ISSUES ON THE NUMERICAL MODELING OF WAVE-INDUCED FORCES ON A SHIP IN FOLLOWING/QUARTERING WAVES

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Abstract:

A series of semi-captive model tests on a patrol boat in following and quartering regular waves have been performed at the Maritime Dynamic Laboratory at SSPA Sweden. The model was fixed in the surge, sway and yaw directions. Hence the wave-induced forces and moments were measured for these directions. The measurements were conducted at different forward speeds, incident wave angles, wave amplitudes and wave periods. The main purpose of this work is to get better insight into hydrodynamics at following and quartering seas, which will help to find improved methods for theoretical calculation of the forces acting upon a hull for the wave directions considered. One of the applications for such improved methods is to calculate the risk for broaching-to for fast ships.

1. INTRODUCTION

Broaching-to is a well-known dynamic instability phenomenon for fast vessels in following waves. The problem was quantitatively analysed by du Cane and Goodrich [1] as early as 1962. However, there is still a lack of methods for quantitative analysis in spite of years of research efforts. Wahab and Swaan [2] established the motion equations describing the surge, sway and yaw motion in following regular waves based upon the Froude-Krylov approach and analysed the broaching mathematically as an eigenvalue problem. They could then identify the positions of a ship along a steep wave for possible on-set of broaching. Umeda has presented several papers on his studies of broaching-to under the last teen years; see the biography in his latest paper [3]. His mathematical model can qualitatively explain the capsizing phenomena associated with broaching-to, but quantitatively overestimate the danger of capsizing.

Actually, broaching is a complex dynamic instability problem, in which many force components are involved. Among them the wave-induced forces constitute the dominant part of the instability mechanism. Numerical modelling together with computer simulations has become an important tool for analysis of this kind of dynamic instability problem.

A series semi-captive model test on a patrol boat in following and quartering regular waves are performed at the Maritime Dynamics Laboratory at SSPA Sweden. The wave-induced forces and moments have been measured at surge, sway and yaw direction for different forward speeds, incident wave angles, wave amplitudes and wave periods. The main purpose of this work is to get better insight into hydrodynamics at following and quartering seas, which will help to find improved methods for theoretical calculation of the forces acting upon a hull for the wave directions considered.

One of the applications for such improved methods is to accurately determine the risk for broaching for fast ships.

In this paper, some test results are presented and compared with theoretical calculations. A discussion is also carried out on issues concerning numerical modelling of the wave-induced hydrodynamic forces and moments, especially with respect to the effect of forward speed of the ship, non-linear wave effects (including influence of steep waves), etc.

2. MODEL TESTS AND THEORETICAL CALCULATIONS

2.1 Model tests

The semi-captive model measurements were carried out in the Maritime Dynamics Laboratory (MDL) at SSPA Sweden. The size of the wave basin is 88 m by 39 m, with wave-maker on two sides. A large multi-motion carriage spans the whole basin. The six-component balance was sited inside the model, which is connected to the carriage through a link, while the model is still allowed to be free to move in the heave and pitch direction respectively. The forces and moments acting on the model were measured. They are Fx (surge force), Fy (sway force), Fz(heave force), Mx (roll moment), My (pitch moment) and Mz (yaw moment), beside the heave and pitch motion.

The model was built to the scale 1:18. The main particulars are shown in the following

Lpp	34 m
B	6 m
T_f	1.99 m
T_a	1.92 m
Displ.	190 m^3
C_b	0.46
LCG	-2.1 m
KG	2.9 m

An evaluation method was used to determine the time history of the wave motion around the ship model for every test in form of a wave equation, based upon the measured wave motion by the wave height meter, see the method description in Hua, Abrahamsson and Byström [4]. The same method was also used to extract the first order motion responses and the first order wave-induced forces and moments.

