A REVIEW OF AVAILABLE METHODS FOR APPLICATION TO SECOND LEVEL VULNERABILITY CRITERIA

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ABSTRACT

The International Maritime Organization (IMO) has begun work on the development of next generation intact stability criteria. These criteria are likely to consist of several levels: from simple to complex. The first levels are expected to contain vulnerability criteria and are generally intended to identify if a vessel is vulnerable to a particular mode of stability failure. These vulnerability criteria may consist of relatively simple formulations, which are expected to be quite conservative to compensate for their simplicity. This paper reviews methods which may be applicable to the second level of vulnerability assessment, when simple but physics-based approaches are used to assess the modes of stability failure, including pure-loss of stability, parametric roll, surf-riding, and dead-ship condition.

Keywords: *stability criteria, dynamic stability, capsize, pure-loss, parametric roll, surf-riding, dead-ship condition*

1. INTRODUCTION

Current stability criteria have been used for several decades to determine the level of safety for both existing and novel ship designs. However, their current state is not representative of the level of our understanding about the mechanisms of dynamic stability failure of ships. Acknowledging this deficiency, the IMO has begun work on the development of next generation intact stability criteria to address problems related to dynamic phenomena and expand the applicability of criteria to current and future ship designs.

The new generation criteria should include a range of fully dynamic aspects in their formulation. A thorough critical analysis of the existing situation at the beginning of the revision of the Intact Stability Code and a description of the present state-of-the-art can be found in a series of works by Francescutto (2004, 2007).

The current framework of new generation intact stability criteria started take shape at the 50th session of IMO Sub-committee on Stability and Load lines and on Fishing Vessels Safety (SLF) with SLF-50/4/4 (2007), submitted by Japan, the Netherlands, and the United States and SLF-50/4/12 (2007), submitted by Italy. A significant contribution was also made in SLF-50.INF.2 (2007), submitted by Germany. These discussions resulted in a plan of development of new generation of criteria (see Annex 6 of SLF 50/WP.2). Following this plan, the intersessional Correspondence Group on Intact Stability developed the framework in a form of the working document that can be found in Annex 2 of SLF 51/4/1 (2008), submitted by Germany. Annex 3 of this paper contains the draft terminology agreed upon for the new criteria that was also developed by the Correspondence Group. The most current version of the framework document can be found in Annex 1 of the report of the Working

Group on Intact Stability (SLF 51/WP.2). To facilitate these discussions, Belenky, et al. (2008) presented a broad review of issues related with development of new criteria, including the physical background, methodology, and available tools.

It is the intention of the authors of this paper to provide a follow-up review, with a particular focus on methods for application to second level vulnerability criteria.

The views and opinions expressed in this paper are solely and strictly those of the authors, mainly for facilitating wider and deeper discussion inside the research community. They do not necessarily reflect those of the delegations that involve the authors, the intersessional correspondence group on intact stability, or the working group on intact stability of the International Maritime Organization. The contents of this paper also do not necessarily indicate a consensus of opinion among the authors, but the authors agree on the need for further discussion of the contents.

2. OVERVIEW OF NEW GENERATION INTACT STABILITY CRITERIA

The major modes of stability failures were listed in section 1.2 of the 2008 IS Code, part A. They include:

- Phenomena related to righting lever variations, such as parametric roll and pure loss of stability;
- Resonant roll in dead ship condition defined by SOLAS regulation II-1/3
- Broaching and manoeuvring-related problems in waves

A multi-tier structure of new criteria has been identified from the ongoing discussion at IMO. Each of the four identified stability failure modes: pure-loss of stability, parametric roll, surf-riding and broaching, and dead ship conditions is evaluated using criteria with different levels of sophistication.

The intention of the vulnerability criteria is to identify ships that may be vulnerable to dynamic stability failures, and are not sufficiently covered by existing regulation (SLF 51/WP.2, Annex 1).

A first level of criteria (vulnerability criteria) could be followed by other stability assessment methods with increasing levels of sophistication. The first level of the stability assessment is expected to be relatively simple, preferably geometry based if possible. The function of this first level is to distinguish conventional ships, which are obviously not vulnerable to stability failure. The second tier is expected to be more sophisticated and assess vulnerability to dynamic stability with enough confidence that the application of direct calculation could be justified. The expected complexity of the second level vulnerability criteria should consist of a spreadsheet-type calculation.

An outline of the process is shown in Figure 1. The methods will most likely be different for each stability failure mode.

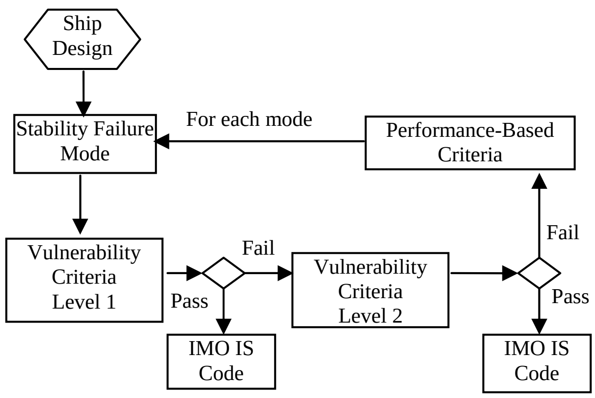

Figure 1. The proposed assessment process for next generation intact stability criteria.

