark on damping at small rolling angles), any significant discrepancy is likely to be sought in the SPH computational method.

2. EXPERIMENTS. TEST CASES.

The experiments were conducted with the tank testing device of the CEHINAV group (see Souto et al 2006 for a detailed description). The standard forced motion configuration of the device, used regularly in the design of anti-roll tanks, was modified by disconnecting the driving electrical engine from the tank holding structure, in order to allow the free motion of the tank.

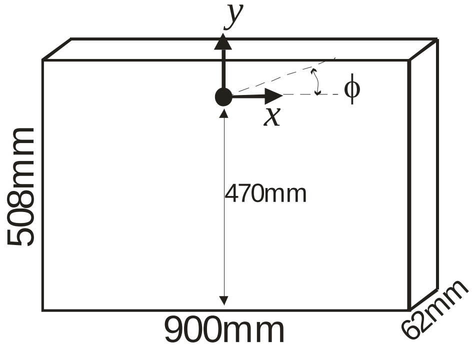

Figure 1. Tank dimensions.

The tank chosen for the simulations is depicted with its dimensions in figure 1. It is narrow along the z-direction, i.e. the direction perpendicular to the paper, in comparison with the horizontal and vertical dimensions. This is intended to have predominantly a two dimensional flow, since the code used for the simulations will be a 2D one, faster than the 3D ones, and which can be more easily modified and validated than a 3D one. The rotating shaft is placed 470mm above the base line of the tank. Having the tank below the rotating shaft makes it possible in this case to have a still stable configuration. Moreover, having the base of the tank at a considerable distance from the rotation axis makes the flow more interesting in terms of propagation of generated waves. There is a horizontal linear guide 600mm long, placed at the rotation centre to simplify some terms in the motion equation (figure 2). This linear guide consists of a controllable electrical engine that laterally moves a weight with a specified motion. This weight is intended to generate the heeling moment and roughly mimic the the wave action on the roll motion.

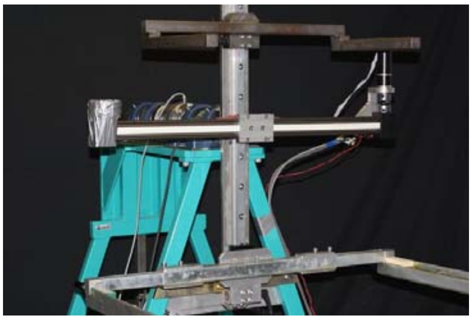

Figure 2. Sliding mass device.

The natural frequency of the rigid system with the empty tank ω_0, as will be later discussed, is 3.26 rad/s. The water depth (H) whose first sloshing frequency matches ω_0 has been chosen for the experiments (92mm). The test matrix is defined by choosing 3 moving weight frequencies $0.9\omega_0$, ω_0 and $1.1\omega_0$ and 4 moving weight motion amplitudes (50, 100, 150 and 200mm). Three different liquids have been used, namely: water, sunflower oil and glycerin. For all cases, the moving mass m is the same, m=4.978 kg. A summary of the test cases is reported in table 1.

It is important to define which are the Reynolds numbers of our experimental flows. The characteristic velocity will be related to the bore front propagation velocity of the equal height dam-break ($\sqrt{gH}$); the characteristic length will be taken as the water depth H. The physical constants of the three liquids and the corresponding Reynolds numbers are documented in table 2.

Table 1. Test matrix (repeated for each liquid: water, sunflower oil and glycerin).

$w/w_0 \setminus A$	50mm	100mm	150mm	200mm
0.9	0.9/50	0.9/100	0.9/150	0.9/200
1.0	1.0/50	1.0/100	1.0/150	1.0/200
1.1	1.1/50	1.1/100	1.1/150	1.1/200

Table 2. Physical properties (units SI) of the liquids: ρ for density, μ, ν for the dynamic and kinematic viscosity. Re for Reynolds number.

