INVESTIGATION OF PARAMETRIC ROLL OF A CONTAINER SHIP IN IRREGULAR SEAS BY NUMERICAL SIMULATION

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ABSTRACT

In the present study, characteristics of parametric roll in irregular seas are investigated numerically. Focus is made on the comparison of statistical properties of parametric roll resonance and wave induced roll resonant roll in stern-quartering waves. NLOAD3D is used for numerical simulation. The program was validated through comparisons with extended susceptibility criteria of ABS guidance and model test data of a 6,000 TEU class container ship(WILS-JIP, 2008). Susceptibility in irregular seas and statistical and spectral properties are also investigated in piratical point of view for various combinations of irregular seas.

Keywords: *parametric roll, non-ergodicity, susceptibility of parametric roll*

1. INTRODUCTION

Parametric roll motion is becoming one of important design concerns as large container ships with wider breadth have appeared and they are expected to be continuously increasing. Parametric roll is one of typical examples of highly nonlinear phenomenon in seakeeping and dynamic stability of ship, change of GZ curve of a ship in waves is regarded as a major contributor to parametric roll, which could be modelled as Mathieu-type instability equation. After the pioneering works of Pauling and Rosenberg(1959) and Pauling(1961) on parametric roll mechanism, there have been a number of studies about analytical and numerical models on parametric roll based on Mathieu-type instability equation. ABS has developed a guide for the ASSESSMENT OF PARAMETRIC ROLL RESONANCE IN THE DESIGN OF CONTAINER CARRIERS(2004), in which the state of the art knowledge on various aspects of parametric roll is well described for identifying and evaluating parametric roll. On the characteristics of parametric roll in irregular waves, the situation is rather complicated than in regular wave cases, more studies are needed to build up data for investigate occurrence and statistical characteristics in practical point of view even if there have been some noticeable studies on probabilistic properties such as non-ergodicity and non-stationarity(Belenky et al., 2003, Bullian, 2006) and identification of parametric roll using effective wave concept(Umeda, 1990, Belenky and Sevastinov, 2003) in irregular waves.

In the present study, characteristics of parametric roll in irregular seas are investigated numerically. Focus is made on the comparison of statistical properties of parametric roll resonance and wave induced roll resonant roll in stern-quartering waves. NLOAD3D is used for numerical simulation. Before implementing the parametric roll motion simulation in irregular seas, the program was validated through comparisons with extended susceptibility criteria of ABS and model test data of a 6,000 TEU class container ship(Hong et al., 2008). As a first step for validation of NLOAD3D, results of regular wave simulation are compared with susceptibility criteria and model tests data as discussed in section 2.1. In irregular waves, firstly the results were compared with those from LAMP for cross-check of equivalency. Then, various aspects of parametric roll in irregular waves were investigated such as susceptibility, identification in oblique waves, and ergodicity and convergence of parametric roll. Susceptibility in irregular seas and statistical and spectral properties are investigated for various combinations of irregular seas.

2. VALIDATION OF NUMERICAL SIMULATIONS

2.1 Model tests

During the model test on wave loads in waves carried out in WILS-JIP project(2006-2008, MOERI), unwanted large roll motions in longitudinal waves were observed for specific wave conditions. Although the tests were not designed to capture parametric roll and the run length was not sufficiently long to reach fully developed state, the results are useful to get an idea for qualitative judgment of parametric roll.

The model was made of 4 segments to capture wave loads. In order to suppress interference between towing system and ship motion, a carefully devised towing system using spread spring and counter weight was employed as shown in Fig. 1(Hong et al., 2008). No critical interference due to spring systems were observed, ship speed was kept constant by using towing weight and spring-wire mechanism.

A series of tests carried out for various headings, ship speeds and waves; 7 headings from 0 to 180 degrees with 30 degrees interval, 4 speeds from 0 to 15 knots with 5knots interval, wave frequencies range from 0.3 to 1.0 rad/s with heights from 3 to 10m. Among the above test conditions, the following cases summarized in the Table 1 were identified as candidate cases of possible parametric roll responses from encounter frequency based predictions. Observation of video records and time series plot of roll response were carried out for determination of occurrences of parametric roll among the pre-selected cases. Fig. 2 shows a typical example of parametric roll observed from the model tests, which corresponds to $\omega = 0.45$ rad/s, h=10m, V=5 knots, following sea.