3. PURE-LOSS OF STABILITY

Differences in the change of stability in waves, in comparison with calm water, were known to naval architects since late 1800s (Pollard & Dudebout, 1892; Krylov 1958).

However, it was uncommon until the 1960s that attempts to calculate the change of stability in waves (Paulling 1961) and evaluate it with a series of model tests (Necheav, 1978; available in English from Belenky & Sevastianov 2007) were made. As a distinct mode of stability failure, pure-loss of stability was identified during the model experiments in San-Francisco Bay (Paulling, et al., 1972, 1975; summary available from Kobylinski & Kastner 2003).

Accurate calculation of the change of stability in waves presents certain challenges, especially at high speed, as the pressure around the hull and waterplane shape are influenced by the nonlinear interaction between encountered, reflected, and radiated waves (Nechaev, 1989).

The physical mechanism of pure-loss of stability is rather simple: if the stability is reduced for a sufficiently long time, a ship may capsize or attain large roll angle (see Figure 2, taken from Belenky, et al., 2008).

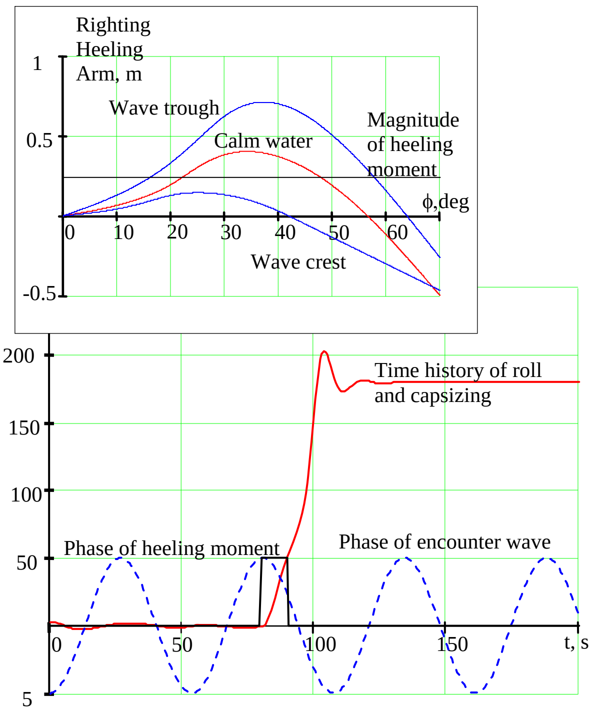

Figure 2. Capsizing due to pure-loss of stability.

This phenomenon is presently addressed, although in a very short and qualitative form, by the MSC.1/Circ.1228 (2007). MSC.1/Circ.1228 does not provide any ship-dependent information concerning the considered failure mode.

The difficulty of developing criteria for pure-loss of stability is not limited to the calculation of the GZ curve in waves. A probabilistic characterization of the associated stability changes is also difficult, due to nonlinear nature of stability changes in waves.

Boroday (1967, 1968) developed a method for the assessment of statistical characteristics for restoring moments in waves, followed by an energy balance-based method for evaluation of the probability of capsizing (Boroday & Netsevatev, 1969). Using energy balance methods for changing stability in waves was also the focus of Kuo & Vassalos (1986). Dunwoody (1989a, 1989b), Palmquist (1994), and Bulian & Francescutto (2006a) considered statistical characteristics of GM and other elements of the righting moment as a stochastic process.

The alternative approach of the "effective wave" was proposed by Grim (1961). In this method, the length of the wave is equal to ship length— the wave is unmovable. However, its amplitude is a random process (Figure 3).

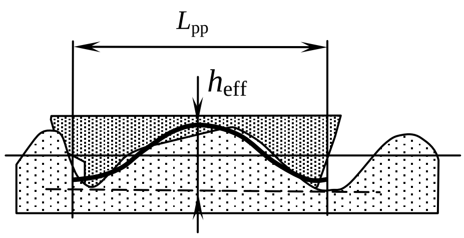

Figure 3. A concept of an effective wave.

The amplitude of the effective wave is calculated in order to minimize the difference between the effective and encountered waves. Umeda, et al. (1993) performed comparative calculations and suggested the effective wave to be of satisfactory accuracy for engineering calculations.

The effective wave approach was used in a number of works; of special interest are those with a “regulatory perspective.” Helas (1982) discussed the reduction of the righting arm in waves. To consider the duration where stability is reduced, the amplitude of the effective wave was averaged over a time sufficient to roll to a large angle, about 60% of natural roll period. As a result, the average effective wave amplitude becomes a function of Froude number. Analyzing results of the calculation performed for different vessels in different sea conditions, recommendations were formulated for a stability check in waves. Small ships and ships with “reduced stability” were meant to be checked using the GZ curve— evaluated for the effective wave with averaged amplitude against conventional calm water stability criteria.

