PARAMETRIC ROLLING OF A CONTAINER VESSEL IN LONGITUDINAL WAVES

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ABSTRACT

A time domain numerical simulation of ship motion in six degrees of freedom is proposed to detect parametric rolling in both regular and irregular longitudinal waves. An investigation into the dynamic stability in waves of a container vessel is made and compared with experimental data. The basic approach of the simulation program involves computation of hydrodynamic coefficients of added mass and damping, restoring coefficients and diffraction and Froude-Krylov excitation forces at each step in time according to the instantaneous waterline and vessel position, using a strip theory method and a pressure integration technique along segments. This article briefly describes the computational technique utilized and makes comparisons between numerical and experimental data on parametric rolling obtained to infer about the levels of fidelity of the proposed methodology. At the end some recommendations are presented to mitigate parametric rolling in actual sea waves.

Keywords: *Parametric rolling, Wave-induced ship motions in waves, Non-linear ship dynamics in waves, Time domain simulation*

1. INTRODUCTION

Since the early fifties, parametric resonance has been already studied and discussed by several investigators: Kerwin (1955), Paulling (1959), Sanchez and Nayfeh (1990), Umeda et al. (1995), Hamamoto and Panjaitan (1996), and safety authorities: IMO (1995). However this particular phenomenom in head waves has been considered mostly of theoretical interest and less worthy of practical concern. More recently evidence of parametric rolling in head seas on a post-Panamax C11 class container ship presented by France et al. (2003) received wide and renewed attention from IMO (2003), demonstrating the practical importance of this phenomenon. Therefore, in 2004 the American classification society ABS (2004) issued the first guide for the assessment of parametric rolling in the design of container vessels.

However, in order to incorporate parametric rolling in ship design and its operation, a deeper understanding of the phenomenon is still required. Hence, first it was decided to compare the empirical roll damping established from the forced rolling model tests with applicable analytical methods. Then, free running model experiments with all the relevant operational conditions were conducted, and the corresponding numerical simulations were later performed for comparison. From experimental results, effects of ship's advance speed on parametric rolling in longitudinal waves were examined. Finally, from time domain numerical simulations, it was also possible to infer about the levels of fidelity of the proposed model which compromises the calculation of the non-linear ship's responses in full six degrees-of-freedom.

2. NUMERICAL FORMULATION

2.1. Occurrence of Parametric Rolling

In classical seakeeping theory the motions of a vessel are caused by direct wave excitation and large roll responses of ships are associated with synchronous rolling in beam seas, where the waves encounter frequency coincides with the ship's natural roll frequency. This is called a linear roll resonant condition. However, there are other non-linear mechanisms in waves, which may also result in extremely pronounced rolling.

For example, for a ship in longitudinal waves (coming either from head or stern) the uneven wave surface together with the pitch and heave motions of the ship results in a time-varying underwater hull geometry. This varying geometry, in turn, results in time varying changes in the metacentric height, i.e., in the static roll stability. The variation of the static roll stability can cause instability if it occurs in the appropriate period, and this instability leads to roll when a small excitation, introduced by wind gust or a rudder movement for example, causes the vessel to take a small roll angle to one side. The roll motion, once started, may grow to large amplitude if the stability variation and the wave height are large. Moreover, the maximum roll response will be limited by roll damping and, in extreme conditions, may result in danger to the ship or its cargo. This phenomenon is referred to as auto parametrically excited rolling which is usually shortened to parametric rolling. The term describes a state of motion that results not from direct excitation by a time-varying external force or moment but from the periodic variation of certain numerical parameters of the oscillating system. In this dynamic situation, the ship motion should be described by coupled roll, pitch and heave, and hence the restoring forces and moments should include effects relevant to these instantaneous motions.

From theory and as validated by model tests for different types of vessels conducted by Hamamoto et al. (1996), Ribeiro e Silva et al. (1998), Neves (2002), Perez-Rojas (2003), France et al. (2003), Hashimoto et al. (2006), low cycle parametric rolling occurs when the following requirements are satisfied:

The natural period of roll is equal to approximately twice the wave encounter period, and the wavelength is on the order of the ship length (between 0.8 and 1.6 times LBP);

The wave height exceeds a critical level;

The roll damping is low.

2.2. Numerical Prediction of Parametric Rolling

Ribeiro e Silva and Guedes Soares (2000), demonstrated that both linearised and non-linear theories could be used to predict parametric rolling in regular head waves. On the linear model (in the form of Mathieu's equation) stability variations were evaluated from the linearised righting arm curves with the wave crest varying longitudinally along the ship hull. However, this model was not accurate enough to predict ship's roll response magnitude under wave-induced parametric resonance conditions, since deck submergence effect on restoring characteristics of the vessel could not be included and therefore the limit cycle behaviour could not be obtained.