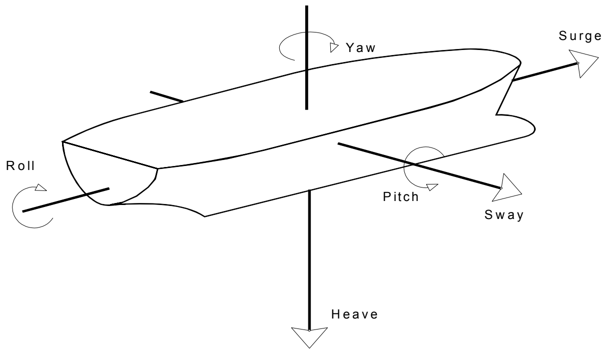

Figure 1. Definition of the six degrees of motion freedoms

2.2 Theoretical calculations

The theoretical model used by Umeda [3] is designated for calculation of wave-induced surge and sway force and yaw moment.

The wave-induced surge force in a regular wave is based upon the Froude-Krylov hypothesis, resulting in the following expression:

$$Fx = -\rho \cdot g \cdot \zeta_w \cdot k \cdot \cos\beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x\cos\beta) \cdot dx \tag{1}$$

where

$$C_1(x) = \frac{\sin\left(k\sin\beta \cdot B(x)/2\right)}{\ldots}$$

and ξ_G is the longitudinal position of the center of gravity relative to the wave trough.
For calculations of the wave-induced sway force and yaw moment, the following expressions was used:

$$Fy = \rho \cdot g \cdot \zeta_w \cdot k \cdot \sin\beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x\cos\beta) \cdot dx$$
$$+ \zeta_w \cdot \omega \cdot \omega_e \cdot \sin\beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x\cos\beta) \cdot dx$$
$$- \zeta_w \cdot \omega \cdot u \cdot \sin\beta \cdot \left[a_{22}(x) \cdot e^{-kd(x)/2} \cdot \cos k(\xi_G + x\cos\beta)\right]_{AE}^{FE} \tag{2}$$

$$Mz = \rho \cdot g \cdot \zeta_w \cdot k \cdot \sin\beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot x \cdot \sin k(\xi_G + x\cos\beta) \cdot dx$$
$$+ \zeta_w \cdot \omega \cdot \omega_e \cdot \sin\beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot x \cdot \sin k(\xi_G + x\cos\beta) \cdot dx$$
$$+ \zeta_w \cdot \omega \cdot u \cdot \sin\beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot \cos k(\xi_G + x\cos\beta) \cdot dx$$
$$- \zeta_w \cdot \omega \cdot u \cdot \sin\beta \cdot \left[a_{22}(x) \cdot e^{-kd(x)/2} \cdot x \cdot \cos k(\xi_G + x\cos\beta)\right]_{AE}^{FE} \tag{3}$$

In the above two formulas, the first terms in (2) and (3) represent the forces and moments due to the Froude-Krylov and diffraction effect. The third term in (2) and the third and forth terms in (3) are considered to be the lifting effect by Umeda [3]. Strip method is used for calculation of the wave-induced sway force and yaw moment. The two-dimensional added mass $a_{22}(x)$ is calculated be panel method at zero frequency.

Observe that the magnitudes of the wave-induced surge and sway force and moment respectively are linearly dependent of the wave amplitude and that these forces and moment vary harmonically with the ship position along a wave. This is a natural consequence of the fact that the derivation of the above expressions is based upon a linear strip approach.

3. RESULTS

All the numerical results are presented in full-scale.

3.1 Wave-induced force on surge, sway and yaw motion

In most numerical models it is assumed that the total longitudinal force on a ship in a following wave consists of two force components. One of them is constant equal to the resistance in calm water and the other is the wave-induced surge force. Table 1 shows the resistance in calm water for three speeds, 10.59, 17.65 and 24.71 knots respectively. Table 2 shows the resistance components and wave-induced surge forces in regular following waves at different forward speeds. The ratio of the wavelength to ship length is equal to one for the measurements presented in Table 2.