Umeda & Yamakoshi, (1993) used the effective wave to evaluate the probability of capsizing due to pure-loss of stability in short-crested irregular stern-quartering waves with wind. The capsizing itself was associated with departure from the time-dependent safe basin. The effect of initial conditions was taken into account using statistical correlation between roll and the effective wave.

Instead of the Grim effective wave, Vermeer (1990) assumed waves to be a narrow-banded stochastic process. Capsizing is associated with the appearance of negative stability during the time duration, and it is sufficient for a ship to reach a large roll angle under the action of a quasi-static wind load. The probability of capsizing is considered as a ratio of encountered wave cycles, where the wave amplitude is capable of a significant and prolonged deterioration of stability.

Themelis & Spyrou (2007) demonstrated the use of the concept of a “critical wave” to evaluate probability of capsizing. Because pure-loss of stability is a one wave event, it is straightforward to separate the dynamical problem from the probabilistic problem. First, the parameters that are relevant to the waves capable of causing pure-loss of stability are searched. Then, the probability of encountering such waves is assessed. To complete the probabilistic problem, the initial conditions are treated in probabilistic sense.

Bulian, et al. (2008) combined the effective wave approach with upcrossing theory, where the phenomenon of pure-loss of stability is associated with an up-crossing event for the amplitude of the effective wave. Such an approach allows the time of exposure to be considered directly and produces a time-dependent probability of stability failure.

An additional advantage of upcrossing theory is the possibility to characterize the “time above the level”, which in this context is the duration of decreased stability. The mean value of this time can be evaluated by a simple formula, but evaluation of other characteristics is more complex (Kramer & Leadbetter, 1967).

In summary, it seems that the second level vulnerability criteria for pure-loss of stability is likely to be probabilistic, due to the very stochastic nature of the phenomenon. For criteria related to this mode of stability failure, the time duration while stability is reduced on a wave could be included. Methods useful for addressing pure-loss of stability may include, but are not limited to, the effective wave, the narrow-banded waves assumption, the critical wave approach, and upcrossing theory. As Monte-Carlo simulations may be too cumbersome for vulnerability criteria, at this time, simple analytical and semi-analytical models seem to be preferable.

4. PARAMETRIC ROLL

Parametric roll is the gradual amplification of roll amplitude caused by parametric resonance, due to periodic changes of stability in waves. These stability changes are essentially results of the same pressure and underwater geometry changes that were the cause in the pure-loss of stability mode.

However, in contrast to pure-loss of stability, parametric roll is generated by a series of waves of certain frequency. Therefore, parametric roll cannot be considered as a one wave event (Figure 4).

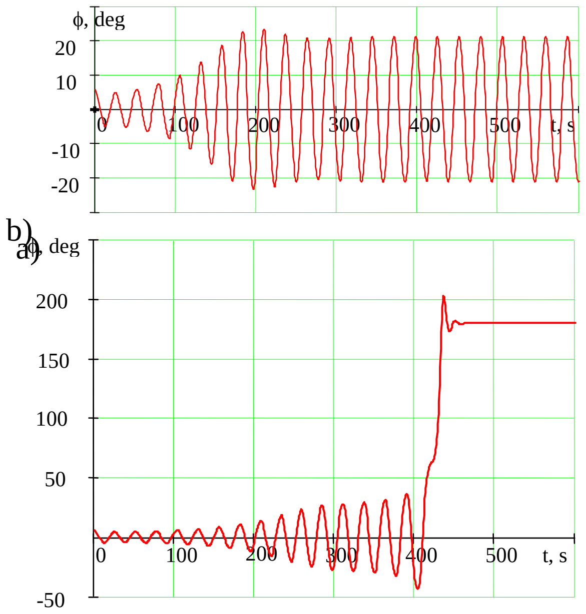

Figure 4. Partial (a) and total (b) stability failure caused by parametric resonance.

Research on parametric roll began in Germany in the 1930s (Paulling, 2007). In the 1950s, the study of parametric roll for a ship was continued by Paulling & Rosenberg (1959). Paulling, et al. (1972, 1975) observed parametric roll in following waves during model tests in San-Francisco Bay (summary available from Kobylinski & Kastner, 2003). Later it was discovered that this phenomenon could occur in head or near head seas as well (Burcher, 1990; France, et al., 2003).

Parametrically excited roll motion is at present widely recognized as a dangerous phenomenon, where the cargo and the passengers and the crew may be at risk. Hull forms have evolved from those considered in the development of the 2008 IS Code, as MSC Res. 267(85)— basically the same ship types used 30-50 years ago for the definition of Res. A. 749(18) (2002). For certain ship types, present hull forms appear to have resulted in the occurrence of large amplitude rolling motions associated with parametric excitation. According to this evidence, there is a compelling need to incorporate appropriate stability requirements specific to parametric roll into the new generation intact stability criteria.

The simplest model of parametric resonance is the Mathieu equation. It can describe the onset of instability in regular waves. Because the Mathieu equation has a linear stiffness term, it is incapable of predicting the amplitude of roll, and not applicable in irregular waves. Nevertheless, it was used by ABS (ABS, 2004; Shin, et al., 2004) to establish estimates for initial susceptibility to parametric excitation, which corresponds more with intended use for the first level vulnerability criteria.

