DEVELOPING THE S FACTOR

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ABSTRACT

The current various proposals at IMO for the *s* factor (probability of surviving a given flooding) make no reference to survival time. The paper shows a direct link of the 'prime' *s* factor with the time to capsize and shows how to utilise experimental data from 30-minute test runs for the *s* factor based on longer duration of tests. Unexpectedly, the extension of tests has a modest effect on the survival factor, and hence – modest effect on subdivision index *A*. Much more important is improving a deficient formulation for the required index *R*, as flooding cases with $s_i = 1$ have an infinite survival time.`

Keywords: *factor s, survival time, and rulemaking*

1. INTRODUCTION

The factor *s* is understood as the conditional probability of surviving a given flooding due to collision damage with sufficient survival time, assumed to be 30 minutes during the original research, related to the static equivalency method (SEM). This method was developed in 1995 in the wake of the sinking of "Estonia" for ro–ro vessels with the large open main deck (vehicle deck). The method evolved from research carried out at Strathclyde University (Vassalos 1996, and 1997) based on a framework presented earlier by Pawłowski (1995).

The current various proposals at IMO for the *s* factor make no reference to survival time, which is a serious drawback. Nowadays sufficient survival time is considered to be 3 hours. Hence a question arises, if the *s* factor, based on half-an-hour duration of tests, is still valid for regulatory purposes. The answer to this question is provided below. The analysis of the problem is not that easy, as survival time is a random quantity.

2. SURVIVAL TIME

In spite of the complexity, probability density function (pdf) of survival time can be relatively easily derived recalling the basic probability theory. For that we shall consider experiments with a damaged ship lasting an indefinite time whose duration time has been divided into 30-minute segments. It is reasonable to assume that probability of surviving in each segment is the same and equals *P*. This probability could vary only in case of progressive flooding. Such a sequence of tests, with a constant probability in each trial, is termed as a *Bernoulli trial process*.

Probability *P* is a prime *s* factor and for a given damage scenario and loading condition is a function of sea states in terms of the significant wave height H_s. The quantity $1-P$ then represents *probability of capsizing*, identical with CDF of critical sea states. The random nature of the critical sea states comes from the random nature of water elevation on the vehicle deck, discussed by Pawłowski (2003). Probability *P* varies across the capsize band from a value 1 at the lower boundary to zero at the upper boundary of the capsize band (uncertain zone), see Figure 1. Above the upper boundary there is an unsafe region with a 100% of capsizal, where the time to capsize, termed also as the survival time, is in minutes and reduces to seconds, when the sea state becomes higher. Below the lower boundary a safe region stretches with a 100% of survival (no capsizal), where survival time is infinite. It is noteworthy that all tests before 1995 failed to identify the existence of the capsize band.

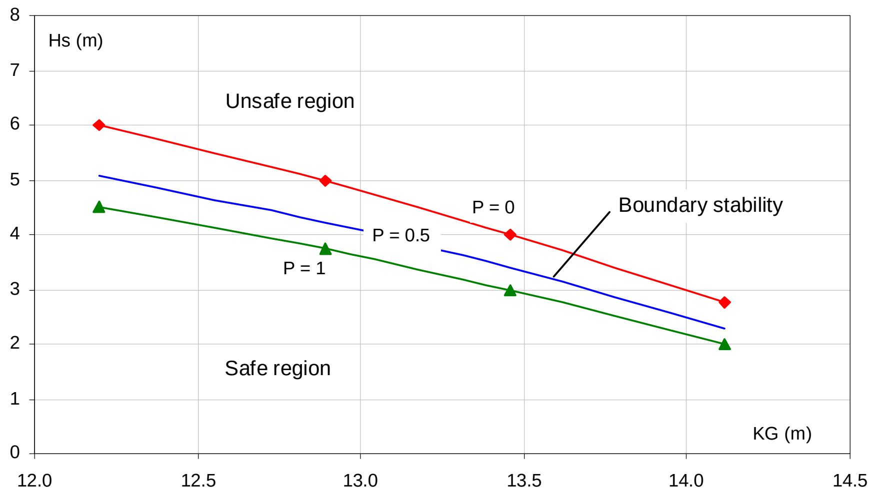

Figure 1. Definition of capsize band and survival boundaries for given flooding versus height of the centre of gravity (for a passenger vessel investigated in the Harder project).

