PROBABILISTIC ANALYSIS OF ROLL PARAMETRIC RESONANCE IN HEAD SEAS

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Abstract

The paper presents some background for an analysis of the risk of severe parametric roll motion for a ship operating in head seas. This background includes a consideration of basic probabilistic qualities of parametric roll in head seas: ergodic qualities and distributions, since these results are necessary to establish a method of prediction of extreme values. The ship motions that generate parametric excitation, heave and pitch, have also been studied in this analysis.

The post-Panamax C11 class container carrier was chosen for analysis, since a vessel of this type is known to have suffered significant damage in an incident attributed to severe parametric roll. It was shown that despite large-amplitude of motion, pitch and heave retain their ergodic qualities and normal character of distribution, while the roll motions are clearly non-ergodic and do not have a normal distribution. The analysis is built upon the numerical simulation of ship motion in head seas using Large Amplitude Motion Program (LAMP).

The paper also considers the effectiveness of anti-rolling devices in mitigating parametric roll by suppressing the parametric excitation. It was shown with numerical simulation that correctly tuned U-tube type of anti-rolling tank has to potential to reduce the occurrence of parametric roll and significantly increase the stability and safety of large modern container carriers.

1. INTRODUCTION AND BACKGROUND

The phenomenon of parametrically induced roll has been known to naval architects for over fifty years [1]. Initially, it was thought to be a phenomenon of following seas and was of significance for smaller, high-speed displacement vessels such as some fishing boats and seagoing tugs. In recent years, however, parametric roll has been observed in large seagoing ships, particularly container ships operating in head seas. A significant recent casualty is described in [1] in which a large number of containers were lost from a post-Panamax container ship caught in a severe storm in the North Pacific. A number of other container losses have occurred and are thought to be attributable to the same cause.

When a ship sails in head or following seas, the geometry of the underwater hull is constantly changing with time as a result of the wave surface along the hull as well as the pitching and heaving motions. In general, stability is greater than that in still water when a wave crest is at bow and stern, and it is diminished relative to still water when a crest is amidships. The effects are most pronounced in waves of length about equal to the ship length, and increase with increasing wave steepness. In pure head or following seas, there is no direct wave-induced rolling moment such as would exist if the waves approached the ship from any other direction.

Nevertheless, if the period of wave encounter is approximately one-half the natural period of roll, a rolling motion can exist even in the absence of a direct roll exciting moment. This is a kind of dynamic motion instability and is a consequence of the equation of rolling motion in the presence of the periodically varying stability. Specifically, rolling motion can be set up if the stability variations occur at the critical period ratio of ½ and there is some arbitrarily small initial disturbance. The disturbance always exists in natural waves because of directional spreading. Under most conditions, the roll is quickly damped out and is of no consequence.

In some conditions of high seas, however, the proper ship speed and heading and for certain hull form characteristics, the rolling motion can grow to large proportions. Capsizings have been recorded, particularly of small high-speed fishing vessels when heavily loaded in following seas. The head sea parametric roll is a more recently identified phenomenon and seems especially likely to occur in the case of large container ships as a result of certain features of their hull form that lead to especially pronounced variations in stability as the ship sails through head or following seas.

This paper describes an ongoing work carried out and sponsored by American Bureau of Shipping. Since the work is ongoing, it is not meant to offer a complete solution but rather to continue the discussion started in [1].

2. NUMERICAL SIMULATIONS

Since the parametric roll phenomenon is caused by time variation of transversal stability, the numerical simulation method must be able to adequately model the changes of geometry of immersed part of the hull due to large waves and ship motions. Following [1], the simulations were made using the Large Amplitude Motion Program (LAMP) with its "approximate body nonlinear" formulation. In this formulation, which is also referred to as the LAMP-2 approach, the hydrostatics and Froude-Krylov forces are computed over the instantaneous wetted hull surface while the perturbation potential, which includes radiation and diffraction effects, is computed over the mean wetted surface. Since it is the nonlinear hydrostatics that is "responsible" for the parametric roll phenomenon, the use of the mean waterline formulation is justified. This formulation provides a substantial speed-up of calculations as compared to the fully body-nonlinear approach, which is especially critical when working with stochastic processes.