More complex 1(.5)-DOF models include nonlinearity in the time-dependent variable stiffness term. This nonlinearity leads not only to stabilization of roll motion, e.g. establishing finite roll amplitude, but also to a significant alteration of instability boundary in comparison of Ince-Strutt diagram— with unstable motions found outside of the Mathieu instability zone (Spyrou, 2005). This should cause concern about relying on the standard linear stability boundaries.

Inclusion of additional degrees of freedom, primarily pitch and heave (Neves & Rodríguez, 2007) results in additional realism. These are especially important for parametric roll in head seas, where pitch and heave may be large. Spyrou (2000) included surging as well. These models, even with their low dimensional character, seem to give reasonable agreement with model tests (Bulian, 2006).

Evaluation of the steady-state amplitude of parametric roll can be carried out using asymptotic methods, such as the method of multiple scales (Sanchez & Nayfeh, 1990). Approximate analytical expressions exist for the determination of the nonlinear roll response curve (e.g. ITTC, 2006). The potential of these methods is discussed in Spyrou et al. (2008).

The introduction of irregular waves results in drastic changes in the way parametric roll develops. Large oscillations may not be persistent anymore: it may stay, but it may also be transitory (Figure 5).

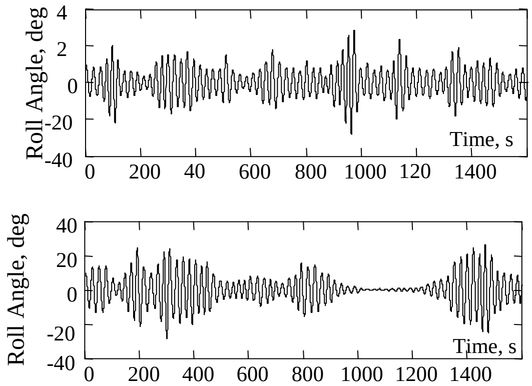

Figure 5. Sample records of simulated parametric roll in long-crested seas (Shin, et al., 2004).

Such behavior has significant influence on the ergodicity of the process— the ability to obtain statistical characteristics from one realization if it is long-enough. Theoretically, the process remains ergodic, as the integral over its autocorrelation function still remains finite, but the time necessary for convergence may be very large. As a result, the term "practical non-ergodicity" is used (Bulian, et al., 2006; Hashimoto, et al., 2006).

Distribution of parametric roll may be different from Gaussian. Hashimoto, et al. (2006) obtained distributions with large excess of kurtosis from model tests in long- and short-crested irregular seas (Figure 6).

Roll damping has more influence on roll amplitude in irregular waves. The damping threshold defines if a wave group can contribute to parametric excitation and therefore, to results in an increase of the roll amplitude. Characteristics of wave groups, such as number, length, and the height of waves in the group, may also have an influence on how fast the roll amplitude increases. In a sense, a regular wave can be considered as an infinite wave group. Therefore, using regular waves could be too conservative for an effective vulnerability criterion, due to the transient nature of this phenomenon (SLF48/4/12).

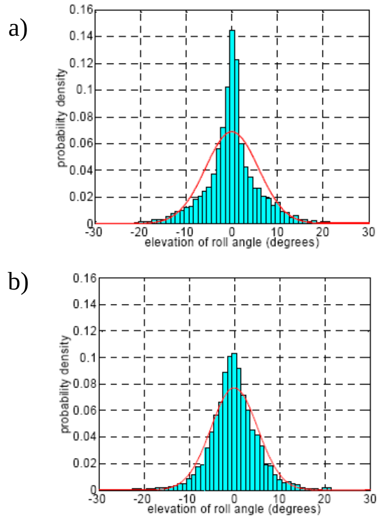

Figure 6. Distribution of roll angles during parametric resonance in (a) long-crested and (b) short crested seas.

Focusing on the instability boundaries in irregular waves, Francescutto, et al., (2002) used the concept of "effective damping," also mentioned in Dunwoody (1989a); that is meant to be subtracted from roll damping. Bulian, et al. (2004) used the Fokker-Planck equation and the method of multiple scales for the same purpose. They utilized Dunwoody's spectral model of GM changes in waves (1989a, 1989b). Furthermore, Bulian (2006) made use of Grim's effective wave concept. Despite overestimating the amplitude of roll, the boundary of instability was well identified.

Application of the Markov processes for parametric roll in irregular waves was considered by Roberts (1982). To facilitate Markovian properties, (dependence only on the previous step), white noise must be the only random input of the dynamical system. Therefore, the forming filter, usually a linear oscillator, must be part of the dynamical system to ensure irregular waves have a realistic-like spectrum. These complexities allow the distribution of time before the process reaches a given boundary to be obtained— the first passage method. This method is widely used in the field of general reliability. Application of this approach to parametric roll was considered by Vidic-Perunovic & Jensen, (2009).