	ρ	μ	ν	Re
Water	998	8.94e-4	8.96e-7	97546
Oil	900	0.045	5e-5	1748
Glycerine	1261	0.934	7.4e-4	118

According to the obtained Reynolds number, the water cases will be fully turbulent and the glycerin ones will be completely laminar. In the oil cases, both regimes will coexist. In order to simulate the water cases, due to the small thickness of the boundary layers and the high resolution it would imply, a free slip condition will be used. We will mainly focus on the water cases in the present paper, using the other liquids' cases as reference regarding the onset of splashing and breaking waves and the influence of these phenomena on the damping effect of the TLD.

3. MODEL

3.1. Analytical model of the SDOF structural system

An analytical model of the SDOF structural system used in the experiments is needed in order to have it incorporated into the structure part of the SPH code. It was obtained by rigorously analyzing the dynamics of the system and by obtaining the coefficients after carefully analyzing a set of tests with the empty tank and thereafter finding a data-consistent damping term model. The analytical model used to describe the behaviour of the system is, in general, as follows:

$$\left[I_o + m\xi_m^2(t)\right]\cdot\ddot{\phi} + 2m\xi_m(t)\dot{\xi}_m(t)\cdot\dot{\phi} - g\cdot S_G\cdot\sin(\phi + m\cdot g\cdot\xi_m(t)\cdot\cos(\phi) = Q_{damp}(t) + Q_{fluid}(t) \quad (1)$$

$$Q_{damp}(t) = -K_{df}\cdot sign(\dot{\phi}) - B_\phi\cdot\dot{\phi} \quad (2)$$

- where:
- ϕ [rad] is the roll angle.
- g $[m/s^2]$ is the gravitational acceleration.
- I_o $[kg\cdot m^2]$ is the polar moment of inertia of the rigid system.
- m $[kg]$ is the mass of the moving weight.
- $\xi_m(t)$ [m] is the instantaneous (imposed) position of the excitation weight along the linear guide (tank-fixed reference system).
- $\dot{\xi}_m(t)$ [m/s] and $\ddot{\xi}_m(t)$ [m/s^2] are the first and second time derivatives of $\xi_m(t)$ [m].
- $S_G = M_R\cdot\eta_G$ [kg·m] is the static moment of the rigid system with respect to the rotation axis.
- M_R [kg] is the total mass of the rigid system.
- η_G [m] is the(signed) distance of the centre of gravity of the rigid system with respect to the rotation axis (tank-fixed reference system).
- $Q_{damp}(t) = -K_{df}\cdot sign(\dot{\phi}) - B_\phi\cdot\dot{\phi}$ [N·m] is the assumed form of roll damping moment with a:
- A dry friction term $-K_{df}\cdot sign(\dot{\phi})$ with K_{df} [N·m] being the dry friction coefficient.
- A linear damping term $-B_\phi\cdot\dot{\phi}$ with
- B_ϕ [N·m/(rad/s)] being the linear damping coefficient.
- $Q_{fluid}(t)$ [N·m] is the fluid moment.

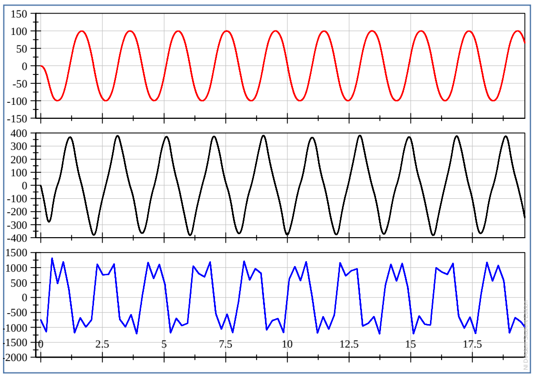

Figure 3. $\xi_m(t)$ top, $\dot{\xi}_m(t)$ middle, $\ddot{\xi}_m(t)$ bottom.

3.2. Analytical model assessment.

3.2.1 General

By using a set of inclining as well as decay tests, the unknown parameters can be determined, including the natural frequency of the rigid system ω_0 . The values of these parameters can be found in table 3. In the case of decay tests $m=0$, whereas in the case of forced rolling tests the motion $\xi_m(t)$ of the shifting mass is imposed during each experiment. During each experiment $\xi_m(t)$ is directly measured whilst its derivatives are obtained from numerical derivation after fitting a least square cubic spline to the moving weight motion signal in order to mitigate the noise influence in the derivatives. See figure 3 for an example of the sliding mass motion curves.