A non-linear numerical model of parametric resonance taking into consideration deck submergence and other non-linearities on restoring moment of ships in regular waves was also proposed by Ribeiro e Silva and Guedes Soares (2000). In that model a quasi-static approach was adopted to study the roll motion, where only the variations on transverse stability in regular waves were considered. For that purpose, an uncoupled roll equation, which included the effects of heave and pitch responses in regular waves, and immersed hull variations due to wave passage on roll restoring term, was used to describe the parametrically excited roll motions. While good agreement in terms of limited response behaviour was found between the time domain simulation of roll motion in longitudinal regular waves and the existing experimental data, simulations of parametric rolling in irregular waves as presented in literature by Bulian and Francescutto (2002), Belenky (2003), and Pereira (2003), could not be performed using that one degree-of-freedom model.

To overcome these shortcomings a non-linear model, coupled in the five degrees-of-freedom was then developed and proposed Ribeiro e Silva et al. (2005) to simulate the time domain responses of a ship in uni-directional long-crested irregular waves. Although that non-linear time domain simulation did not take into account for all hydrodynamic phenomena, the code was more sophisticated than the previous one and was capable of predicting parametric rolling responses in a scenario more closely related to the ship's conditions that may be found at sea.

The computer code presented by Ribeiro e Silva (2008) extends the existing model by adding the 6th degree-of-freedom and changing the pressure integration method which is now less computational demanding and therefore more efficient in engineering practical terms of view.

2.3. Mathematical Model

In this work unrestrained rigid body motions of a vessel with advancing speed are considered. The dynamics of oscillatory ship motions is governed by Newton's second law, which represent the equilibrium between the internal forces due to inertia, gravity, and the external forces acting on the ship, given by:

$$[M]\{\ddot{\xi}\} = [F^{Dyn}] + [F^{Sta}] \quad (1)$$

These static and dynamic forces $[F]$ and motions $\{\xi\}$ are represented on a inertial coordinate system (see Figure 1) fixed with respect to the mean position of the ship, $X = (x, y, z)$, with z in the vertical upward direction and passing through the centre of gravity of the ship, x along the longitudinal direction of the ship and directed to the bow, and y perpendicular to the latter and in the port direction. The origin is in the plane of the undisturbed free surface.

According to Figure 2, the translatory displacements in the x, y, and z directions are respectively the surge ξ_1, the sway ξ_2, and the heave ξ_3. The rotational displacements about the x, y, and z axes are respectively the roll ξ_4, the pitch ξ_5, and the yaw ξ_6.

Under the assumptions presented before all hydrodynamic forces are linear, and combining these with the mass forces one obtains six linear coupled differential equations of motion, given by:

$$\sum_{j=1}^{6} \left\{ \left(M_{kj} + A_{kj}\right)\ddot{\xi}_j + B_{kj}\dot{\xi}_j + C_{kj}(t)\xi_j \right\} = F_k\, e^{i\omega_e t} \quad (2)$$

If the ship travels along a prescribed path β at an initial steady velocity U (see Figures 1 and 2), she will encounter the regular wave crests with a frequency of encounter, given by:

$$\omega_e = \omega - kU\cos\beta \quad (3)$$

where the surface elevation of a regular wave is given by:

$$\eta_w = \eta_w^a \cos k\left[x\cos\beta - \left(c - U\cos\beta\right)t\right] \quad (4)$$

According to St. Dennis principle in irregular seas, it is possible to describe the equations of motion given by the sum of sinusoidal waves yielding to an irregular wave profile, given by:

$$\eta_w = \sum_{n=1}^{N} \eta_{w_n}^a \cos\left[\begin{array}{l} \dfrac{\omega_n^2}{g} x\cos\beta - \\ \left(\omega_n - \dfrac{\omega_n^2}{g} U\cos\beta\right)t \\ + \varepsilon_n \end{array}\right] \quad (5)$$

where N is the number of component waves, ω_n the circular frequency, ε_n the random phase angle, and $\eta_{w_n}^a$ the amplitude of the n-th component waves which are given by the wave spectrum $S(\omega)$.