As seen, the resistance components in waves at the three speeds 10.59, 17.65 and 24.71 knots are very close to the corresponding ones in calm water. On the other hand the calculated wave-induced surge force is independent of the ship speed according to (1), while the measured wave-induced surge force has considerable dependence of forward speed.

Table 2 shows that the wave-induced surge force increases with ship speed up to 17.65 knots. The wave-induced surge force at 17.65 knots is about 45% higher than at 3.53 knots. However, the wave-induced surge force gets a drop from 17.65 to 21.18 knots.

For comparison, the calculated wave-induced surge force is $7.412*10^4$ N/m per wave amplitude according to (1) while the highest measured one is $4.954*10^4$. This means that the calculated result is overestimated with more than 50% for a large range of speeds for this specific wavelength. However, the discrepancy decreases with increasing ratio of the wavelength to ship length. When the wavelength is greater than twice the ship length, the calculated wave-induced surge force becomes reasonably accurate, see Hua, Abrahamsson and Byström [4].

The effect of incident wave angle on the resistance component and wave induced surge force is shown in Table 3. The measured result is obtained at the ship speed of 17.65 knots in waves with the wavelength twice the ship length. The incident wave angles increases from 10 degrees to 80 degrees under the model measurement. The wave amplitudes are between 1.66 and 2.19 m for all the measured cases. The resistance component shown in Tab.3 is increasing with the incident wave angle.

The wave induced surge force as function of wave amplitude in waves with wavelength twice ship length is shown in Figure 2. The incident wave angle is 30 degrees and ship speed 17.65 knots. As seen, the force is weakly non-linear in relation to the wave amplitude.

Figure 3 shows the calculated and measured amplitude of the regular wave-induced sway force as function of wavelength and Figure 4 for the wave-induced yaw moment The incident wave angle is 30 degrees and the Froude number is 0.5. Relative to the results of the model tests, the calculations generally underestimates the wave induced forces and moments.

Figure 5 shows a time series of the wave-induced sway force. The incident wave angle is 30 degrees and the wavelength is twice the ship length. The Froude number is 0.5. The x-axis shows distance from the ships center of gravity to the wave crest divided by the wavelength. The non-dimensional distance 0.5 means that the ship center of gravity is located at the wave trough. Figure 6 shows the wave-induced yaw moment versus distance from the ships center of gravity to the wave crest divided by the wavelength.

Table 1. Ship resistance in calm water

Speed (knots)	10.59	17.65	24.71
Resistance (N)	2.792E4	1.196E5	1.769E5

Table 2. Surge force in regular waves with $\lambda/Lpp=1$

Speed (knots)	Wave ampl. (m)	Resistance (N)	Surge force (N)	Surge force /Wave ampl
3.53	0.692	7,31E3	2.343E4	3.386E4
7.06	0.626	1.412E4	2.226E4	3.556E4
10.59	0.628	2.881E4	2.633E4	4.240E4
17.65	0.621	1.156E5	3.077E4	4.954E4
21.18	0.627	1.632E5	2.118E4	3.378E4
24.71	0.618	1.814E5	2.354E4	3.810E4

Table 3. Surge force at 17.65 knots, regular waves $\lambda/Lpp=2$

wave angle (deg.)	Wave ampl (m)	Resistance (N)	Surge force (N)	Surge force /Wave Ampl
10	1.936	1.145E5	2.076E5	1.072E5
20	1.662	1.214E5	1.218E5	7.329E4
*30**	*1.224*	*1.157E5*	*0.993E5*	*8.897E4*
40	1.852	1.273E5	1.858E5	1.003E5
50	1.936	1.336E5	1.288E5	6.653E4
60	1.913	1.334E5	1.001E5	5.233E4
70	1.846	1.370E5	6.270E4	3.397E4
80	1.836	1.359E5	3.678E4	2.003E4

* At this case, the encounter frequency becomes near zero. There is some degree of uncertainty in this measured data.