Table 1. Test cases of occurrences of parametric rol.l

Wave freq.	heading	Wave height	Wave slope	Ship speed	Encounter freq.(nom)	Encounter freq.(mes)
0.35	180	3	167.6	0knots	0.35	0.3495
0.35	180	3	167.6	5knots	0.3821	0.3818
0.35	180	5	100.5	5knots	0.3821	0.3818
0.35	180	7	71.8	5knots	0.3821	0.3825
0.35	180	10	50.3	5knots	0.3821	0.3804
0.4	180	3	128.3	0knots	0.40	0.3985
0.4	180	3	128.3	5knots	0.4420	0.4430
0.4	180	5	77.0	5knots	0.4420	0.4409
0.4	180	7	55.0	5knots	0.4420	0.4426
0.4	180	10	38.5	5knots	0.4420	0.4404
0.48	180	18.24	14.7	5knots	0.5404	0.5378
0.5	180	19	13.0	5knots	0.5656	0.5617
0.48	45	15.8	16.9	5knots	0.4373	0.4386
0.4	30	3	128.3	5knots	0.3637	0.3605
0.45	30	3	101.4	5knots	0.4040	0.4038
0.35	0	7	71.8	5knots	0.3179	0.3172
0.35	0	10	50.3	5knots	0.3179	0.3176
0.4	0	3	128.3	5knots	0.3580	0.3578
0.4	0	5	77.0	5knots	0.3580	0.3591
0.4	0	7	55.0	5knots	0.3580	0.3560
0.4	0	10	38.5	5knots	0.3580	0.3553
0.45	0	3	101.4	5knots	0.3969	0.3956
0.45	0	5	60.8	5knots	0.3969	0.3980
0.45	0	7	43.4	5knots	0.3969	0.3937
0.45	0	10	30.4	5knots	0.3969	0.4003
0.46	0	20.57	14.1	5knots	0.4045	0.4093
0.5	0	7	35.2	5knots	0.4344	0.4351
0.5	0	10	24.6	5knots	0.4344	0.4362
0.55	0	10	20.4	5knots	0.4707	0.4728

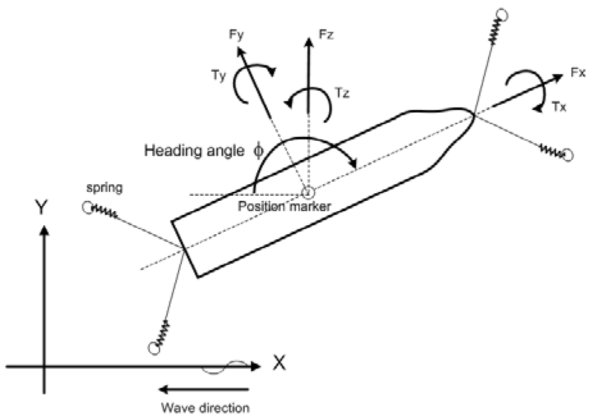


Figure 1. Coordinate system of towing and mooring device.

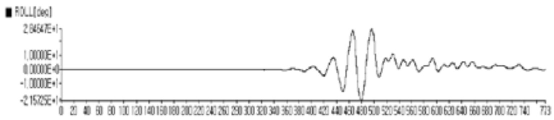


Figure 2. An example of measured parametric roll($\omega = 0.45$ rad/s, h=10m, V=5 knots).

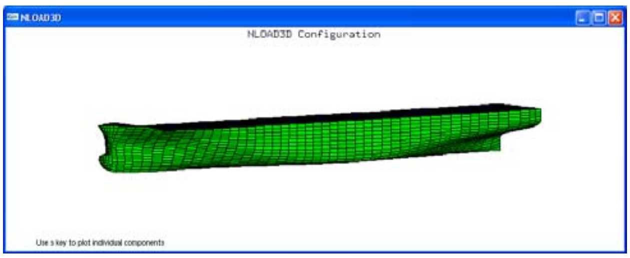


Figure 3. NLOAD3D panel configuration.

2.2 NLOAD3D

NLOAD3D is a simulation system gives numerical solution of the nonlinear wave-body hydrodynamic problem and the integration of the ship motion in time domain. This solution is based on an approximate nonlinear time-domain approach(Weems et al., 2000; Lin et al., 1999; Shin et al., 1977) that combines body-nonlinear hydrostatic and Froude-Krylov forces with a 3-D body linear solution of the wave-body disturbance potential, nonlinear models for viscous and appendage force, and a time-domain nonlinear solution of the equation of motions. The above technical features of the NLOAD3D makes it possible to perform analysis of parametric roll motion in regular and irregular waves as well like other state of the art seakeeping codes such as LAMP and PRECAL.

2.3 Susceptibility and regular wave simulation

Model Ship

The object ship for the present study is a 6,000TEU class container carrier. Main particulars of the model ship are given in Table 1. The model ship is the same model used in the WILS-JIP(2007), in which a series of systematic model test data for wave loads analysis are available.

Table 2. Main particulars of the model ship.

Item	Prototype	Calc. model
LOA	300.25 m	300.25 m
LPP	286.3 m	286.3 m
B	40.3 m	40.3 m
D	24.1 m	24.1 m
Displacement	95276.1 ton	92501 ton
LCG from AP	136.018 m	136.212m
Draft at AP	13.127 m	13.127 m
Draft at FP	12.973 m	12.973 m
KG	17.816 m	17.955m
GM	1.154 m	1.1539
kxx	15.8813 m	15.8813 m
kyy	68.8799 m	68.8799 m
kzz	68.5515 m	68.5515 m

Fig. 3 shows panel representation for numerical analysis.