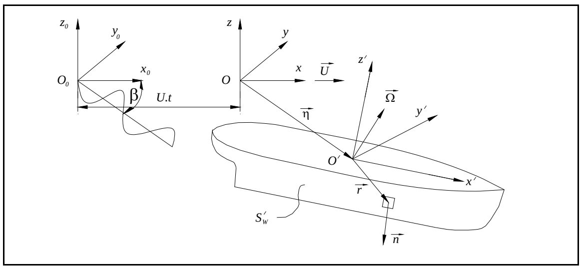

Figure 1. Definition of the earth-fixed coordinate system (X_0) and the inertial coordinate system (X), and mathematical relation between these two coordinate systems.

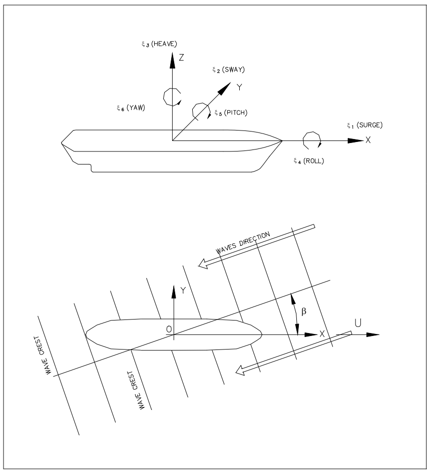

Figure 2. The coordinate system and six modes of motion, and definition of the ship heading angle.

In general, the forces acting on the ship's hull consist of control forces from rudder, and active fins, environmental forces from wind and waves and reaction forces due to ship motions. In this study water is assumed incompressible, inviscid (although viscous effects are considered when roll damping is calculated), and deep.

When the problem of wave-induced parametric rolling is concerned, only the forces due to wave excitation and reaction forces due to wave-induced ship motions are taken into account. Other forces are assumed to be cancelled by each other, which means, the ship and relative course of the ship to the wave direction is kept constant during the time domain simulation for a given loading condition. Hence, the wave excitation forces consist of incident wave forces (or Froude-Krylov forces), diffraction forces, and the reaction forces of restoring forces and radiation forces. In respect to surge motion, also a semi-empirical methodology has been adopted to estimate the added mass and the Froude-Krylov wave excitation forces in surge mode. These methods are the ellipsoid method and the Hutchison e Bringloe (1978), respectively.

The investigation of the transverse stability performances under parametric rolling on longitudinal waves represents a complex problem. In an approximate way then radiation and wave excitation forces are calculated at the equilibrium waterline (see Figure 3) using a standard strip theory, where the two-dimensional frequency-dependent coefficients of added mass and damping are computed by the Frank´s close fit source-distribution method, and the sectional diffraction forces are evaluated using the Haskind-Newman relations as presented by Salvesen et al. (1970).

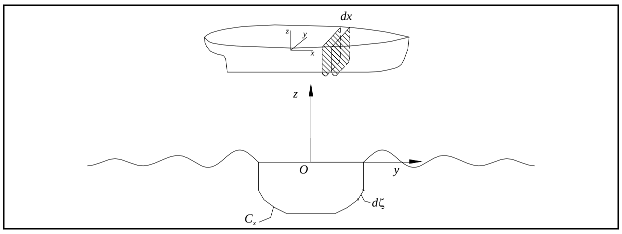

Figure 3. Definition of hydrostatic and hydrodynamic properties by integration along the length of the hull (strip theory).

For a time domain simulation of ship motions instabilities in waves, it is necessary to predict the body motions and forces at discrete time steps. The process by which this is done in the present investigation is now outlined.

At each time step the underwater part of the hull is calculated, together with its geometric, hydrostatic and hydrodynamic properties. The forces and moments, calculated using these instantaneous properties, are used to derive consequential translational and rotational motions. These motions are applied to the hull and the time step incremented. This process is cyclic, the previous time step providing conditions for the current time step. As shown on equation (2), a quasi-static approach is adopted to calculate the non-linear restoring coefficients in heave, roll, and pitch motions in waves, in which calculations of significant variations on these restoring coefficients are calculated over the instantaneous waterline.

As illustrated in Figure 4, the two-dimensional hydrostatic forces and moments calculations are made using the pressure integration technique along each segment (C_x) of each transverse section of the ship hull, rather than using area and volume integration of the ship offsets. The original theoretical approach to the pressure integration technique outlined by Schalck and Baatrup (1990) has been adopted in conjunction with a practical method to generate the segments required to calculate the hydrostatic pressure distribution under either a regular or irregular wave profile.

At this point it should be mentioned that a more sophisticated wave-induced parametric rolling model could be adopted, where added masses and damping coefficients would be also calculated with consideration to the instantaneous submerged hull body under the wave surface. However, the hydrodynamic component of the parametric excitation is insignificant in comparison with quasi-hydrostatic excitation caused by the incident wave potential and the wave-induced heave and pitch motion.