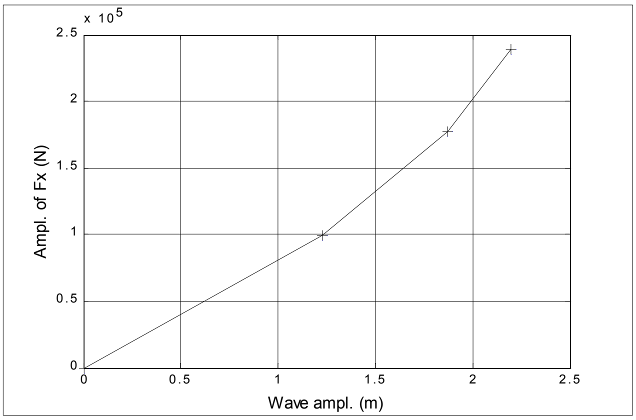

Figure 2. The wave induced surge force as function of wave amplitude in waves with wavelength twice the ship length. The incident wave angle is 30 degrees and ship speed 17.65 knots.

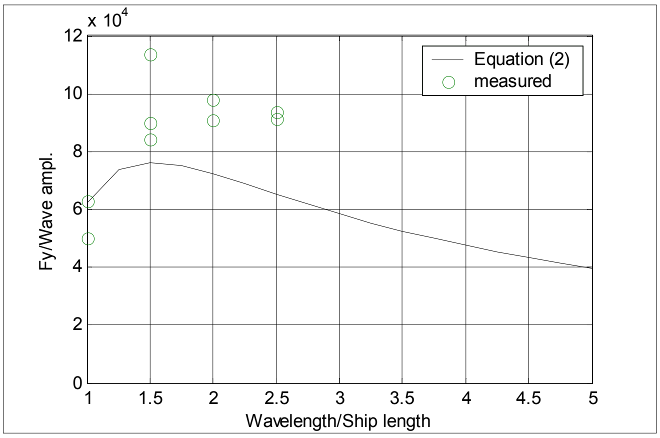

Figure 3. Wave induced sway force in quartering waves as function of wavelength. The incident wave angle is 30 degrees and the Froude number 0.5.

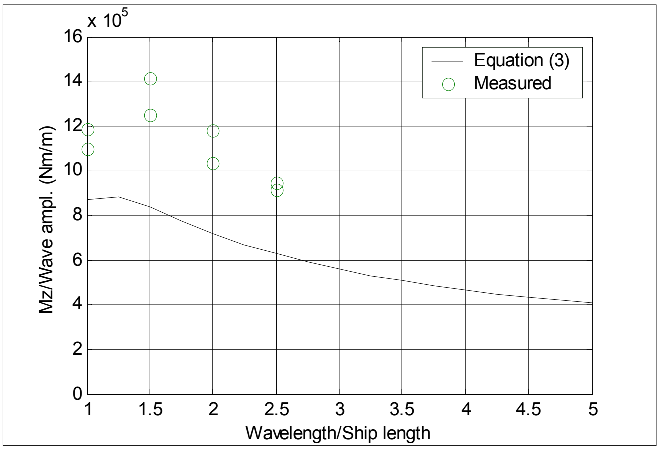

Figure 4. Wave induced yaw moment in quartering waves as function of wavelength. The incident wave angle is 30 degrees and the Froude number 0.5.

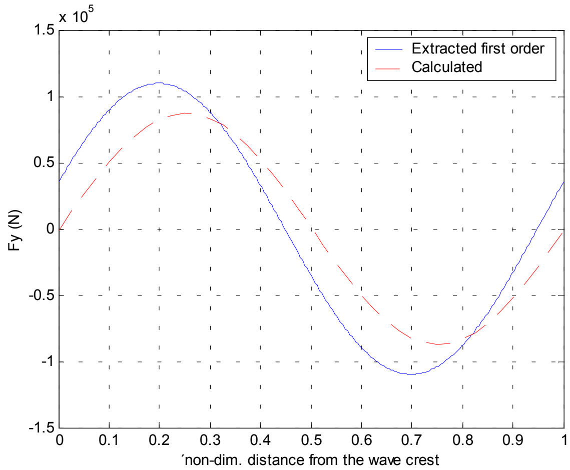

Figure 5. Comparison of the measured and calculated wave-induced sway force in a quartering wave. Fn=0.5.