Roll damping value was calibrated to match experimental value obtained from free roll decay test. Fig. 2 shows an example of calibration results. The upper figure denotes roll decrement ratios and the lower figure denotes fitted time series data.

Susceptibility

In the present work, the full procedures of the ABS guide were implemented and simple method was adopted for calculating hydrostatics. Modification of the procedure was made for considering the model test conditions of WILS-JIP in which parametric roll phenomena were observed under relatively small and moderate wave heights conditions.

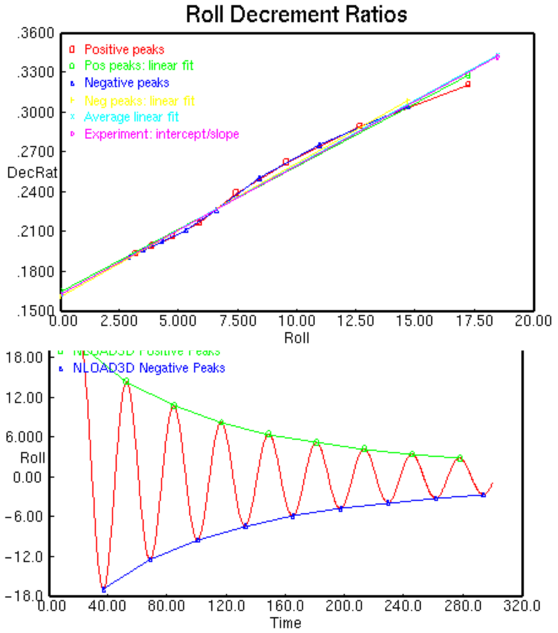


Figure 4. Comparison of roll decrement ratios and free roll simulation results for ship speed, 5knots.

As well described in the ABS guide(2004), equation of roll motion in longitudinal waves of high amplitude can be expressed in the following form, typical Mathieu-type equation.

$$\ddot{\phi} + 2\delta\dot{\phi} + \frac{W \cdot GM(t)}{I_{xx} + A_{44}} = 0$$

$$\ddot{\phi} + 2\delta\dot{\phi} + \left(\omega_m{}^2 + \omega_a{}^2 \cos(t)\right) \cdot \phi = 0 \qquad (1)$$

$$\frac{d^2\phi}{d\tau^2} + 2\mu\frac{d\phi}{d\tau} + (\overline{\omega}_m{}^2 + \overline{\omega}_a{}^2 \cos(\tau)) \cdot \phi = 0$$

Introducing dimensionless parameter as described in eq.(17) of the guide, the final form is obtained as follows(Shin et al., 2004).

$$\frac{d^2x}{d\tau^2} + (p + q\cos(\tau)) \cdot x = 0 \qquad (2)$$

Where, p represents shift of natural frequency while q denotes amplitude of forcing term for given wave frequency and amplitude.

$$p = (\overline{\omega}_m^2 - \mu^2), q = \overline{\omega}_a^2 \qquad (3)$$

The final form of the frequency criteria has the following form.

$$\frac{1}{4} - \frac{1}{2}q - \frac{q^2}{8} + \frac{1}{3}\cdot\frac{q^4}{128} \le p \le \frac{1}{4} + \frac{q}{2} \qquad (4)$$

The final form of the damping threshold criteria has the following form.

$$\mu\frac{\omega_0}{\omega_E} < 0.5 \cdot q \cdot k_1 \cdot k_2 \sqrt{1 - k_3^2} \qquad (5)$$

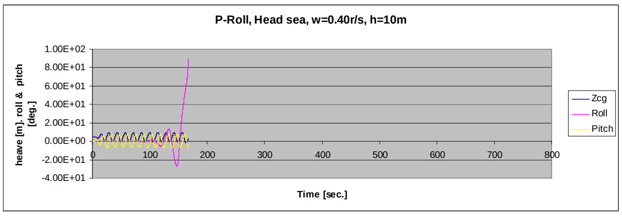
Details of $k_{1,2,3}$ are given in Shin et al.(2004).

<u>Parametric roll simulation in regular waves</u>

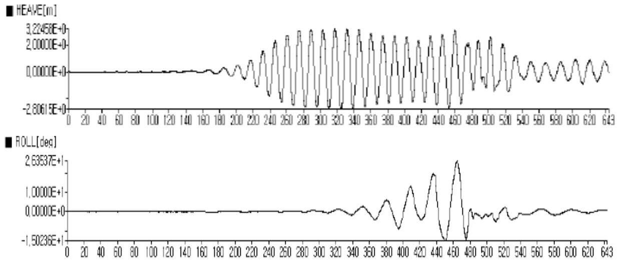
There are some factors affecting discrepancy between model tests and numerical simulation.