Time-domain Monte-Carlo simulations (Brunswig, et al., 2006) are increasingly used as computers become more powerful and are integral part of any naval architect's office. At the same time, caution has to be exercised when using Monte-Carlo simulations, due to statistical uncertainty and other methodological challenges that may limit its use for second level vulnerability criteria. Nevertheless, having in mind the difficulties associated with prediction of amplitude for parametric resonance in irregular waves, Monte-Carlo method should not be dismissed. Themelis & Spyrou (2007) used explicit modelling of wave groups as a way to separate problems related to ship dynamics from the probabilistic problem. This method exploited the growth rate of parametric roll due to the encounter of a group, i.e. by solving the problem in the transient stage. Thereafter, the probability of encounter of a wave group can be calculated separately. In principle, this may allow enough simplification to make Monte-Carlo methods into feasible and robust tool for the second level vulnerability criteria for parametric roll.

In summary, it seems that the second level vulnerability criteria for parametric roll may be related to the size of the instability area in irregular seas. The group wave approach seems to be promising, as a method to build a criterion based on amplitude.

5. SURF-RIDING AND BROACHING

Broaching is the loss of controllability of a ship in astern seas, which occurs despite maximum steering effort. Stability failure caused by broaching may be "partial" or "total". It can appear as heeling during an uncontrollable, tight turn. This is believed to be intimately connected with the so called surf-riding behaviour. Moreover, it can manifest itself as the gradual, oscillatory build-up of yaw, and sometimes roll, amplitude. The dynamics of broaching is probably the most dynamically complex phenomenon of ship instability. Well known theoretical studies on broaching from the earlier period include Davidson (1948), Rydill (1959), Du Cane & Goodrich (1962), Wahab & Swaan (1964), Eda (1972); Renilson & Driscoll (1981), Motora, et al.(1981); and more recently, e.g. by Umeda & Renilson (1992), Ananiev & Loseva (1994), and Spyrou (1996, 1997, 1999). Broaching has also been studied with experimental methods. Tests with scaled radio-controlled physical models have taken place in large square ship model basins; e.g. Nicholson (1974), Fuwa (1982), De Kat & Thomas (1998).

Surf-riding is a peculiar type of behaviour where the ship is suddenly captured and then carried forward by a single wave. It often works as predecessor of broaching and its fundamental dynamical aspects have been studied by Grim (1951), Ananiev (1966), Makov (1969), Kan (1990), Umeda (1990), Thomas & Renilson (1990), and Spyrou (1996). Key aspects of the phenomenon are outlined next. In a following or quartering sea, the ship motion pattern may depart from the ordinary periodic response. Figure 7 illustrates diagrammatically the changes in the geometry of steady surge motion, under the gradual increase of the nominal Froude number (Spyrou 1996). An environment of steep and long waves has been assumed.

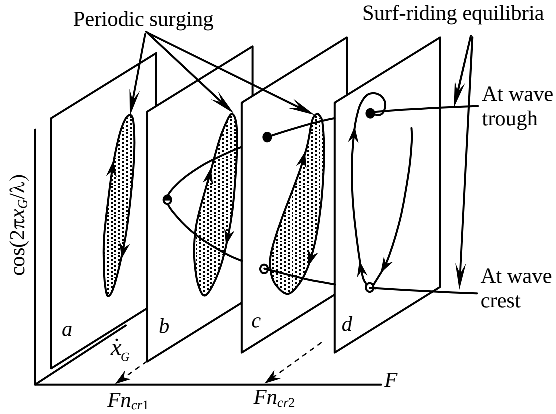

Figure 7. Changing of surging/surf-riding behavior with increasing nominal Froude number.

For low *Fn*, there is only a harmonic periodic response (plane (a)) at the encounter frequency. However, as the speed approaches the wave celerity, the response becomes asymmetric (plane (c)). The ship stays longer in the crest region and passes quickly from the trough. In parallel, an alternative stationary behavior begins at some instance to coexist. This results from the fact that the resistance force which opposes the forward motion of the ship may be balanced by the sum of the thrust produced by the propeller and the wave force along the ship's longitudinal axis. This is surf-riding and the main feature is that the ship is forced to advance with a constant speed equal to the wave celerity. It has been shown that surf-riding states appear in pairs: one nearer to wave crest and the other nearer to trough. The one nearer to the crest is unstable in the surge direction. Stable surf-riding can be realised only in the vicinity of the trough and, in fact, only if sufficient rudder control has been applied.

Surf-riding is characterised by two speed thresholds: the first is where the balance of forces becomes possible (plane(b)). The second threshold indicates its complete dominance in phase-space, with sudden disappearance of the periodic motion. It has been pointed out that, this disappearance is result of a global bifurcation phenomenon, known in the nonlinear dynamics literature as *homoclinic saddle connection* (Spyrou, 1996). It occurs when a periodic orbit collides in state-space with an unstable equilibrium— the surf-riding state near to the wave crest. A dangerous transition towards some, possibly distant, undesirable state is the practical consequence.

To be able to show the steady motion of a ship overtaken by waves as a closed orbit, one should opt to plot orbits on a cylindrical phase-plane. On the ordinary flat phase-plane the bifurcation would appear as a connection of different saddles ("heteroclinic connection", see Figures 8 and 9 based on Makov). This is characterised by the tautochronous touching of the "overtaking wave" orbits, running from infinity to minus infinity, with the series of unstable equilibria at crests. This has been confirmed numerically by Umeda (1999). Therefore, no matter what terminology is used, there is consensus and clear understanding, rooted in dynamics, about the nature of phenomena.