In brief, LAMP is a time domain simulation system based on a 3-D potential flow panel solution of the wave-body interaction problem and incorporating flexible models for control systems, green-water-on-deck, viscous forces, and other effects. The latter feature is very important for adequate modeling of parametric resonance, since the viscous roll damping model can be tuned based on the results of the model test [1]. As was shown in [1], roll damping, including nonlinear terms in roll, is a key factor in predicting roll amplitude in the regime of parametric resonance.

A more complete description of the LAMP System is given in another paper at this conference [2], while [3] and [4] contain details on its theoretical background. Reference [5] describes recent developments and applications of the LAMP System.

3. SHIP CONFIGURATION

The ship chosen for the present analysis is the C11 class container carrier that lost many containers in a casualty involving large roll motions in severe head seas, and was the subject of the experimental and numerical study described in [1]. Fig. 1 presents a general view of her hull geometry.

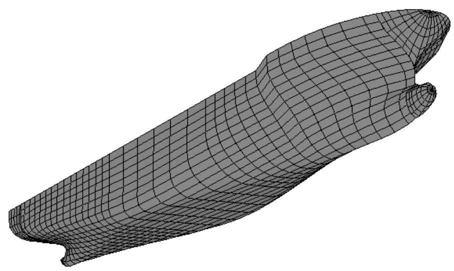

Fig. 1. Hull of post-Panamax container carrier

The ship has large bow flare and an overhanging stern, exactly the geometry features that can invoke parametric roll. The large bow flare and overhanging stern result in significant changes to the waterplane as the ship pitches and heaves in waves. When the encounter period is close to ½ of the natural roll period, these changes provide the parametric excitation leading to a rise of roll motions [1]. How large the roll becomes is determined by nonlinear factors, including the shape of the *GZ* curve and nonlinear roll damping. That is why body nonlinear hydrostatics and a correct model of roll damping are critical for evaluating extreme parametric roll motions. Here an empirical model of roll damping, tuned and validated with experimental results, is used [1].

4. PARAMETRIC ROLL IN REGULAR WAVES

Before proceeding with parametric roll of a ship, we would like to consider the Mathieu equation, which is the simplest mathematical model of parametric resonance. It is a linear ordinary differential equation with periodic coefficient

$$\frac{d^2\phi}{d\tau^2} + (p + q\cos\tau)\cdot\phi = 0 \qquad (1)$$

where p is related to the square of the ratio of the forcing frequency to the natural frequency and the parameter q plays the role of the amplitude of the parametric excitation. Depending on the values of parameters p and q, the solution of Mathieu equation might be decaying, periodic (also known as Mathieu function) or rising infinitely. An example of the rising (or "unstable") solution is illustrated by phase trajectory shown in Fig. 2.

An Ince-Strutt diagram in fig. 3 shows zones where combinations of the parameters p and q in the Mathieu equation result in such an unstable solution. There are several zones of instability: the first zone starts from p=0.25 and corresponds to a natural period exactly twice the period of parametric excitation.

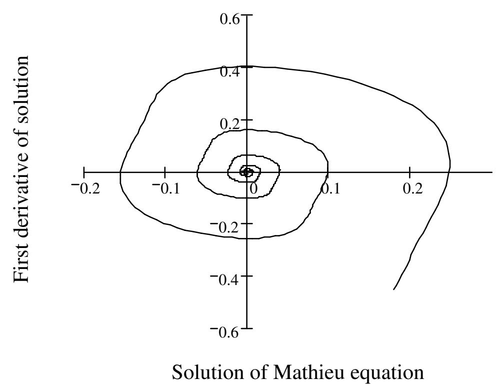

Fig. 2. Phase trajectory of unstable solution of the Mathieu equation (p=2.5, q=0.2, damping factor 0.05, initial displacement 0.01)

Adding a linear damping to Mathieu equation does not limit the amplitude of its solution. Instead it "lifts" the zone boundaries and creates a "threshold" for the amplitude of parametric excitation (q) which results in a rising solution.