- Simulation model is an approximate model, so wave scattering and steady wave pattern are not considered.
- Roll damping is another source of inaccuracy in resonant response.
- Limitation of run length in the model tests may affect the result.

Fig. 3 shows an example of head sea regular wave case, wave frequency 0.4 rad/s and wave height of 10m. As clearly seen in the figure, development of parametric roll resonance is growing very rapidly both in simulation and model test. Parametric roll motion was suppressed in the initial developing stage in the model test to prevent capsizing. Since the WILS-JIP model tests were not designed to capture parametric roll, qualitative identification of parametric roll is rather considered in this study. So, development of roll motion with natural frequency is regarded as parametric roll both in simulation and model test in this study.



(Simulation)



(Model test)

Figure 5. Comparisons of the simulation with the experiments: Head Sea Wave frequency=0.40 rad/s, h=10m, 5knots.

Fig. 6 summarizes comparison of susceptibility between rule criteria, numerical simulation and model tests. In the figure blue region denotes susceptibility domain by criteria. Right-hand side direction means increasing wave height while downward direction means increasing encounter frequency. Each cell in the figure corresponds to a pair of wave height in m and encounter frequency in rad/s. Orange and pink coloured cells represent parametric roll occurred in the model tests in following and head sea conditions, respectively. While red coloured cell denotes parametric roll occurred both in head and following seas condition. Yellow, grey and green coloured cells denote model test cases but no parametric roll was observed. Symbols "o" and "x" denotes whether the parametric occurred or not in the simulation, bold face symbol represents following sea case, normal symbol represents head sea case.

As shown in the figure the susceptibility criteria well predict parametric rolling and it shows good correlation with model tests and numerical simulations. It also can be seen that better correlation is obtained between the simulation and model tests in following sea cases, in which steady wave pattern effect is less than the cases of head sea.

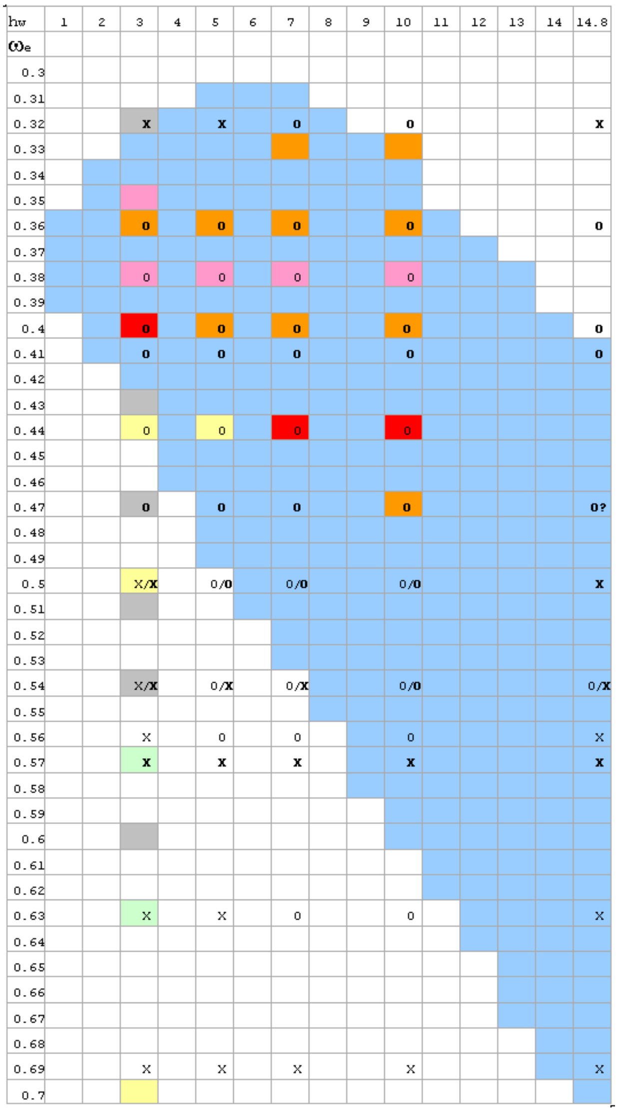


Figure 6. Comparisons of susceptibility between criteria, model tests and numerical simulations.

3. PARAMETREIC ROLL IN IRREGULAR SEAS

It is not easy to define criteria to identify the occurrence of parametric roll in irregular sea condition because it is hard to characterize main parameter to induce parametric roll in irregular waves deterministically. A series of numerical simulation will be useful to figure out relationship between characteristic incident wave height and period(such as significant wave height and zero-upcrossing period or modal period), heading angle, ship speed and occurrence of parametric roll.

In the present study, NLOAD3D is employed for simulation of parametric roll in waves. A series of simulations was carried out for various conditions of ship speed, wave headings and combinations of wave heights and periods to investigate main parameters associated with correlation to susceptibility in irregular waves. Firstly NLOAD3D results in irregular waves were compared with those from LAMP for cross-check, comparison results showed equivalency of statistics from both codes. Various aspects of parametric roll in irregular waves were investigated not only for susceptibility, identification in oblique waves but for ergodicity and convergence of parametric roll.