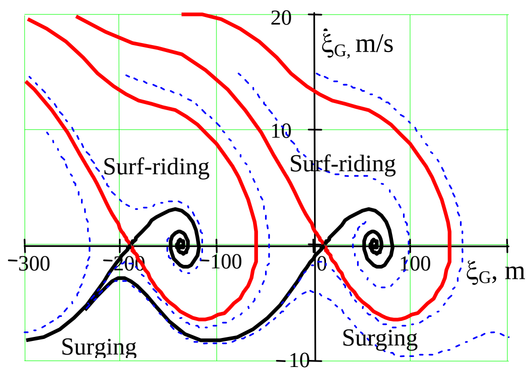

Figure 8. Phase plane with surging and surf-riding, $Fn_{cr1}<Fn<Fn_{cr2}$.

The initial attraction into surf-riding, caused by the unstable surf-riding equilibrium near the crest, should be followed by capture of the ship at the stable surf-riding equilibrium near the trough. However, this is a violent transition and, if the rudder control is insufficient, it will end in broaching. The effectiveness of rudder control in relation to the inception of this behaviour, and also characteristic simulations, can be found in the literature (Spyrou, 1996, 1999).

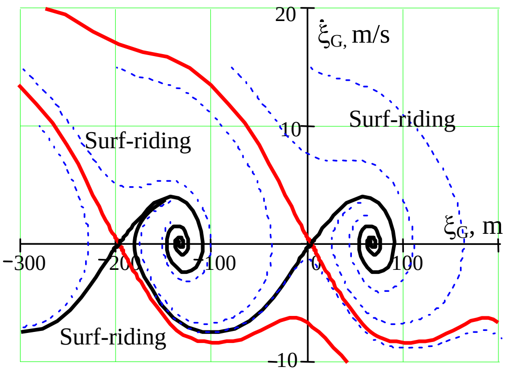

Figure 9. Phase plane with surf-riding only Fn>Fn$_{cr2}$.

From the above, one sees that, susceptibility to this type of broaching could be assessed by targeting three sequential events: the condition of attraction to surf-riding; the condition defining an inability to stay in stable surf-riding, thus creating an escape; and the condition of reaching dangerous heeling during this escape— manifested as tight turn. For the first condition, we already have deterministic (analytical and numerical) criteria based on Melnikov and other methods (Ananiev, 1966; Kan, 1990; Spyrou, 2006). For the second, criteria have been discussed, linking ship manoeuvring indices with control gains (Spyrou, 1996, 1998). The third condition can be relatively easily expressed as the balance of the roll restoring versus the moment of the centrifugal force that produces the turn. A proper probabilistic expression of these conditions is still wanting. However, the critical wave approach may provide a very practical way of dealing with this (Themelis & Spyrou, 2007). Efforts in similar spirit were reported by Umeda (1990) and Umeda, et al. (2007).

In reviews of broaching incidents, those that had happened at low to moderate speeds (i.e. far below the wave celerity of long waves) are usually distinguished from the more classical high-speed events. These incidents could have some special importance, because they seem to be relevant also to vessels of larger size, which would be otherwise unlikely to be prone to direct broaching. Consideration of the nature of such a broaching mechanism— direct broaching, i.e it doesn't require the ship to go through surf-riding, has shown that, under these conditions, a bifurcation can arise for a critical value of the commanded heading, a result basically of the horizontal plane dynamics, creating stable sub-harmonic yaw (Figure 10).

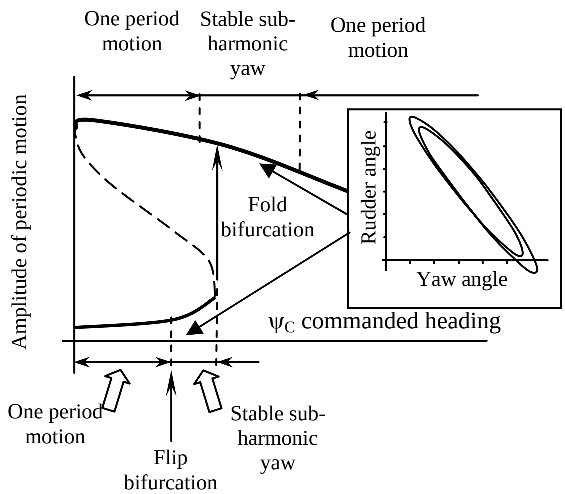

Figure 10. Direct broaching.

Further increase of the commanded heading was found to cause a rapid increase of the amplitude of yaw oscillation, leading shortly to a turn backwards of the steady yaw response curve and, inevitably, a sudden and dangerous jump to resonant yaw (Spyrou, 1997). The transient behaviour looks like a growing oscillatory yaw which corresponds closely to what has been described as cumulative-type broaching.