Table 3. Mechanical parameters of the rigid system.

Quantity	Units	Value
S_G	$kg \cdot m$	-29.2
I_o	$kg \cdot m^2$	26.9
K_{df}	$N \cdot m$	0.540
B_ϕ	$N \cdot m / (rad/s)$	0.326
ω_0	rad/s	3.26

3.2.3 Free decays without fluid: comparison between simulations and experiments

Figure 5 shows a comparison between an experimental decay and a simulated decay using the parameters reported in Table 3. The agreement is excellent in the range of roll angles above about 2-3deg. For smaller oscillations, the assumed damping model underestimates the actual damping of the system and therefore the simulated time histories will be slightly under-damped in the tail region. A better modelling of the friction damping or a modification of the damping characteristics of the system would be therefore necessary in the case of interest in the region of small amplitude roll motions.

3.2.2 Forced roll without fluid: comparison between simulations and experiments

A series of tests with the empty tank and the moving mass have been performed in order to check the capability of the model to reproduce the experimentally measured rolling motion of the system. The test matrix was described in section 2. Figure 4 shows an example of comparison between experiments and simulations. The match for the decay and forced roll tests is good enough to ensure that the model of the rigid system is adequate to proceed with the liquid coupling simulations.

3.3. SPH formulation

3.3.1 General

The SPH formulation used for this paper is the weakly compressible one (Monaghan, 1994), in which the closure of the system formed by the compressible mass and momentum conservation equations is achieved by means of an equation of state (EOS) that correlates pressure with density. This EOS is defined in such a way that the density variations are very small, thus modelling a quasi-incompressible fluid.

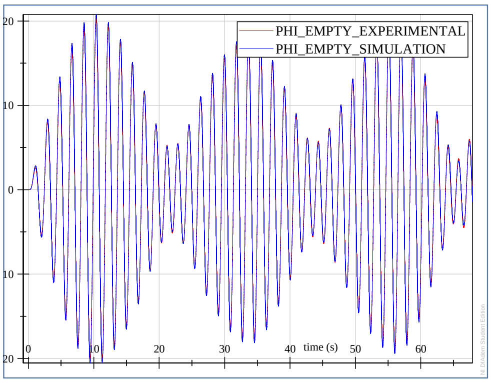

Figure 4. Comparison between roll angle of the empty tank in experiments and simulations. Empty tank, nominal maximum shift 200mm, nominal oscillation period 1.764s ($1.1\omega_0$).

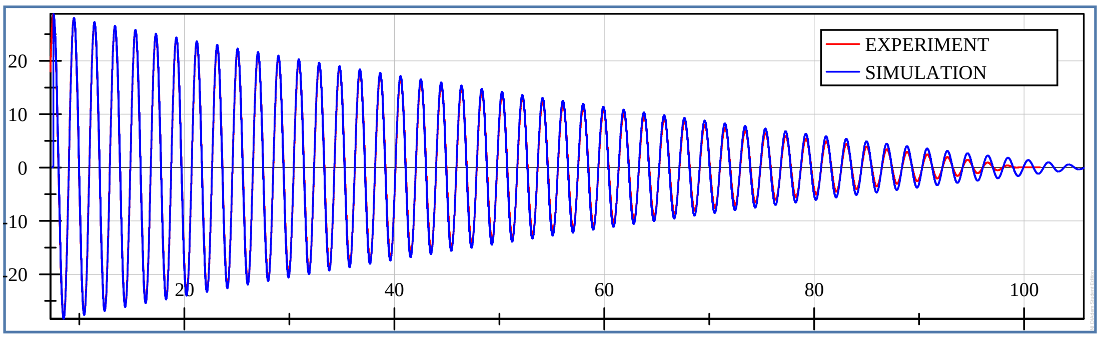

Figure 5. Example of comparison between experimental and simulated free decay angles.

Another important aspect of the computation is the use or not of MLS normalized kernels (Dilts, 1999) for performing the interpolations as well as the periodic reinizialization of the density field with such interpolation in order to reset the compatibility between the volume, the mass and the density field (Colagrossi and Landrini, 2003). Using normalized kernels becomes quite important when fluid fields are evaluated close to the boundaries and density reinitialization is important to remove some oscillations in the pressure field. In the present case there is no need to evaluate any specific field (pressure for instance) because we are interested only in the motion of the tank which acts as an integrator of all the fluid loads. Therefore, we think using those techniques would not contribute much in this particular case.