3.1 Susceptibility

The following simulations have been carried out for investigation of occurrence of parametric roll in head and following irregular seas. Roll natural period is 34.3 seconds.

- Hs: 3.5, 5.5, 7, 8.5m
- Tp(=5 Hs1/2): 0.8 ~ 1.2 Tp

Table 3. Susceptibility for following sea, Vs=5 knots.

	Modal Periods[sec.]				
Hs[m]	0.8Tp	0.9Tp	Tp	1.1Tp	1.2Tp
3.5	7.48	8.42	9.35	10.29	11.22
Te/Tr	0.28	0.31	0.33	0.36	0.38
5.5	9.38	10.55	11.73	12.90	14.07
Te/Tr	0.33	0.36	0.40	0.43	0.46
7	10.58	11.91	13.23	14.55	15.87
Te/Tr	0.37	0.40	0.44	0.48	0.52
8.5	11.66	13.12	14.58	16.04	17.49
Te/Tr	0.40	0.44	0.48	0.52	0.56

Table 3 shows an example of test cases in following sea condition with ship speed of 5 knots. Coloured cell denotes parametric roll occurred in the simulations. From 160 cases of simulations, it was found that parametric roll occurred for the following conditions shown in Table 4.

In the Table, Lw and Lpp denote wave length corresponding to modal period and ship length, respectively. Tpe, Tz and Tr represent encountering modal and zero-upcrossing periods, roll natural period, respectively. In head sea, no occurrence of parametric roll was observed for ship speed greater than 10 knots. In following sea, no parametric roll was observed for ship speed greater than 15knots. Relatively large range of wave periods causes parametric roll in irregular longitudinal waves, it seems that wave length and encounter wave period are both important in irregular waves. It is expected that coincidence of either of modal period or zero-upcrossing periods with half of roll period is important because the modal period provides a large excitation while zero-upcrossing period gives frequent excitations. Comparison with regular wave susceptibility shows better correlation is obtained for modal period than zero-upcrossing period compared with regular wave susceptibility.

Table 4. susceptibility of parametric roll in longitudinal irregular waves.

Hs	Lw/Lpp	Tpe/Tr	Tze/Tr	Vs	Heading
5.5	0.91~1.08	0.424~0.463	0.301~0.330	0	0/180
7~8.5	0.61~1.67	0.308~0.510	0.219~0.363	0	0/180
8.5	1.40~1.67	0.424~0.466	0.302~0.332	5	180
5.5	0.75~1.08	0.431~0.465	0.307~0.331	5	0
7~8.5	0.61~1.67	0.365~0.563	0.260~0.401	5	0
5.5	0.91~1.08	0.475~0.535	0.338~0.381	10	0
7~8.5	0.61~1,67	0.448~0.628	0.319~0.447	10	0

-Regular wave susceptibility
H=5.5m: Te/Tr=0.41~0.67
H=7m: Te/Tr=0.41~0.68
H=8.5m: Te/Tr=0.44~0.75

Following sea conditions with same encounter wave periods give more frequent parametric roll than head sea conditions. One of reasons is that wave energy is accumulated in following sea while the energy is distributed in head sea, which corresponds to strength of forcing. Finding range of wave periods and wave heights for irregular sea state will practically contribute to prediction of parametric roll in real sea states.

3.2 Identification of parametric roll and linear resonant roll in irregular seas

In stern quartering sea condition, there are two possibilities of resonant roll motions. The one is linear resonant roll and the other one is parametric resonant roll. In this section, it will be shown that normalized coherence could be a useful measure for determining parametric roll or not from resonant roll responses in irregular seas. The concept of normalized coherence is based on the fact that the coherence is a kind of measure whether the relationship between input and output is linear or not. Parametric roll motion is a typical example of nonlinear ship motion response while resonance of roll is mainly dominated by linear parameters such as mass and restoring coefficients.

The following simulations were carried out for investigating correlations between linear resonant roll, parametric roll and coherence.

Case #1, 2: Hs=8.5m, Tz=10.5s, Vs= 5knots, 15 degrees,

Case #2-1,2: Hs=8.5m, Tz=10.5s, Vs= 25knots, 30 degrees,

Cases #1 and #2 were simulated to investigate effects of linear and nonlinear roll simulation. Case #1 corresponds to the case nonlinear roll simulation which can capture parametric roll while case# 2 did not consider change of GM. Cases# 2-1,2 are the cases for linear resonant roll motion.