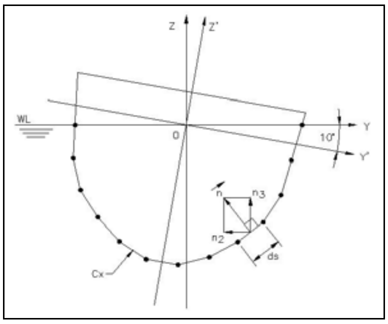

Figure 4. Definition of geometric properties of a transverse section of the underwater part of the hull.

The hydrodynamic component depends upon the overall submerged hull form, while the quasi-static hydrostatic component is strongly dependent upon wave passage and hull-shape (i.e. variations about the still waterline). For this reason and others associated with larger computational efforts only the quasi-hydrostatic component of the parametric excitation is taken into account in here.

In particular the roll added inertia and radiation coefficients are taken to be linearly proportional to roll acceleration and velocity, respectively. Hence, hydrodynamic memory effect due to roll motion and consequently its effect, expressed as roll damping, is practically negligible at frequencies lower than 0.5 [rad/s] (see Figure 5).

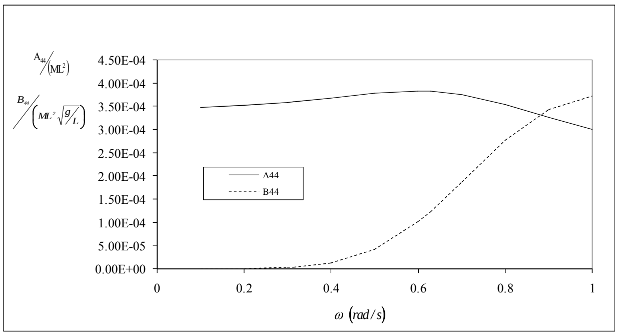

Figure 5. Added mass and damping coefficients of roll motion.

The development of the computer code SHIPATSEA, to calculate the roll restoring moment under parametric rolling conditions in longitudinal waves, represented an important task. However, in order to use this numerical model with adequate levels of confidence, it will be necessary also to confirm its validity by means of thorough and extensive experimental measurements.

3. SELECTION OF CASE STUDY

The study presented herein adopts a typical form of a container vessel from which experimental results on parametric rolling still to be reported were available.

3.1. Outline of Model Experiments

Model experiments were conducted at the Seakeeping basin of "Hamburgische Schiffbau-Versuchsanstalt GmbH" - HSVA (length: 300 [m], width: 18 [m], and depth: 6 [m]), equipped with a flap type wave-maker and an overhead towing carriage. A scaled model of the container vessel was used here, and her principal particulars and body lines are presented in Table 1 and Figure 6, respectively.

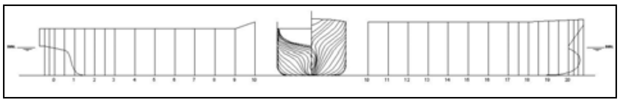

Figure 6. Hull form of the container vessel.

Table 1. Container vessel particulars.

Length between perp.	L_{pp} [m]	117.6
Breadth	B [m]	20.2
Deck height	D [m]	15.7
Draught	T [m]	8.1
Displacement	Δ [ton]	12,714
Block coefficient	C_b	0.65
Midships coefficient	C_m	0.97
Long. position of CG	LCG (%L_{pp})	0.00
Vert. position of CG	VCG [m]	8.73
Pitch gyration radius	r_{yy} [m]	29.35
Roll gyration radius	r_{xx} [m]	7.19
Metacentric height	GMt [m]	0.50
Natural roll period	T_{44} [s]	21

Figure 7. A photograph of the scaled model of container vessel.

The hull is vertical sided amidships above the waterline and presents large beam variations at aft and fore ends. The ship model of wooden and glass reinforced plastic construction was manufactured on a scale 1:24. The free-running ship model was propelled by an electric motor with a constant revolution control system. The hull is appended with three pairs of bilge-keels, and a rudder which is controlled by an auto-pilot. In addition, surge, sway, heave, roll, pitch, and yaw were obtained from the data of ship position recorded by an optical tracking sensor fixed to the moving carriage.

Prior to parametric rolling tests, forced rolling model tests were conducted at different speeds in calm water to assess roll damping. Then, parametric rolling tests were conducted for both head and following regular waves, where variations of wavelength, wave heights and ship's advance speed were tested. From the model test results it was possible to determine the wave height, wavelength and ship's advance speed for which parametric rolling would start, i.e. most critical wave encounter condition.