3.3.2 Boundary conditions.

Other important point is the viscosity and the eventual need for an artificial viscosity term. In the first SPH approaches to free surface incompressible flows (Monaghan, 1994), the main aim was to solve inertia and gravity driven flows. An artificial viscosity term was included. This term prevented particles crossing their trajectories as well as providing some additional diffusion that increased the stability of the time integration. The approach was later seen to model accurately viscous laminar flows (Monaghan 2005) and works as a very dissipative (often too dissipative) term in high Reynolds number flows which are simulated with a much lower numerical Re. A very promising approach, not yet implemented by the authors, relies on Riemann solvers to resolve the interaction between particles; this makes a big difference in the viscous interactions (Le Touzé *et al.*, 2008).

3.3.2 Boundary conditions.

The implementation of the boundary conditions will be achieved by using Ghost Particles (Colagrossi and Landrini, 2003), which work very well for a rectangular domain like the present one. For more details on the SPH formulation with application to the assessment of localised values like the wave impact pressures, see Delorme *et al*, (2009).

4. RESULTS OF THE SIMULATIONS.

4.1. General

The natural period of the system is approximately 1.927s. The liquid height has been chosen so that the linear theory first sloshing period will also be 1.927s. Simulations regarding off-resonance conditions have been performed but the most interesting cases correspond to the resonance ones, and in particular those for which the differences between the three fluids in the dampening effects are worth noting, which correspond to the smallest moving mass amplitudes: 50mm and 100mm. For 50mm the roll angles are large enough to obtain results of reasonable accuracy. However, as discussed in section 3.2.2, for these cases inaccuracies have been observed concerning the modelling of damping, and such inaccuracies could have influenced also the results from SPH simulations. Therefore, the 100mm amplitude cases will be in principle the ones that merit further study. The resonance cases of higher amplitudes show in turn other important features, like for instance that some fluids are able to limit the structure motions whilst others, due to their viscosity (and consequently due to their different dissipation mechanism), do not provide under the same conditions enough damping to put an upper limit the roll angle in resonance conditions.

4.2 Convergence analysis.

A convergence analysis in terms of the particle number has been performed using 800, 3200 and 12800 particles. The convergence to a specific solution is very good during the simulation range. A comparison for the glycerine case simulation for resonance condition with 100mm weight displacement is shown in figure 6.

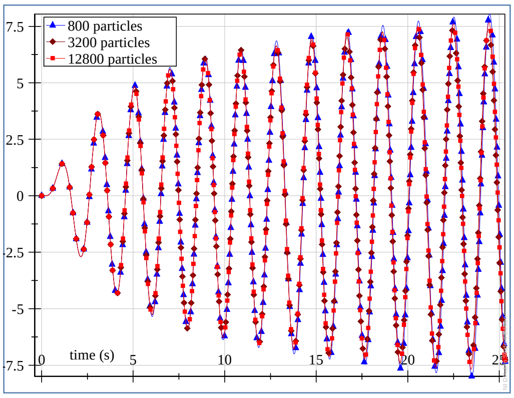

Figure 6. Convergence analysis.

4.3 Resonance cases:

4.3.1 General

The magnitude whose value will be discussed is the reduction ratio in amplitude between the partially filled tank and the empty tank roll angles (percentage) in a specific period of time. A ratio close to 100% will mean that the liquid has no dampening effect. A ratio close to 0%, will mean that the resulting amplitude is very small. The ratio will be established taking the maximum values of the partially filled tank roll angle and the empty tank roll angle. The values of this ratio are reported in table 4, with a row corresponding to the SPH simulations.

Table 4. Amplitude reduction ratio (amplitude partially filled tank / amplitude empty tank).

A	50mm	100mm	150mm	200mm
Glycerin	28.4	43.0	58.7	66.2
Oil	13.1	31.4	51.4	61.9
Water	6.4	23.6	46.7	59.3
SPH	10.4	32.2	45.5	58.5

The differences in this ratio for the three liquids are most substantial for the small amplitude cases, as expected. We will describe with detail the 100mm amplitude case, which is the most significant.