As shown in Fig. 7 coherence of linear roll has values close to 1 within the frequency range where wave energy is concentrated but relatively small values are obtained for the case of parametric roll within the same frequency range. "Linear roll" represents the results from linear ship motion option in NLOAD3D. So, test conditions for the case#1 and case#2 are same but simulation option is different. There is a noticeable difference of coherence functions between twos cases, linear roll is insensitive to filtering number while parametric roll is quite sensitive. Figs. 7~8 show this trend. Fig. 9 shows comparison of linear resonance case. As shown in the figure, no noticeable difference is found between two calculation options.

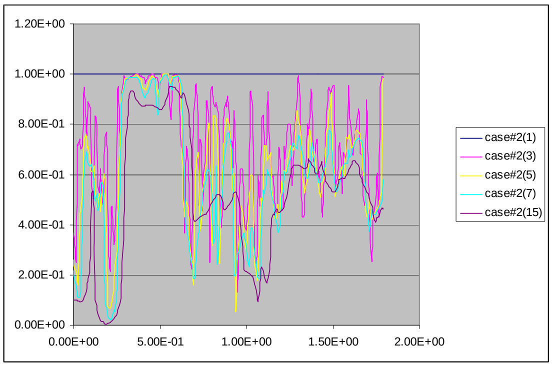


Figure 7. Coherence of linear roll motion for Hs=8.5m, Tz=10.5s, Vs= 5knots, 15 degrees.

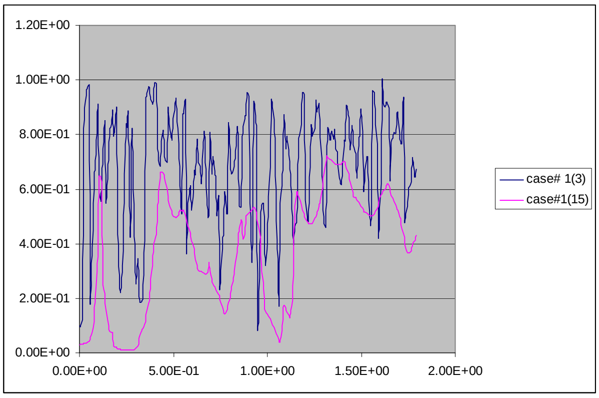


Figure 8. Coherence of nonlinear roll motion for Hs=8.5m, Tz=10.5s, Vs= 5knots, 15 degrees.

The following IPR(Index of Parametric Roll) could be defined for quantitative assessment of parametric roll measure in irregular waves.

$$IPR = \int_{\omega_1}^{\omega_2} H(\omega) S_\zeta(\omega) d\omega / m_0 \qquad (2),$$

$$H(\omega) = S_{xy}(\omega) S_{yx}(\omega) / (S_{xx}(\omega) S_{yy}(\omega))$$

$$m_0 = \int_{\omega_1}^{\omega_2} S_\zeta(\omega) d\omega$$

where, $H(\omega)$, $S_\zeta(\omega)$, $S_{ij}(\omega)$ are coherence function, wave spectrum and cross spectrum, respectively. In case of linear roll response whatever it is resonated or not the IPR is higher than 0.8 while IPR is about 0.5 or less in case of parametric roll. Further extensive numerical experiments could give more reliable standard of IPR.

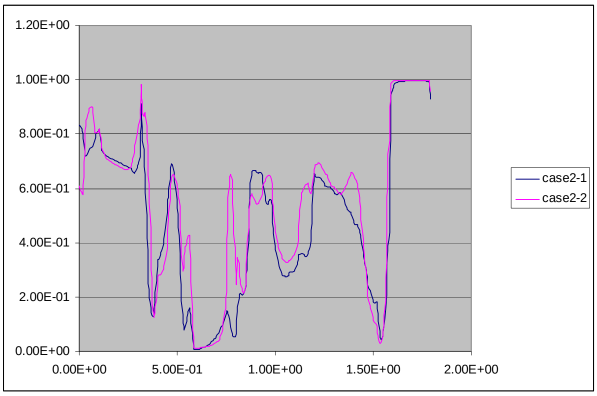


Figure 9. Comparison of coherence of linear resonant roll motion for Hs=8.5m, Tz=10.5s, Vs= 25knots, 30 degrees(LFILTR=15).

3.3 Ergodicity

Ergodicity of parametric roll has been one of important issues because reliable quantitative estimation of parametric roll is of great concern in practical design point of view. Convergence tests for parametric roll have been carried out for the case of Hs=8.5m, Tz=10.34 sec., heading=30 degrees and ship speed of 5 knots. 50 runs were conducted to investigate difference between each run and accumulated convergence characteristics. 30 minutes is taken as the duration of each realization.

As seen in Figs. 10, correlation between each run is very good for wave and heave, relative error of each run to averaged value in RMS^2 is at most less than 10%. RMS^2 was considered as parameter for ergodicity by Belenky(2003) because squared value magnify the statistical fluctuation. The results for statistical nature of parametric roll in the present study are almost same as previous studies(Belenky, 2003, Bulian et al., 2006) in qualitative point of view. Their opinions were 'clearly non-ergodic' or 'ergodic but practically non-ergodic'. Their conclusions imply that we should take unrealistically very long simulation time to get reliable statistical results when we consider parametric roll phenomenon. Parametric roll motion statistics such as RMS, significant double amplitude, average of 1/10th highest values, etc. fluctuate quite a lot between each runs. Highest deviation from average values in RMS exceeds 50% but most of results from RMS to significant values are inside 25% band.