If probability of surviving one segment is P, probability of surviving n segments is given by:

$$F = P^n, \tag{1}$$

where $n = t/t_0$, t is time (in minutes) measured from the completion of flooding, and $t_0 = 30$ minutes. During the first half an hour segment $p_1 = 1-P$ ships capsize, during the second segment $p_2 = P^2-P = P(1-P)$ ships capsize, during the third one $p_3 = P^3-P^2 = P^2(1-P)$, and so on. Probability of capsizing after n segments equals obviously $1-P^n$. This is identical with the CDF, shown in Figure 2 for $P = 0.4$ and 0.8, since Σp_k for the first n consecutive segments equals $1-P^n$.

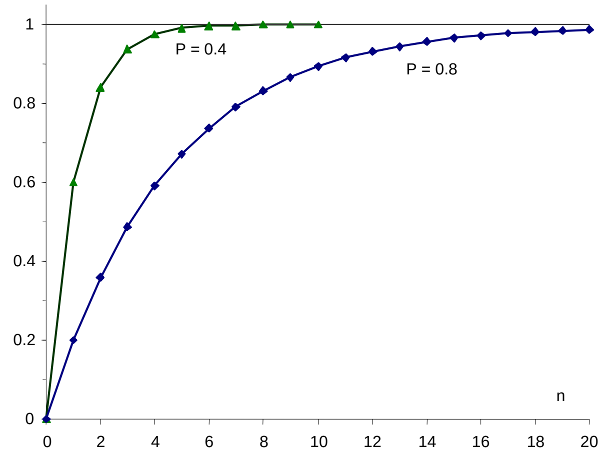

Figure 2. CDF for survival (capsizal) time.

The curves in Figure 2 are broken (segmented) instead of stepwise, as probability of capsizal in each segment is uniformly distributed. Probability density functions corresponding to these CDFs are therefore histograms, shown in Figure 3, that agree very well with those obtained from experiments. The fraction of capsized ships in consecutive segments equals $p_k = P^{k-1}(1-P)$. Mass probabilities related to these segments form a geometric sequence with the ratio P.

Knowing the frequency of ships that capsized in each segment the average time of capsizal (survival) can be calculated. Assuming that capsizing is equally distributed over each segment, the mean survival (capsizal) time is given by the following expression:

$$t_s = 15\,(p_1+3p_2+5p_3+\ldots) = 15\,\Sigma(2n-1)\,p_n,$$

with $p_n = P^{n-1}(1-P)$, where the summation is taken from $n = 1$ to infinity. After performing simple mathematics, shown by Pawłowski (2004 and 2007), the following expression results for the mean survival (capsizal) time:

$$t_s = 15\cdot\frac{1+P}{1-P}\ (\text{min}), \tag{2}$$

The higher the probability of survival P, the higher the mean survival time is, clearly seen also in Figure 2 and Figure 3, providing more time for evacuation. The mean survival time in Figure 2 equals area above the CDF up to the asymptote, whereas in Figure 3 equals the centre of gravity of area under the pdf. If P approaches 1, survival time tends to infinity, which agrees with common sense. Table 1 provides values of the mean survival time ts as function of P, based on equation (2).

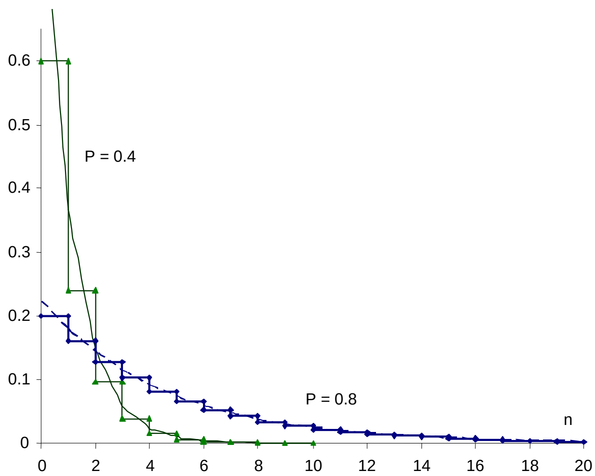

Figure 3. Distribution of probability for capsized ships and its analytic approximation.