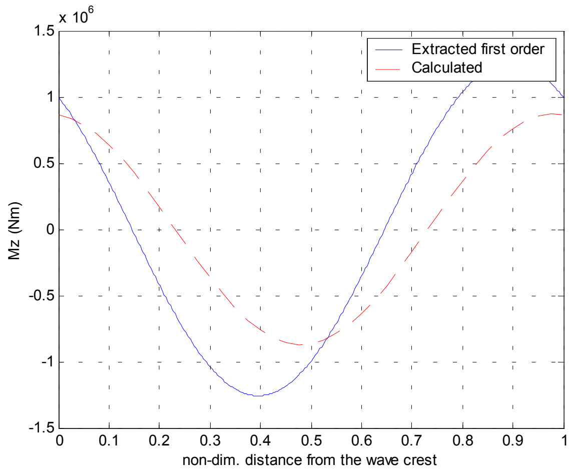

Figure 6. Comparison of the measured and calculated wave-induced yaw moment in a quartering wave. Fn=0.5.

3.2 Superposition in an irregular wave

Ocean waves are irregular. Theoretically, an irregular wave can be assumed to be the sum of a number of regular waves. Superposition principle is often adopted in probabilistic seakeeping analysis. It is therefore interesting to investigate its validity in practice for calculation of the wave-induced sway force and yaw moment in irregular waves.

A simple way to test the validity of the superposition principle is to arrange the model measurement in a wave consisting of two regular wave components. The result from the test with the two-component wave was compared to two different test runs each using one of the components of the two-component wave. The incident wave angle is 30 degrees and the Froude number is 0.3. Figure 7 shows the wave measured by the wave height meter and the wave re-constructed from the wave equation. The two regular wave components in the wave equation are also shown in the same figure. The wavelengths of the two wave components are 2 and 2.5 times the ship length respectively. We define here these two waves as Wave No.1 and Wave No.2 respectively.

Figure 8 shows the time histories of the extracted heave and pitch motion together with the original. The extracted heave and pitch motion means here that each of the two motions consists of two harmonic components corresponding to the two regular wave components. The same kinds of result for Fx, Fy and Mz are shown in Figure 9.

Based upon the extracted components of the wave, heave and pitch motion, sway force and yaw moment, it becomes easy to calculate the transfer functions of both amplitude and phase for these two waves. Table 2 shows the transfer functions of heave, pitch, Fy and Mz extracted from the two-component wave and the ones from the model tests in two regular waves corresponding to the two regular wave components. The second column 'Wave No.1' in Table 4 contains the transfer functions for Wave No.1 obtained from the wave of two regular wave components. The third colon 'Reg. Wave No.1' contains the corresponding transfer functions but obtained from a regular wave. Same results are shown in the forth and fifth colons for Wave No.2.

As can be seen, the transfer functions of the heave and pitch amplitudes are generally in good agreement from the two kind model tests. But the phase transfer functions are less good and the maximal difference in phase angle is about 12 degrees, found for the heave motion in the second wave. The discrepancies between the amplitude transfer functions of the wave-induced sway forces and yaw moments obtained from the two kind model tests are between 10% and 24% relatively, while the differences between the phase transfer functions are up to about 10 degrees.