4.3.2 Amplitude = 100mm

In this case, the reduction ratio ranges from 33.2 for water to 60.6 for glycerin (table 4, analysis performed from 0 to 35s, end time in the empty tank experiment). The time evolution of the roll angle for the three liquids can be appreciated in figure 7 and in more detail in figure 8. In figure 7 it can be also appreciated that the tank roll angle increase is stopped by the effect of the three liquids, contrary to what happens with the empty tank.

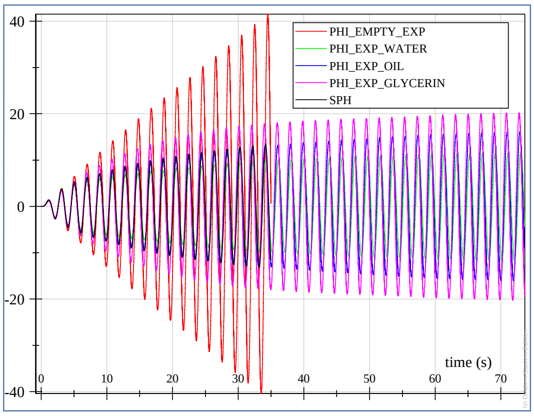

Figure 7. Roll angle. Nominal maximum shift 100mm, nominal oscillation period 1.927s.

There is a substantial difference between water and glycerin and the dynamics are quite different as well, as can be seen in figure 9 (bottom and top). On the other hand, there is not such a big difference between the global dynamics of the oil and water cases, but the differences in the dampening characteristics are very substantial. As can be appreciated in figure 9, in the water case there is the formation of a bore that will develop into a a plunging breaking wave, whereas in the oil case, a mild spilling wave is formed with presumably significantly less dissipation. A specific dye was used for the sunflower oil instead of the fluorescine used for water and glycerin and that cannot be dissolved in oil. This is the reason for the different shade of the oil picture in figure 9.

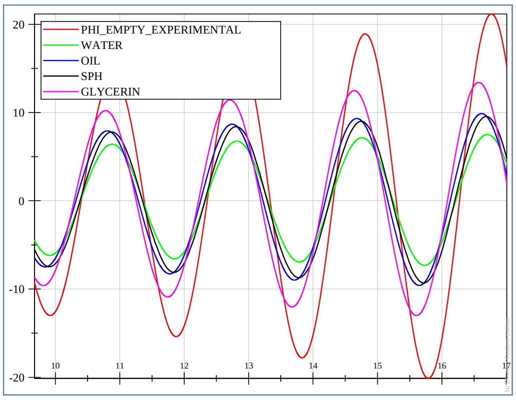

Figure 8. Roll angle (zoom of figure Figure 7).

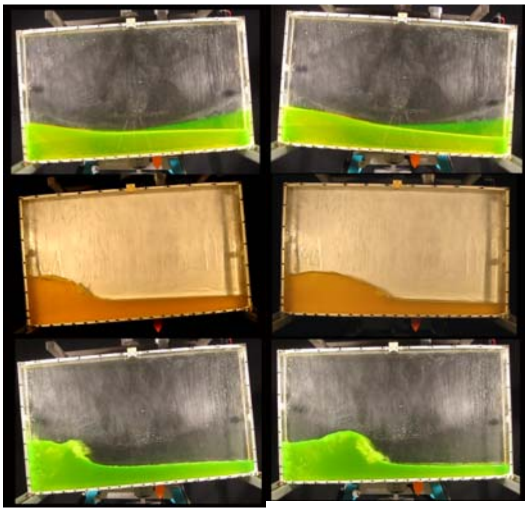

Figure 9. Maximum shift 100mm, oscillation period 1.927s. *t/T0*=8.66 (left) and *t/T0*=8.85 (right). Glycerin (top), oil (middle), water (bottom). The orange color arrow signals the moving mass position.