A stepwise distribution of survival (capsizal) time can be easily approximated by a continuous distribution of the non-dimensional variable $x = t/t_0$, as clearly seen in Figure 2. The CDF in terms of x, denoted by F, is obtained simply replacing n by x. Therefore,

$$F = 1-P^x = 1-e^{x\ln P} = 1-e^{-\lambda x}.$$

where $\lambda = -\ln P$. Differentiating F relative to x yields the pdf. We then get for the pdf the exponential law of distribution:

$$f = \lambda e^{-\lambda x}. \tag{3}$$

As can be seen in Figure 3, the continuous pdf for capsizal time passes through midpoints of individual segments, smoothing the histogram.

The mean non-dimensional survival (capsizal) time $x_s = t_s/t_0$ related to the exponential distribution of capsizal is given by the integral:

$$x_s = \int_0^\infty x\,f(x)\,dx = \lambda\int_0^\infty x\,e^{-\lambda x}dx = 1/\lambda.$$

Hence,

$$t_s = t_0/\lambda = -t_0/\ln P. \tag{4}$$

Making use of a very handy approximation for the natural logarithm: $\ln P \approx 2(P-1)/(P+1)$, equation (4) becomes identical to equation (2).

3. THE CONDITIONAL PROBABILITY OF SURVIVAL

As the sea state H_s at moment of collision is random (unknown beforehand), the resultant probability s that a ship with given loading

Table 1. Mean survival time t_s (in minutes) versus the prime s factor P.

P	1/3	0.4	0.5	0.6	2/3	0.7	3/4	7/9	0.8	0.85
t_s	30	35	45	60	75	85	105	120	135	185

condition (KG-value) and compartment flooded will not capsize after damage within 30 minutes can be obtained by averaging the prime survival factor $P = P(H_s)$ with respect to sea states at the moment of collision:

$$s = E(P) = \int_{H_s} P(H_s) f_c(H_s)\, dH_s, \tag{5}$$

which follows from the Bayes theorem for total probability, where $P(H_s)$ is the probability of survival during 30 minutes at given loading condition and compartment flooded, and $f_c(H_s)$ is the probability density function of sea states at the moment of collision. Integration in equation (5) takes place across the capsize band (vertically in Figure 1) from the lower to the upper boundary. Outside the capsize band the integrand vanishes. The probability of surviving P starts with a value 1 at the lower bound of H_s at the uncertain zone and terminates with a value of zero at the upper bound (see Figure 1). That is to say, the probability $P(H_s)$ monotonically *decreases* across the capsize band. Hence, $P(H_s) = 1-F(H_s)$ is the tail of CDF for the critical sea states.

Applying integration by parts in equation (5), the following is obtained:

$$s = PF_c \Big|_0^\infty + \int_{H_s} F_c(H_s) f(H_s)\, dH_s, \tag{6}$$

since $P' = -f(H_s)$. The first term vanishes and we get eventually

$$s = \int_{H_s} F_c(H_s) f(H_s)\, dH_s. \tag{7}$$

Equation (7) has a similar structure as equation (5). However, in the latter the resultant factor s is expressed by averaging the CDF of sea states at the moment of collision with respect to critical sea sates, whereas in the former – vice versa – the elementary factor s is averaged with respect to sea states at the moment of collision. By virtue of the mean value theorem integral (7) equals F_c, taken at a certain point, denoted by H_{sm}, somewhere inside the range of integration. That is to say, the resultant factor s equals the probability that the sea state at the moment of collision will not exceed *some* mean value of the critical sea states for given loading condition and damage scenario. The quantity H_{sm} need not necessarily be identical with the expected (mean or average) value of the critical sea states but none the less it is very close to it.

The mean critical sea state can be easily defined if the function $P(H_s)$ is known – it simply equals the area under this curve. However, the mean critical H_s has to be defined during model tests, which is virtually impossible to do. In such a case it is far more convenient to replace it by a median value. That is, the critical sea state (or the critical KG-value) is defined as such in which in 50% of runs the ship capsizes and in 50% survives. In routine calculations the critical median sea state is obtained from the SEM, as discussed by Pawłowski (2004), and Vassalos (1996 and 1997).