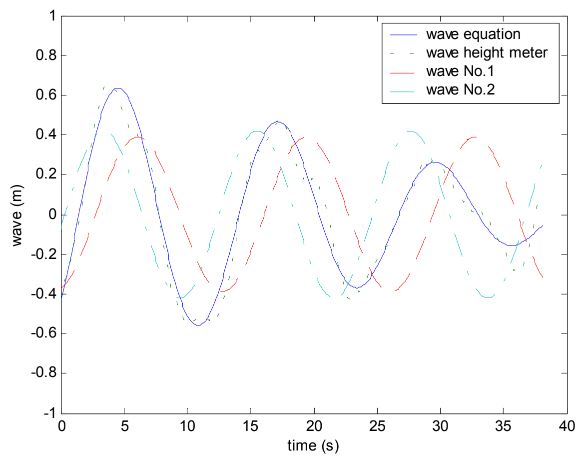

Figure 7. Wave motion given by the wave equation in comparison with the original measured by the wave height meter for a wave consisting of two regular wave components.

Table 4. Transfer functions for heave, pitch, Fy and Mz

*	Wave No.1	Reg. Wave No.1	Wave No.2	Reg. Wave No.2
Wavelength (m)	2 Lpp	2 Lpp	2.5 Lpp	2.5 Lpp
Heave amp. (non)	0.814	0.805	0.896	0.857
Heave phase (deg.)	11.96	18.62	14.76	27.07
Pitch amp. (non.)	0.832	0.770	0.927	0.821
Pitch phase (deg.)	100.8	110.8	109.0	113.6
Fy amp. (N/m)	1.2622E5	1.1006E5	1.1896E5	0.9596E5
Fy phase (deg.)	-104.8	-95.6	-93.6	-91.5
Mz amp. (Nm/m)	9.5493E5	8.6739E9	8.0015E5	6.7053E5
Mz phase (deg.)	150.5	161.3	165.3	167.4

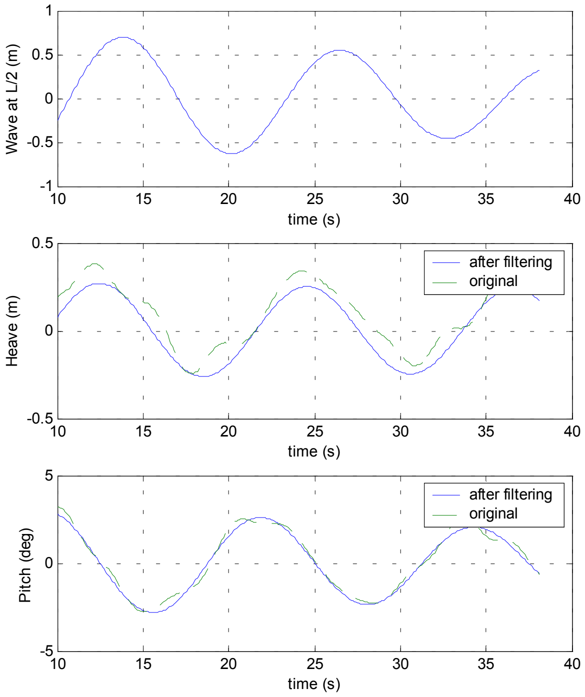

Figure 8. Wave motion at amidships and heave and pitch motion in a following wave.

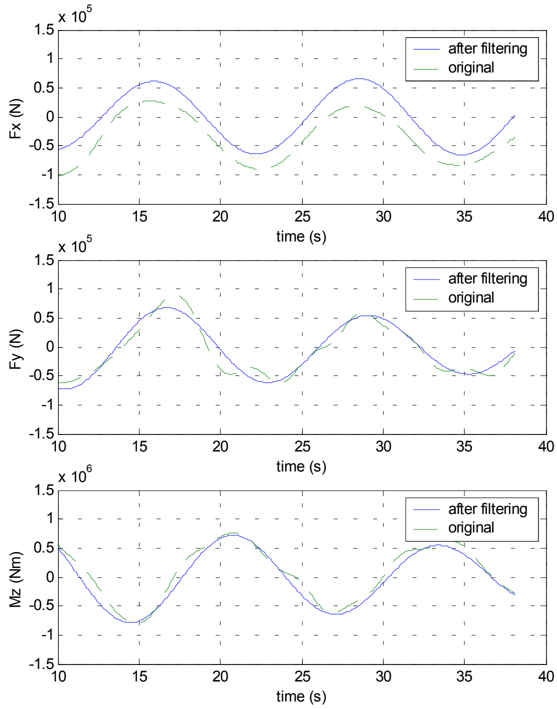

Figure 9. Wave-induced surge and sway force and yaw moment.