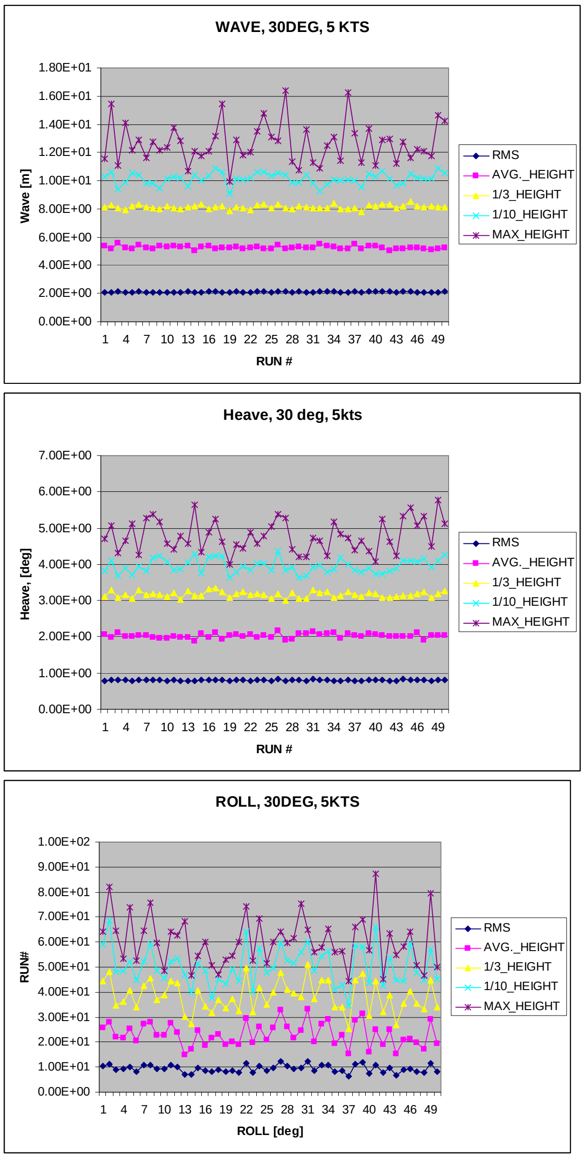


Figure 10. RMS, Average, significant, 1/10th and maximum heights of several realization for Hs=8.5m, Tz=10.34 sec., Heading=30 deg. and Ship speed of 5 knots (wave, heave, roll).

Fig. 11 compares accumulated average of RMS square. More numerical experiments are needed to make concrete conclusions but present results show that average of 5~10 realizations gives converged value within 10% relative error bound to average of 50 runs. This implies that average of 5 or 10 realizations could be a good index to measure parametric roll quantitatively in irregular seas.