The averaging process in equation (7) can be done in a more direct way. The function $F_c(H_s)$ can be expanded into a Taylor's series around $H_{s\,m}$ – the mean value of the critical sea state, unknown beforehand. Taking three terms of this expansion, we get

$$s = \int_{H_s} \left[F_c + f_c(H_s - H_{s\,m}) + \tfrac{1}{2} f_c'(H_s - H_{s\,m})^2\right] f(H_s)\, dH_s,$$

where F_c, f_c, and f_c' are calculated at $H_s = H_{s\,m}$. The above yields then

$$s = F_c(H_{s\,m}) + \tfrac{1}{2} f_c' V(H_s), \tag{8}$$

as the second term vanishes by definition, where $V(H_s)$ is the variance of the critical sea states. Since $f_c' = F_c''$ is negative, the resultant s factor is therefore somewhat smaller than $F_c(H_{s\,m})$. Further, since the median value for asymmetrical distributions is somewhat smaller than the mean value, therefore for the sake of simplicity it can be taken eventually that

$$s = F_c(H_s = H_{s\,50\%}). \tag{9}$$

For example, for a triangular distribution, the mean value is ⅓ of the extension, whereas the median equals $(1-\sqrt{0.5}) = 0.29$ of the extension. Some degree of approximation is justified, as the CDF of sea states at the moment of collision, shown in Figure 4, is known with a limited accuracy. Bearing this in mind, the detailed run of the prime *s* factor – the function $P(H_s)$ is not very necessary, since what is needed for the calculation of the resultant s factor is the knowledge of the median value of critical sea states $H_{s\,50\%}$.

Equation (9) says that having determined (by physical model tests or numerical simulations) the critical sea state $H_{s\,50\%}$ for given damage case and loading condition (*KG*-value), the resultant factor $s = F_c(H_{s\,50\%})$, essential for the probabilistic subdivision regulations, can be obtained from the CDF of sea states occurring at the moment of collision $F_c = F_c(H_s)$. This factor simply equals the probability that the mean critical significant wave height, taken as the median $H_{s\,50\%}$, is not exceeded at the moment of collision. For this purpose, it is sufficient to use the sea state distribution proposed by IMO, as shown in Figure 4, yielding $s > 0.3$. Approximations of the said distribution can be found in Pawłowski (2004, and 2007). In routine calculations, the critical sea state $H_{s\,50\%}$ (with a 50% protection) provides the SEM.

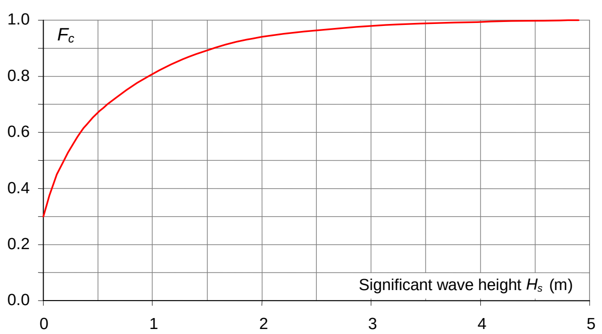

Figure 4. IMO distribution of sea states occurring at the moment of collision.

It is noteworthy that the distribution of sea states at the moment of collision is different from the distribution obtained from regular weather statistics. In a large majority of cases, collisions happen in the proximity of ports, in confined waters and in fog, typically associated with calm weather. It is understandable that in such circumstances sea states are on the whole lower than at the open sea or under normal operating conditions, and – because of that – probably not much different for various sea regions. If the sea state distributions do differ for certain regions, this would provide space for regional deviation in formulae for the *s* factor.

4. ACCOUNTING FOR A LONGER SURVIVAL TIME

In the previous section it has been shown how to derive the *s* factor based on 30-minute tests. That is to say, such a factor provides probability of surviving given flooding at given loading condition with a *minimum* survival time equal to 30 minutes.

If probability of surviving over 30 minutes, denoted by *P*, is known, then probability of surviving one hour equals P^2, as argued in section 2. Hence, the factor *s* corresponding to survival time equal to *at least* one hour is obtained by averaging P^2 over the sea states at moment of collision. Therefore:

$$s = E(P^2) = \int_{H_s} P^2 f_c(H_s)\, dH_s. \tag{10}$$

Applying as before integration by parts, the following is obtained:

$$s = -\int_{H_s} F_c(H_s)(P^2)'\, dH_s = \int_0^1 F_c(P)\, d(P^2), \tag{11}$$

where ′ means differentiation with respect to H_s, and $P = 1 - F(H_s)$ is the tail of CDF for the critical sea states. At the second identity the function $F_c(H_s)$ becomes $F_c(P)$, as $H_s = H_s(P)$. The function *s* is positive, as $(P^2)' = 2PP'$ is negative, yet better seen at the second identity.