During all these tests the model was ballasted to the load condition presented in Table 1, which corresponds to conditions on arrival of a trip with 388 TEU embarked, assuming each TEU weights 14 metric tonnes with a VCG of 45% of its height. As it can be observed from "Germanischer Lloyd" (GL) Stability Booklet (1997), on this load condition, the vessel is statically stable, and is also compliant with all the applicable IMO (2002) stability criteria.

3.1 Numerical Simulations

3.2.1 Roll Damping Prediction

From Ribeiro e Silva et al. (2000, 2005) it is known that when roll damping is tuned to model test results, a very good correlation of the roll motion can be achieved between model tests and the numerical analyses. Particularly, the magnitude of the roll response during parametric rolling is dictated in large part by the amount of viscous damping in the roll degree-of-freedom, where damping improves the stability character of the system by reducing the instability regions in size. To account for these damping effects in this study, it was decided to compare the empirical roll damping established from the forced rolling model tests with applicable analytical methods. Hence, it was found that for slender hull forms the Miller's roll-damping method (1974), could also be used to accurately calculate the total roll-damping coefficient at different advance speed (see Figure 8).

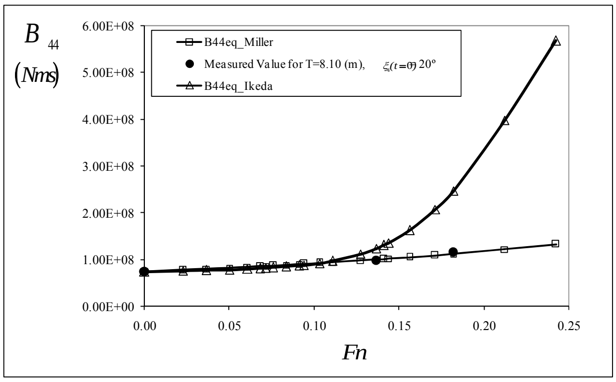

Figure 8. Comparison of semi-empirical methods of prediction of roll damping by Ikeda's (1978), and Miller's (1974) with scaled model measurements.

3.2.2. Regions of Instability

It must be noted that for a specific loading condition, low cycle parametric rolling occurs when the three requirements presented on section 2.1 are satisfied. In respect to stability variations, as shown in Figure 9 these were calculated based on either still water or linearised values of GMt (according to linearisation procedure defined by Ribeiro e Silva et. al (2005).

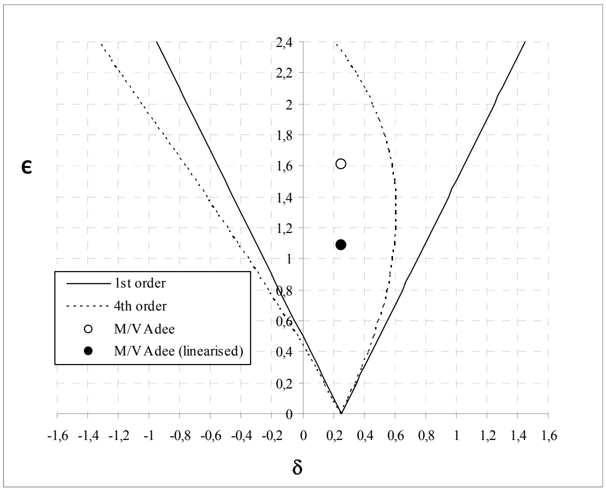

Figure 9. Low cycle regions of instability for container vessel (Ince-Strutt chart).

For this container vessel, the requirements for low cycle parametric rolling to occur lead to a wavelength of 163.5 [m] and a ship's roll natural period of 21 [s], which then results on a wave frequency of 0.614 [rad/s] and an ahead speed of 2.4 [kts]. From Intact Stability Booklet issued by GL the loading condition referred above for the container vessel has been confirmed as viable and the ahead speed of 2.4 knots fits into the operational range of speeds. As shown on Figure 9, these operational conditions also fall into a Mathieu type zone of instability so that critical wave-induced parametric rolling conditions could be identified prior to model tests.

3.2 Regular Waves Tests Comparison

3.3.1. Regular Head Waves

For comparisons, the same critical speed, and wave conditions were used in the simulations as in the model tests. The experimental and numerical results of SHIPATSEA computations for regular head waves (Hw = 6 [m], Tw = 10.23 [s], μ = 180º and U = 2.4 [kts] are presented on Figures 10 and 11, respectively. Plotted side-by-side in these figures are time traces of the wave surface elevation (at the origin), surge, sway, heave, roll, pitch, and yaw motions during low-cycle parametric resonance. The same scales for abscissas and ordinates axis have been adopted as much as possible. However due to practical reasons, the initial time instant of the experimental records has been truncated up to time instant 278 [s] where the first regular waves with the predefined amplitude of 3 [m] arrived at the origin. In respect to sway, and yaw modes, these should not be directly compared to experimental results, since the numerical model has no manoeuvrability capabilities embedded yet, and therefore, forces and moments exerted by the rudder and propulsion system of the model in order to maintain course and speed cannot be taken into account during simulations.