4. DISCUSSION

As known, the major part of the resistance of a high-speed ship in calm water is wave resistance. The effect of the non-linear free surface condition on this resistance component is considerable. Therefore, the hydrodynamic interaction between the velocity-potential of the incident wave and the speed-induced one can also be considerable and affect the total surge force on a high-speed ship in a following wave. That may explain why the calculation of the wave-induced surge force based on Froude-Krylov hypothesis becomes insufficiently accurate for short waves with wavelength near the ship length.

The linear strip approach based calculation of the wave-induced sway and yaw moment in (2) and (3) respectively are underestimated in comparison with the model measured. The discrepancies may be caused by neglecting the three dimensional effect. More probably, the cross flow due to the wave-induced particle velocity may be less accounted in the derivation so that the lifting effect becomes underestimated in (2) and (3).

The wave-induced forces and moment in following waves have been treated as a linear problem in most studies. But, the fact is that the surfing and broaching problem takes place usually in steep wave with the ratio of the wave amplitude to wavelength greater than 0.05, see Du Cane and Goodrich [1]. The feature of a steep wave comparing with a small wave is that the wave crest becomes higher, the wave front steeper and the particle velocity on the wave crest greater. Suppose the ratio of the wave amplitude to wavelength is 0.05, the height of the wave crest can then be estimated according to the second order wave theory

$$\eta(x)\big|_{x=0} = a \cdot \cos(k \cdot x) + \frac{1}{2} \cdot k \cdot a \cdot^2 \cos[2 \cdot (k \cdot x)]$$
$$= a \cdot \left(1 + \frac{1}{2} \cdot a \cdot k\right)$$
$$= a \cdot \left(1 + \frac{5 \cdot \pi}{100}\right)$$

That means that the real wave crest over the still water line is at least 15% greater that the one given by the linear wave theory. The particle velocity on the wave crest can be estimated to be least 37% greater.

At the initial phase of a surfing and broaching scenario, the ship stern is located near or in the wave crest and the main body on the wave down slope so that the ship can be pushed forward into the surfing condition. Certainly, the steep wave effect can increase the magnitude of the wave-induced forces and moment and in turn speed up the instable scenario. However, the actual test series were performed in small waves, i.e. the ratio of the wave amplitude to wavelength is small.

As shown by the result from the two kind tests, the wave-induced sway force does not exactly follow the supposition principle, nor the wave-induced yaw moment. Because of the limited number of the performed tests and technical difficulty, the non-linearity of the problem has not been solely investigated.

5. CONCLUSION

The test result shows clearly that the linear strip approach based formulas are not sufficiently satisfying for calculation of the wave-induced surge and sway force and yaw moment for the actual tested model when the wavelength is shorter than twice the ship length. More sophisticated numerical model is needed for better consideration of the hydrodynamic interaction between the velocity potential of the incident wave and speed-induced velocity potential.

Most mathematical models for ship motions in following waves are formulated based upon the separate treatments of the involved hydrodynamic phenomena associated with ship forward motion and manoeuvre motion in calm water and wave-induced motions respectively. The fact is that the broaching problem takes place while the vessel runs at high speed in a steep wave. Since the separate treatment is based on the assumption of low speed and small wave amplitude and motion magnitudes, its validity is not unquestionable.

Few similar tests have been conducted. It requires model tests of several hulls for validation of numerical models for analysis of surfing and broaching problem of fast vessels in following and quartering waves. Further tests should also be carried in steep waves with wave amplitudes least 5% of the wavelengths, since broaching-to usually occurs to ships in steep waves.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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