Instability could be an intrinsic feature of the yaw motion of a ship overtaken by very large waves. It has been pointed out that, starting from Nomoto's equation, the equation of yaw in following waves becomes Mathieu-type (in Spyrou (1997) the derivation and discussion on this and its connection with the above scenario was presented). The amplitude of the parametric forcing term is the ratio of the wave yaw excitation to the hull's own static gain— maneuvering index *K*. The damping ratio in this equation receives large values, unless there is no differential control. Due to the large damping, the internal forcing that is required in order to produce instability is much higher (steeper waves) than that at the near zero encounter frequency (i.e. for the mechanism of broaching through surf-riding). Simple criteria connecting the hull with rudder control have accrued from this formulation and could be directly in the vulnerability criteria. A probabilistic formulation has not yet been attempted.

6. DEAD-SHIP CONDITIONS

Both the term and the concept of dead ship conditions took its root during the steamer era. When a ship looses power, it becomes completely uncontrollable— a significant disadvantage in comparison with ship equipped with sails. The worst possible scenario is a turn into beam seas, where the ship is then subjected to a resonant roll combined with gusty wind. Traditional ship architecture, with a superstructure amidships, made this scenario quite possible. Significant changes in marine technology have led to more reliable engines and a variety of architectural types. Nevertheless the dead ship condition— zero speed, beam seas, resonant roll, and wind action, maintains its importance.

The present Intact Stability Code (MSC.267(85), 2008), as well as its predecessor (IMO Res. A749(18), 2002), contains the "Severe Wind and Rolling Criterion (Weather Criterion)." This criterion contains a simple mathematical model for ship motion under the action of beam wind and waves. However, some parameters of this model are based on empirical data. Therefore, it may not be completely adequate for unconventional vessels. Thus MSC.1/Circ.1200 (2006) and MSC.1/Circ. 1227 (2007) were implemented, to enable alternative methodologies to be used for the assessment of the Weather Criterion on experimental basis.

The problem of capsizing represents the ultimate nonlinearity: the dynamical system transits to motion about another stable equilibrium. When the wave excitation amplitude approaches a critical value, a number of nonlinear phenomena take place. The boundary of the safe basin becomes fractal (Kan & Taguchi, 1991) as a result of the self-crossing branches of an invariant manifold (Falzarano, et al., 1992). Considering the deterioration of the safe basin (Figure 11), Rainey and Thompson (1991) proposed a transient capsize diagram. It uses the dramatic change of the measure of integrity of the safe basin as a boundary value.

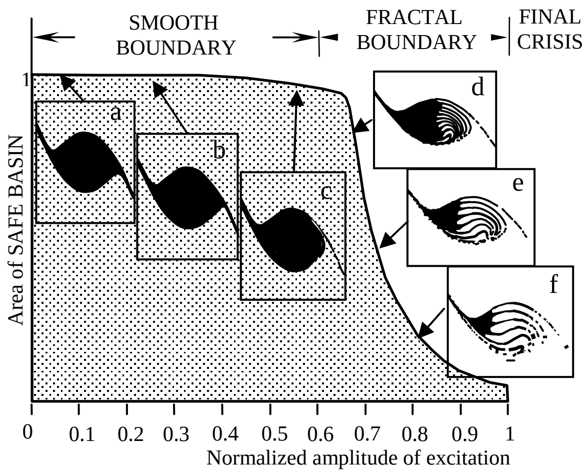

Figure 11. Deterioration of safe basin with increasing of wave amplitude.

Another obvious way to improve criterion is to consider more realistic, e.g. stochastic, models for the environment. This does not necessarily means that the new criterion must be probabilistic. The improvement will be made by adjusting parameters of deterministic criterion to reach similar level of safety for all the vessels which meet it.

Studies of probabilistic methods for these criteria were carried out starting in the 1950s (Kato, et al., 1957). As the conventional weather criterion uses an energy balance approach, an attempt to consider it from probabilistic point of view seems logical (Dudziak & Buczkowski 1978; a brief description is available from Belenky & Sevastianov, 2007). Similar ideas are behind the Blume criterion (Blume, 1979, 1987), which uses a measure of the remaining area under the GZ curve. Bulian & Francescutto (2006b) and Bulian, et al. (2008) developed a method where the righting arm is locally linearized at the equilibrium, due to the action of a steady beam wind. The probability of large roll is calculated using an "equivalent area" to account for the nonlinearities of the righting arm.

The piecewise linear method (Belenky, 1993; Paroka & Umeda, 2006) separates probability of capsizing into two less challenging problems: upcrossings through maximum of the GZ curve and capsizing if such upcrossing has occurred (Figure 12). The probability of upcrossing is well defined in upcrossing theory. While the probability of capsizing after upcrossing can be found using the probability of roll rate at upcrossing that lead to capsizing; this is a linear problem in the simplest case. Statistical linearization can be used to evaluate parameters of the piecewise linear model. Simplicity and a direct relation with time of exposure are among the strengths of the piecewise linear method.

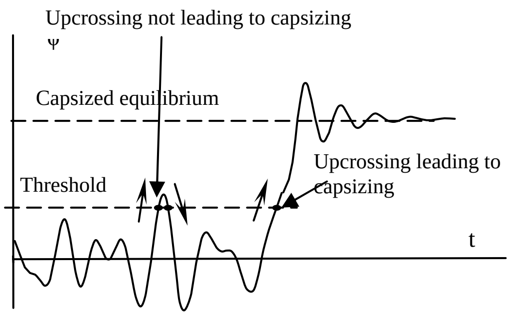

Figure 12. Separation principle in a piecewise linear method.