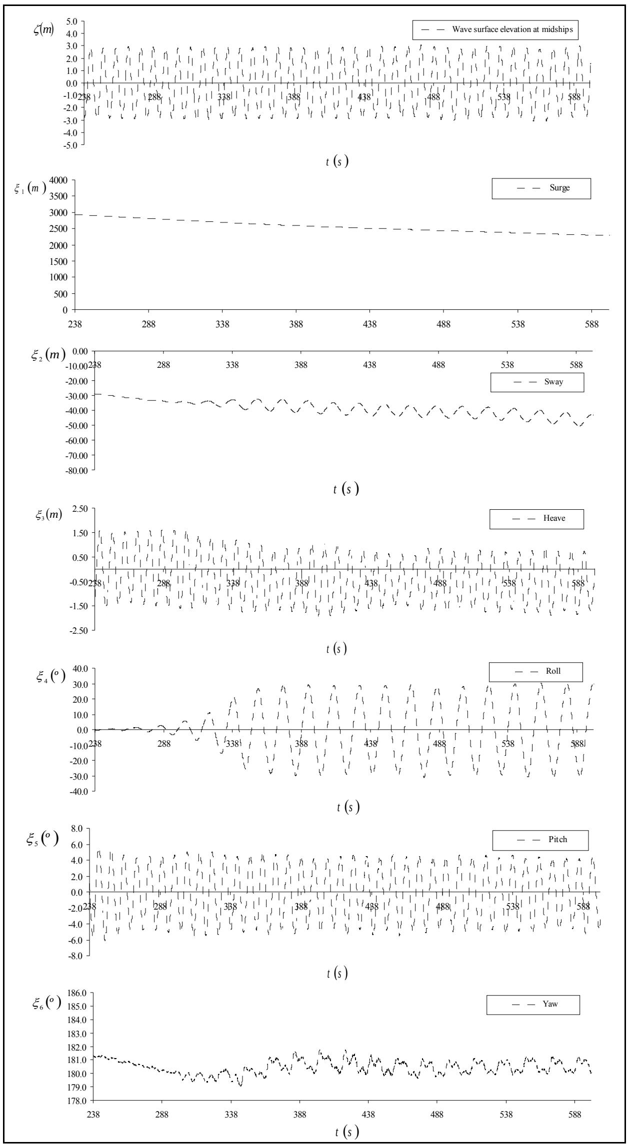

Figure 10. Experimental measurements of parametric rolling in regular head waves (Hw = 6 [m] and Tw = 10.23 [s]).