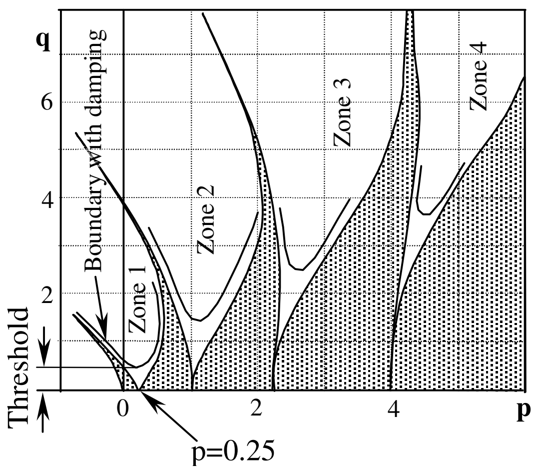

Fig. 3. Ince-Strutt diagram

This is why it is possible to use Mathieu equation for modeling the occurrence raised motions caused by parametric excitation, but not for evaluating how large the parametric oscillations might develop. To do so, nonlinear damping or stiffness terms must be added to "stabilize" the rising oscillations.

Numerical simulations for the ship in head, regular waves can be used to illustrate the mechanics of parametric roll. For waves with an encounter period away from ½ the roll natural period or with insufficiently large amplitude, the roll motion will remain small even if a fairly large roll perturbation is introduced. For waves in the proper frequency range, a sufficiently large wave amplitude will induce large rolling motion with an arbitrarily small roll perturbation.

The development of roll motion shown in Fig. 4 is typical for the parametric resonance regime: the roll motion takes relatively long time to start, increases rapid, and finally stabilizes at steady-state amplitude.

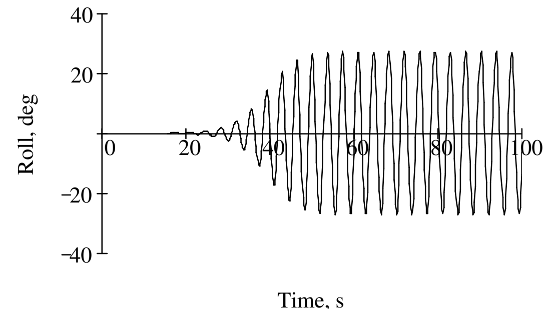

Fig. 4. Development of parametric roll in regular waves (wave amplitude 4.2 m, frequency 0.44 s^{-1}, speed 10 knots)

Fig. 5 shows a steady-state regime of parametric roll along with time-synchronized heave and pitch motions. It is very clear that roll period is twice that of heave and pitch, which corresponds to the first zone of instability in the Ince-Strutt diagram (Fig. 4).

It is peculiar that the steady-state roll motions shown in Fig. 5 are not sinusoidal. This can be seen especially clearly in Fig. 5, which plots the phase trajectory of the roll in the parametric roll regime.

The deviation from the sinusoidal form can be explained by two factors. First, there is the nonlinearity of the damping and restoring terms with roll: at an amplitude close to 30 degrees, their influence is likely to be significant (see *GZ* curve in [1]). Secondly, the parametric response, even in the simplest case described with the Mathieu equation, might be far from sinusoidal (see Fig. 3).