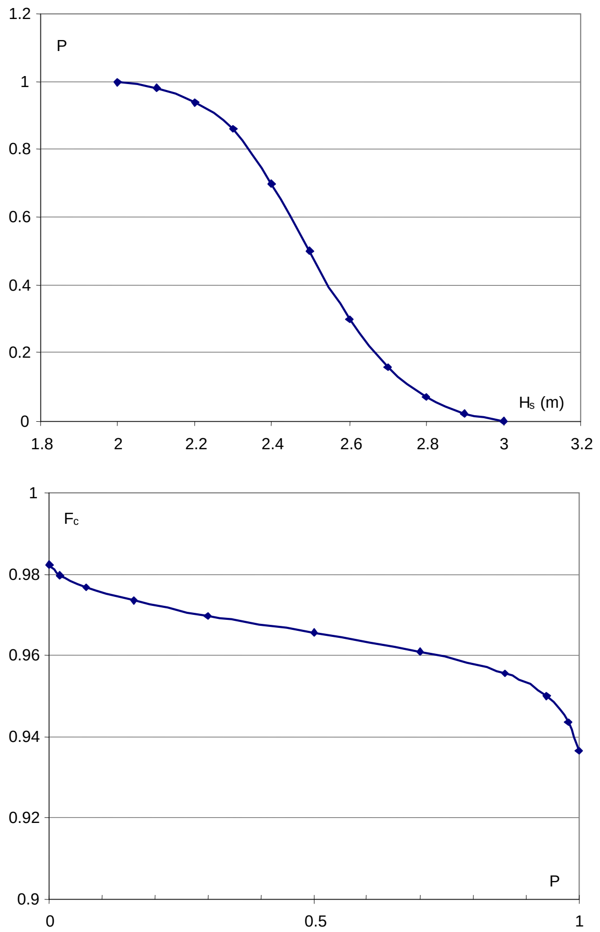

Figure 5. Probability of surviving P (top) and the function F_c versus P (bottom).

A typical run of the function $F_c(P)$ is shown in Figure 5; its detailed run depends on the run of probability of surviving P over the capsize band and the function $F_c(H_s)$ – distribution of sea states at moment of collision. As can be seen, the function $F_c(P)$ monotonically decreases from a value F_c taken for the lower boundary of the capsize band (here 2 m) at $P = 0$, to a value F_c taken for the upper boundary of the capsize band (here 3 m) at $P = 1$.

To abbreviate transformations of equation (11), it is worth reinterpreting equation (7) and noticing it can be written in the form:

$$s = \int_0^1 F_c(P)\,dP.$$

The above represents mean height of the area under the graph shown in Figure 5, which roughly equals a value at $P = ½$. Therefore, $s = F_c(H_s = H_{s\,50\%})$, which agrees with equation (9). In other words, the mean value theorem says that the factor s equals F_c (i.e. CDF for sea states at the moment of collision) calculated for such a sea state H_s for which $P = ½$. Applying this theorem again to equation (11) we immediately get that the factor s equals F_c calculated for the sea state H_s for which $P^2 = ½$. This gives $P = \sqrt{0.5} \approx 0.71$. Therefore,

$$s = F_c(H_s = H_{s\,71\%}), \tag{12}$$

where $H_{s\,71\%}$ denotes the sea state with a 71% protection. In other words, $H_{s\,71\%}$ is the median sea state for one-hour tests.

The above can be easily generalised. The factor s corresponding to survival time equal to at least n half an hour segments is obtained by averaging P^n over the sea states at moment of collision. Therefore:

$$s = E(P^n) = \int_{H_s} P^n f_c(H_s)\,dH_s = \int_0^1 F_c(P)\,d(P^n), \tag{13}$$

which yields

$$s = F_c(H_s = H_{s\,p}), \tag{14}$$

where $H_{s\,p}$ is the sea state with a protection $p = 0.5^{1/n}$ or – in other words – the median value for n half-hour tests. Table 2 provides these values for various duration of tests in terms of quantiles for 30-minute tests.