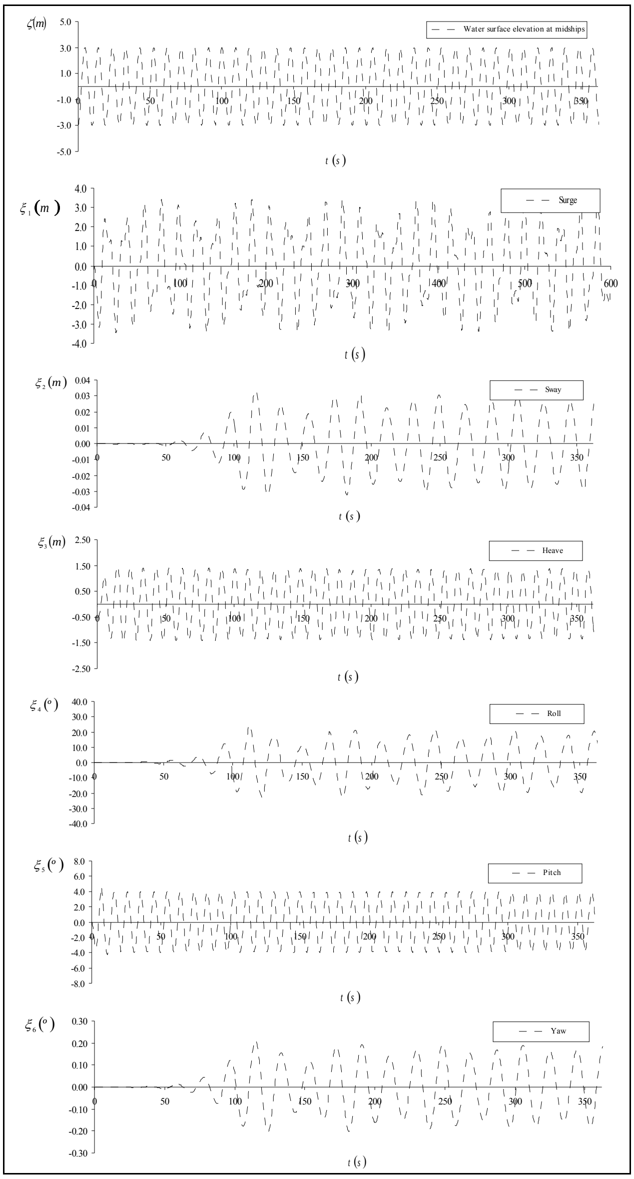

Figure 11. Numerical simulations of parametric rolling in regular head waves (Hw = 6 [m] and Tw = 10.23 [s]).

It can be seen that both measured and simulated wave amplitudes are constant, as a results of this, vertical plane modes, i.e. surge, heave and pitch motion amplitudes are also constant during the stationary phase. Measured and simulated heave motion is about 1.3 and 1.4 [m], and pitch is about 5 and 4º. On the transverse plane, time traces show constant wave and increasing sway, roll and yaw amplitudes up to their maximum amplitude during transient phase, where maximum measured and simulated roll motion is 29.8 and 24º, respectively.

3.3.2. Regular Following Waves

Experimental and numerical results of SHIPATSEA computations for regular following waves (Hw = 4.7 [m], Tw = 9.51 [s], μ = 0º and U = 4.7 [kts]) are presented on Figures 12 and 13, respectively. Similarly to head waves, the experimental records have been truncated up to time instant 247 [s] where the first waves with the predefined amplitude arrived at the origin. From time instant 247 to 347 [s] it can be observed that the model is governing to attain a 0º course. Again, sway, and yaw modes should not be directly compared.

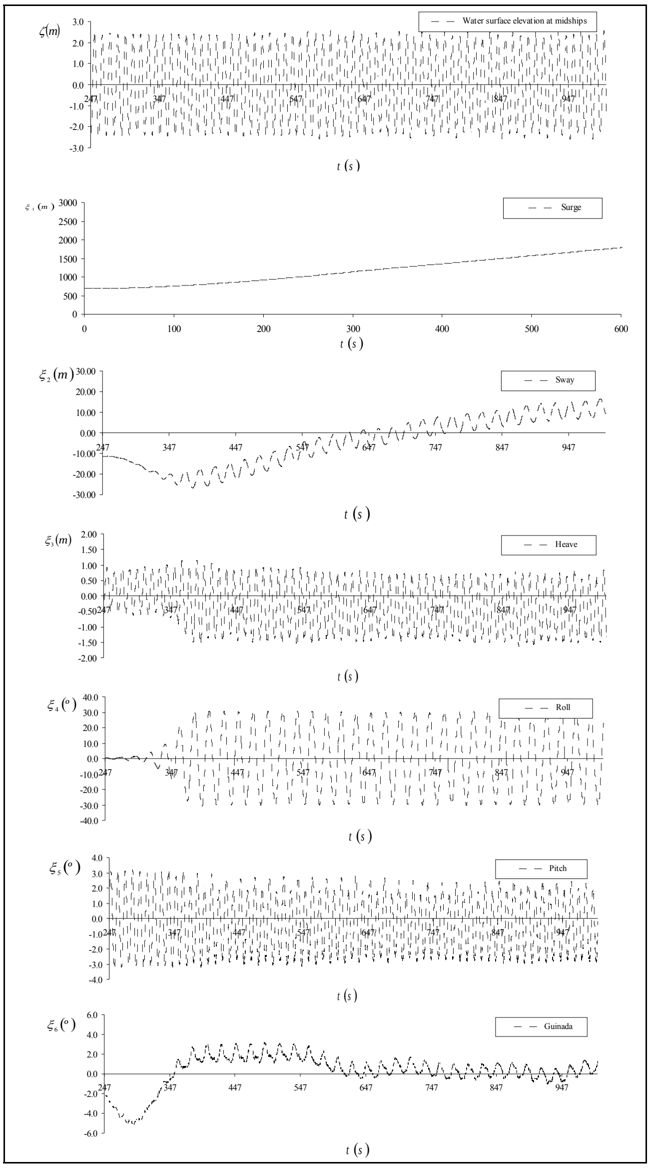

Figure 12. Experimental measurements of parametric rolling in regular following waves (Hw = 4.7 [m] and Tw = 9.51 [s]).