In principle, a critical wave approach can be used for dead ship conditions as well. However, the outcome of capsizing vs. non-capsizing strongly depends on initial conditions. Therefore, a critical wave group approach similar to Themelis & Spyrou (2007) may be also considered as a candidate for vulnerability criteria.

7. THE CHOICE OF ENVIRONMENTAL CONDITIONS

If vulnerability criterion is deterministic and based on a regular wave assumption, the wave is characterized with height and length. These characteristics must then be related to a corresponding sea state and further on with a safety level.

This formulation is not new— this approach is used for evaluation of extreme loads. A typical scheme for calculation of extreme loads is based on long-term assumption, so a number of sea states are considered. An operational profile is usually assumed based on existing experience; it includes the fraction of time that a ship is expected to spend in each sea state. Short-term probability of exceedance is calculated for each sea state; then the formula for the total probability is used to determine the life-time probability of exceedance of the given level. This level is typically associated with significant wave height and zero-crossing, or mean period. These data are then directly used to define a regular wave that is expected to cause extreme loads.

Similar ideas underlie the reference regular wave in the Weather Criterion in the 2008 IS Code, which was determined from the significant wave steepness, based on the work of Sverdrup & Munk (1947).

With all of the similarities, a direct transfer of methods used for extreme loads to stability applications may be difficult. Structural failure is associated with exceeding certain stress levels, actual physical failure is not considered in Naval Architecture. Stresses do not have their own inertia, they follow the load without time lag. The same can be observed about the loads, they occur simultaneously with a wave. They do not depend on initial conditions; even in a nonlinear formulation and do not have multiple responses for the given position of a ship on a wave. Nevertheless, these motion nonlinearities affect only short-term probability, while the scheme as a whole may be applicable.

Determining the equivalence between regular and irregular waves represents a problem by itself. The use of significant wave height or steepness has been a conventional assumption, but it does not have a theoretical background behind it. The general problem of equivalency between regular and irregular roll motions was considered by Gerasimov (1979; a brief description in English is available from Belenky & Sevastianov, 2007) in the context of statistical linearization with energy conservation, resulting in an energy statistical linearization.

The idea of a regular wave as some sort of equivalent for sea states is attractive because of its simplicity. However, the physics of some stability failures may be sufficiently different in regular and irregular waves. As it was mentioned in the section regarding parametric roll, a regular wave is an infinitely long wave train. Roll damping in this case has a very small influence on the response amplitude, while in irregular seas damping has a significant influence on the variance of response. Therefore, the regular equivalent of irregular waves cannot be applied universally. The application of different environmental models is possible for different stability modes. Surf-riding/broaching is an example of a one-wave event where the concept of a critical wave may fit well.

If some of vulnerability criteria are made probabilistic, the next important choice is time scale: long-term or short term. Here, short-term refers to a time interval where an assumption of quasi-stationarity can be applied. Usually this is an interval of three to six hours, where changes of weather can be neglected. Long-term consideration covers a larger interval: a season, a year, or the life-time of a vessel.

The short-term description of the environment is simpler and can be characterized with just with one sea state or spectrum. However, if chosen for vulnerability criteria, justification will be required as to why a particular sea must be used. This choice is important because sea states which are too severe may make the criterion too conservative and diminish its value. Therefore, special research is needed in order to choose the sea state “equivalent” or “representative” for a ship operational profile. This may naturally result in ship-specific sea-state to use for assessment.

An alternative to the selection of a limited set of environmental conditions may be the use of long-term statistics considering all the combinations of weather parameters available from scatter diagrams, or appropriate analytical parametric models (see, e.g. Johannessen, et al., 2002).

8. SUMMARY AND CONCLUDING COMMENTS

Vulnerability to pure-loss of stability is caused by prolonged exposure to decreased stability on a wave crest, and is likely to be characterized by probabilistic criteria. Methods for a first attempt for vulnerability criteria include the effective wave, a narrow-band or envelope presentation, the critical wave approach, and upcrossing theory.

Vulnerability criteria for parametric roll would be more useful if defined in irregular seas. This criterion may be based on size of instability area, while the wave group approach seems to be a good candidate for amplitude-based criterion.

Vulnerability to broaching through surf-riding may be judged by the vulnerability to surf-riding. The critical wave approach seems to be the best fit; however, other methods are not excluded. Vulnerability criterion for direct broaching is likely to be deterministic.

Vulnerability to stability failure in dead ship conditions may be judged using a variety of methods, as it was the main area of application for both probabilistic and deterministic methods. The modified weather criterion, piecewise linear method, and critical wave or wave group approach may be among the first to be considered.

A rational choice of environmental conditions for vulnerability criteria is at least as important as the criteria. An unrealistic environmental condition may lead to incorrect results, even if the criteria are technically correct. However, several possibilities do exist: a regular wave equivalent for life-time risk, a short-term sea state deemed "representative" for a specific ship operational profile, and a long-term approach using a scatter diagram for a representative part of the World Ocean.

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