Table 2 Median values for various duration of tests

one-hour runs	$P = 0.5^{1/2} = 0.71$
1.5 hour runs	$P = 0.5^{1/3} = 0.79$
two-hour runs	$P = 0.5^{1/4} = 0.84$
2.5 hour runs	$P = 0.5^{1/5} = 0.87$
three hour runs	$P = 0.5^{1/6} = 0.89$

The aforementioned considerations can be neatly summarised. The s factor based on n times longer tests than the routine half-hour tests, denoted by $s_{(n)}$, equals

$$s_{(n)} = F_c(H_s = H_{s\,0.5^{1/n}}). \quad (15)$$

For example, $s_{(3)} = F_c(H_{s\,79\%})$, $s_{(4)} = F_c(H_{s\,84\%})$, $s_{(6)} = F_c(H_{s\,89\%})$, and so on. To calculate them we need to know corresponding quantiles of the critical sea states. SEM, however, provides only one of them – the median value $H_{s\,50\%}$. How to get the other is briefly discussed by Pawłowski (2004 and 2007).

The above observations can be strengthen by the observation that equation (13) is the mean height of the area under a graph of the function $F_c(P)$ made against P^n. As can be seen from Figure 6, the mean height approximates well by a value at $P^n = ½$.

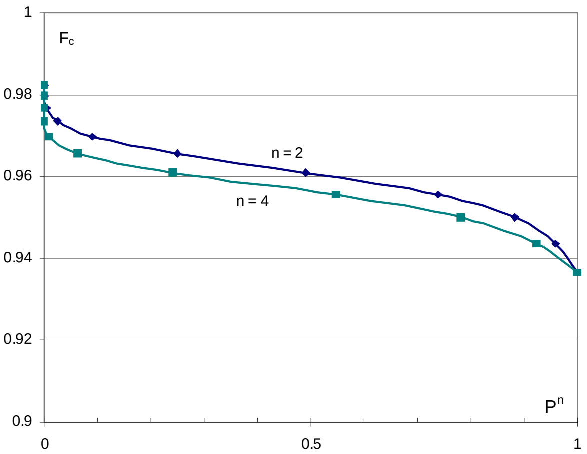

Figure 6.

We can see unexpectedly that the extension of survival time has no more than a modest effect on the survival factor, and hence – a modest effect on subdivision index A. Extending time merely causes a shift from $H_{s\,50\%}$ to higher quantiles of the critical sea states when making readings from the CDF of sea states at the moment of collision for the factor s. The said shift is obviously towards smaller sea states (see Figure 1), therefore the factor s drops. Changes, however, are not dramatic, except cases with poor stability (see Figure 4), with low critical sea states. In the extreme, the shift can be of the order 0.5 m. Irrespective of the definition a minimum value of $s = 0.3$, if IMO distribution of the sea states is used.

It is worth mentioning that $s_{(n)} \equiv E(P^n)$ is not equal to $s_{(1)}{}^n$, as one could think in the first moment, since $E(P^n) > [E(P)]^n$. The difference occurs for partial surviving factors, when the range of sea states at the moment of collision comprise for given KG-value the entire capsize band, that is, when H_s for the upper boundary of the capsize band is smaller than 4 m. Taking $s_{(n)} = s_{(1)}{}^n$, where $s_{(1)}$ is the current 'half-hour' s factor, leads in such cases to a large *underestimation* of the surviving factor. The two quantities equal each other, when the entire range of integration in equation (13) lies above 4 m (i.e., above the highest sea state that can occur at the moment of collision) or for the ship with marginal stability. In the two extreme situation the factor $s = 0.3$ or 1. It assumes zero only when the ship is unable to reach the final stage of flooding, either due to sinking or capsizing before the completion of flooding.

5. CONCLUSIONS

We can conclude that at the end of the day what matters for the safety of the ship is not so much the survival time used for definition of the s factor but the probability of surviving a potential collision by the ship, termed as the index of subdivision A. The level of the index required by the current regulations should be significantly increased, as demonstrated by Pawłowski (2005). The recently adopted formulation for R demonstrates lack of real understanding of what the probabilistic framework was meant to provide and, therefore, out of touch with reality.

6. ACKNOWLEDMENTS

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