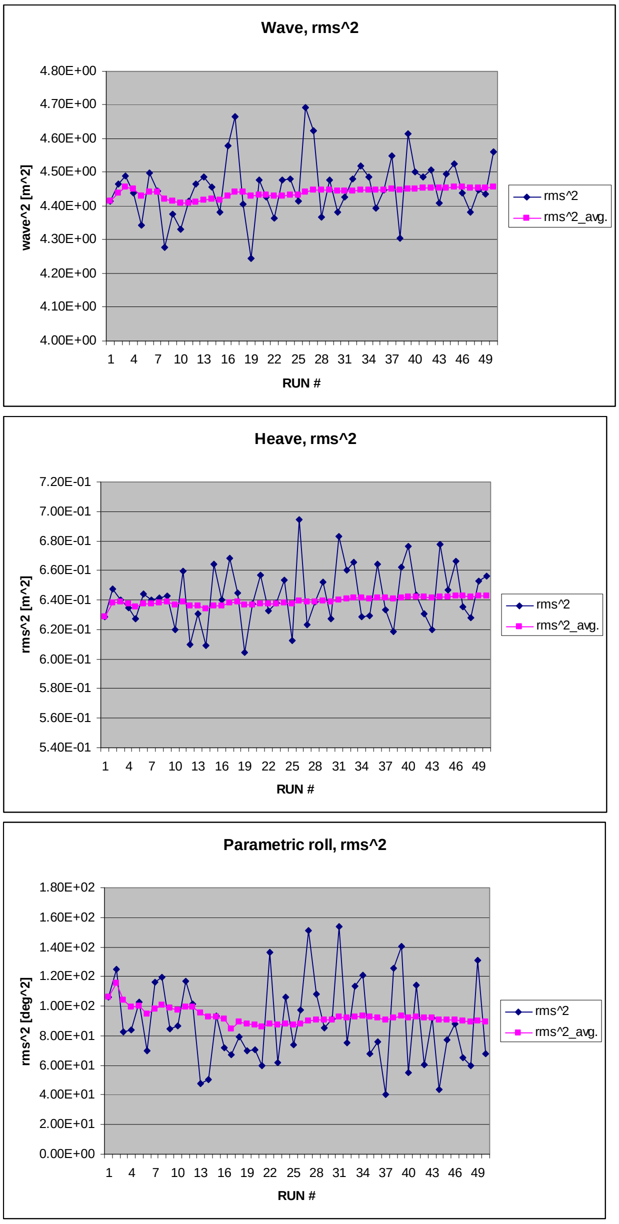


Figure 11. RMS, RMS^2 and their accumulated averages for Hs=8.5m, Tz=10.34 sec., Heading=30 deg. and Ship speed of 5 knots (wave, heave and roll).

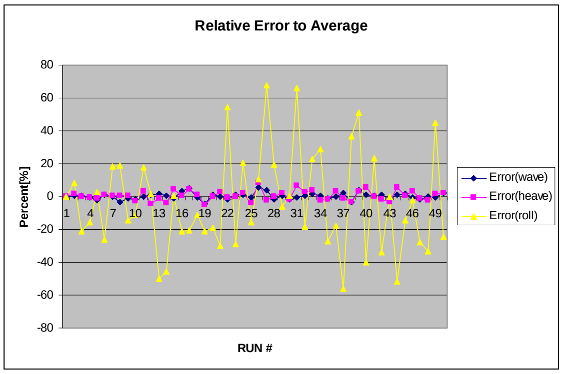


Figure 12. Relative errors of each realizations to accumulated average values(RMS^2).

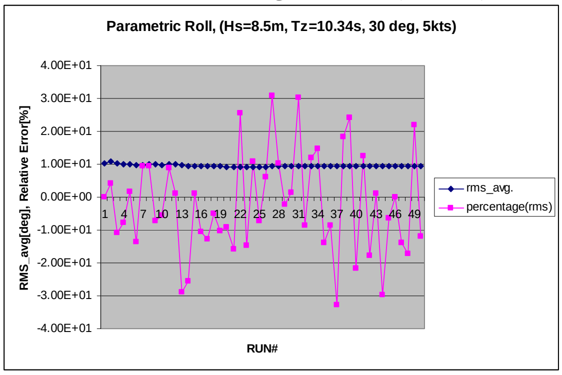


Figure 13. Relative errors of each realizations to accumulated average values(RMS).

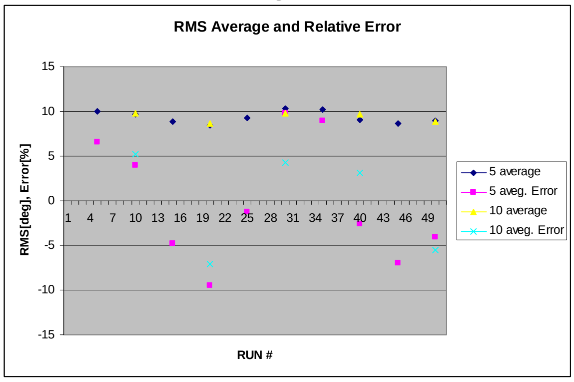


Figure 14. Average of RMS of Parametric Roll and its Relative Error.

Fig. 14 compares relative errors of 5 and 10 averaged RMS values. Most of values are with in 10% error bound for 5 averaged, 5% error bound for 10 averaged cases. Considering parametric roll is highly nonlinear and its nature is very sensitive to damping mechanism, allowance of 10% in RMS level is quite reasonable and practical to define criterion as quantitative measure.

4. CONCLUSIONS

The following conclusions could be drawn from a series of numerical simulations of parametric roll in regular and irregular waves.

- The susceptibility criteria generally well predict parametric rolling and it shows good correlation with model tests and numerical simulations. Better correlation is obtained between the simulation and model tests in following sea cases, in which steady wave pattern effect is less than the cases of head sea.
- Relatively large range of wave periods cause parametric roll in irregular longitudinal waves. Comparison with regular wave susceptibility shows better correlation is obtained with modal period than zero-upcrossing period.
- Normalized coherence could be a useful measure for determining parametric roll or not from resonant roll responses in irregular seas.
- In spite of some previous studies' conclusion on practically non-ergodic characteristics of parametric roll, accumulated average value of 30 minute length of realizations gives converged result. It could be a practical guide to measure quantitative parametric roll under the condition of fixed length of simulation(say, 30 minutes) and number of simulations(say, 5 or 10).

5. ACKNOWLEDGMENTS

This study is a part of the research "Development of Simulation Technology for Dynamic Stability of Ships" supported by Korean Ministry of Knowledge and Economy. The first author appreciates financial support of American Bureau of Shipping for this study during his stay at ABS.

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