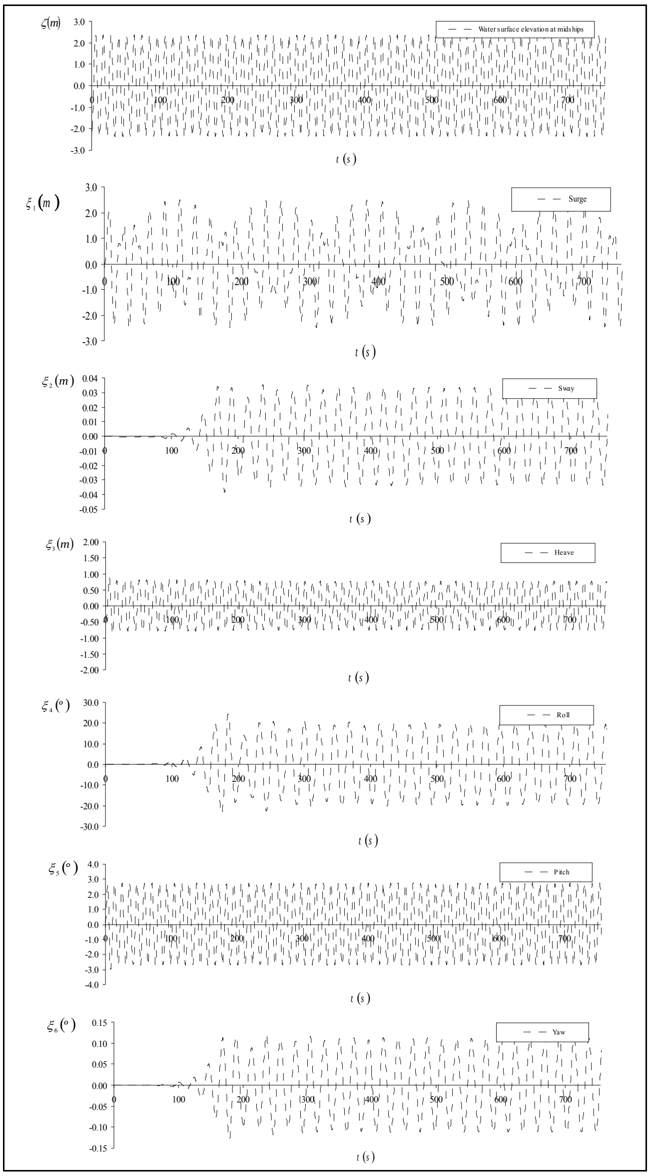

Figure 13. Numerical simulations of parametric rolling in regular following waves (Hw = 4.7 [m] and Tw = 9.51 [s]).

In SHIPATSEA the wave meets the ship at the beginning of the simulation in such a way that approximately 160 [s] are needed for the transient oscillation. On stationary phase, maximum measured and simulated roll is 30.2 and 24º, respectively. Measured and simulated heave is about 1.0 and 0.8 [m], and pitch is about 3 and 2.8º, so that heave and pitch are comparable. However, after time instant 347 [s], a larger difference is found for the negative and positive extrema of the pitch motion. It is believed that this hull's emergence effect is related to the propeller forces, but the reason for this should be further investigated.

4. CONCLUSIONS

The motions of a container vessel in parametric rolling in regular longitudinal waves have been studied with use of scaled model experiments and time domain numerical simulations. The experimental results and subsequent comparison with numerical simulations provide new insights and improved understanding of the parametric rolling mechanism. Also, the present work revealed several aspects of time domain simulation method, which are quite valuable for the validation of the method, and allows to devise possible computer applications to predict the parametric roll behaviour of the real vessel in actual sea waves. Moreover, it can be assumed that in other ship's loading conditions, providing there are some experimental measurements of roll damping characteristics, the SHIPATSEA predictions will be also good. Therefore it can be concluded that the SHIPATSEA results in conjunction with model testing can be used to predict the parametric rolling behaviour of the real vessel in arbitrary conditions.

The results presented in this paper show that this container vessel has a non negligible risk of encountering condition in which parametric rolling in regular waves can occur. As expected, larger roll angles developed in the vicinity of the frequency ratio ½. Therefore, under these conditions, roll angles exceeding 30º to each side can rapidly be produced, resulting eventually in cargo loss or ship damage. No capsizing due to parametric rolling in both head and following waves was obtained on experiments or numerical simulations.

5. ACKNOWLEDGMENTS

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