The SPH results show that the water amplitude reduction ratio error is around 9% in this case. The simulations were run with 800, 3200, and 12800 particles and the convergence in terms of the roll angle curve was good as was commented in section 4.2. It is important to note that the numerical Reynolds number is two orders of magnitude less than the physical one, when comparing with water. It seems that this could justifiy the results being more similar to those for oil but it is not yet clear at this stage.

When analysing individual frames, the dynamics as simulated by SPH is more similar to that of water, as can be seen in figures 10 and 11 which can be easily assimilated to the water case amongst the three of figure 9. Velocity field samples are plotted in figures 10 and 11. It can be seen that the gradients in the breaking area are significant. This is an indication of the violence of the impact of the jet with the free surface, in which a substantial amount of energy is dissipated.

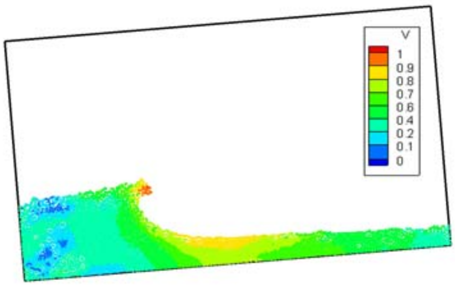

Figure 10. SPH velocity field, *t/T0*=8.66, (see fig. 9).

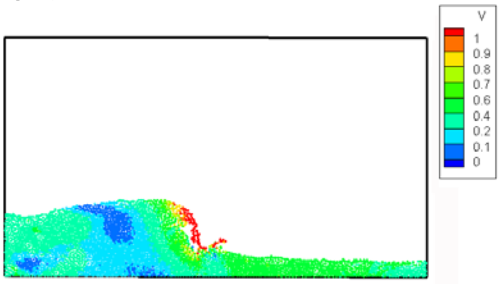

Figure 11. SPH velocity field. *t/T0*=8.85 (see fig. 9).

5. CONCLUSIONS AND FUTURE WORK

The roll motion response of a single degree of freedom (SDOF) structural system to which a rigid rectangular partially filled liquid tank has been attached has been considered. The SDOF system has been described analytically and this description tested experimentally by applying to the system periodic excitations, finding that the response is accurately reproduced by the considered analytical model.

The antiroll tank performance has been assessed both experimentally and numerically with weakly compressible SPH. In order to characterise the wave breaking effects on the response curves, tests have been performed with liquids of different viscosity, the increasing viscosity preventing the onset of breaking waves. It is not completely clear at this stage how to exactly quantify the influence of wave breaking on the damping characteristics of the three liquids in the cases studied in this paper. Nevertheless, from the analysis of the experiments described in the paper, it seems substantial. The capabilities of SPH to treat this coupling problem have been assessed. From the comparisons with the experiments, it seems SPH is able to capture part of the dissipation effects due to wave breaking which is reflected in reasonably accurate damping reduction ratios. Nevertheless, it is not at this stage possible to separate those effects from the shear ones in the boundary layers and in the bulk of the fluid, for which the SPH approach is not yet completely consistent with the Reynolds numbers of the experiments. Further work has to be done in these regards. It would be very interesting to compare these results with those obtained with other methods. To achieve this, application to angular motions of the techniques used for linear TLDs will be assessed as well as the possibility of treating this problem with a commercial CFD.

The work presented in this paper has dealt with a very controlled situation, with a simple dynamical system that can be analytically modelled with a level of accuracy such that the underlying analytical model for the dynamic system can be considered practically "exact" in engineering terms. According to this, the majority of discrepancies between experiments and simulations with the partially filled tank could be attributed to the SPH simulation (apart from the case of small amplitude oscillations where the inaccurate modelling of damping could have been a not negligible source of differences). Taking into account the fact that the agreement between experiments and simulation was, in general, good, we could conclude that the SPH approach, although still needing improvements on some aspects, could serve as a practical tool for the assessment of tanks' performances. The next steps that could be followed along this path are:

- Introduction and consequent experimentation / simulation of cases with the tank equipped with baffles as dissipation means.
- Implementation of a mathematical model for ship motions. In such case SPH could be used as a means to study, e.g., the effect of water on deck, in addition to the study of anti-rolling devices. Moreover, effects like parametric excitation induced by vertical ship motions on the fluid could also be studied.

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