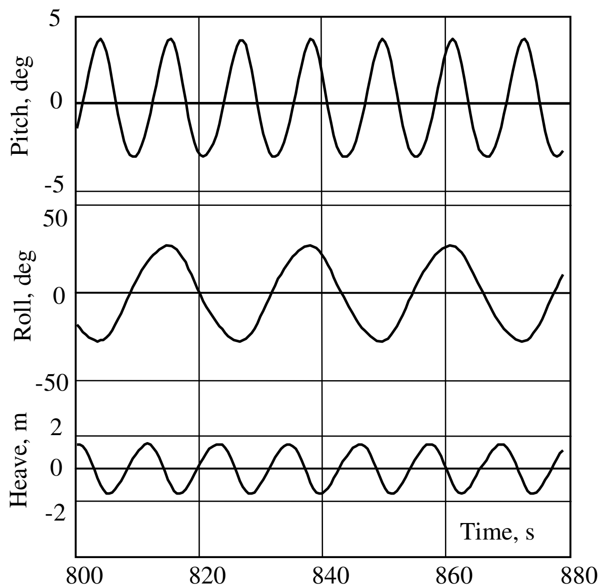

Fig. 5. Steady state parametric roll, pitch, and heave

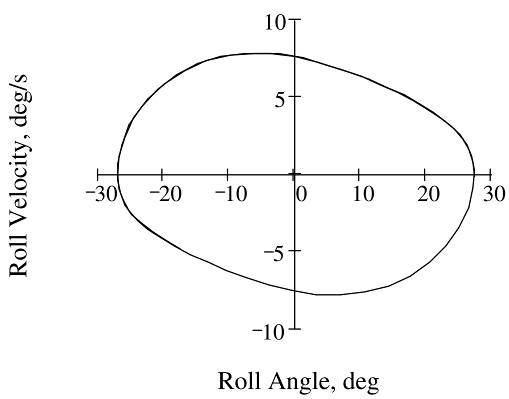

Fig. 6. Phase trajectory of parametric roll

Some deviation from sinusoidal form can be observed in the heave and pitch motions as well, but they are visually much smaller. The phase trajectories of heave and pitch are shown in Figs. 7 and 8, respectively.

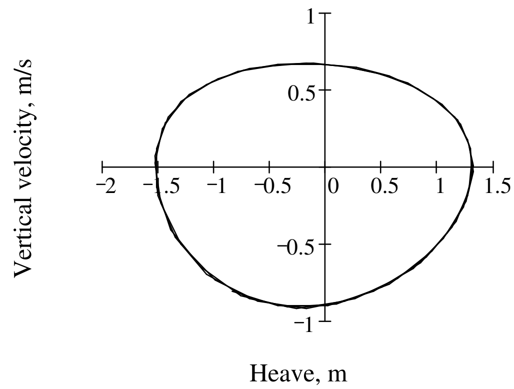

Fig. 7. Phase trajectory of heave in the regime of steady state parametric roll

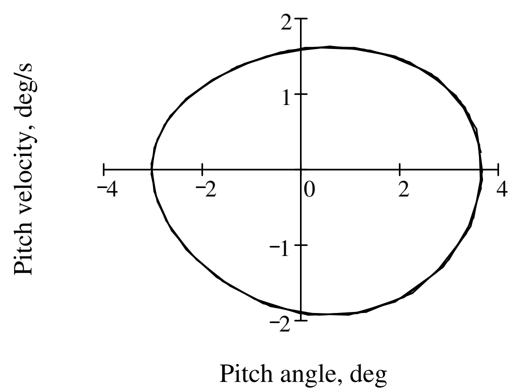

Fig. 8. Phase trajectory of pitching in the regime of steady state parametric roll

5. PARAMETRIC ROLL IN IRREGULAR WAVES

The large amplitude of the roll response encountered in a parametric resonance regime might significantly influence the probabilistic characteristics of rolling. The deviation from sinusoidal form of the steady state parametric roll, as discussed above, might also be considered a sign of non linearity.

The conventional models of ship behavior in irregular seas used by most seakeeping and stability applications are not always valid.

These models assume ergodicity (a quality of stochastic process that allows estimation of statistics using one long realization) and a normal distribution of rolling. However, these assumptions do not always have a solid background.

Observations and records have validated the assumption of normal distribution and ergodicity of waves at sea. If a ship is considered to be a linear system [6], the Weiner-Khinchin theorem states that the ship response will also be normal and ergodic. If, however, nonlinearity is involved, then this assumption no longer holds.

Numerical simulations [7, 8, 9] and model testing [9] have shown that large amplitude roll cannot be considered ergodic. Roll distribution, however, might be assumed normal for low-built ships; if a ship has high freeboard and *GZ* has S-shape, roll distribution might not be Gaussian.

5.1 Model of Irregular Waves

In the irregular wave analysis, only head long-crested seas are considered so that roll is excited only by coupling through pitch and heave. Following [1], a JONSWAP spectrum, shown in Fig. 9, was created for a wind velocity of 30 m/s, a fetch of 100nm, and peak enhancement factor (γ• • of 1.39. This spectrum produces a significant wave height (H1/3) of about 9 m.

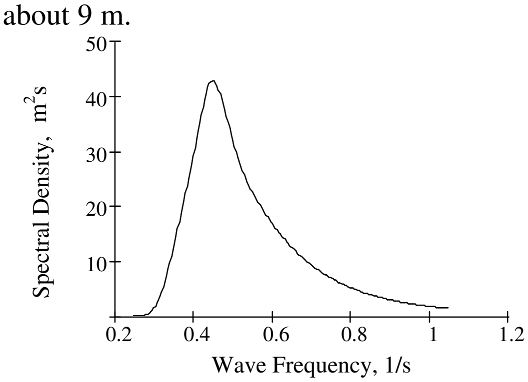

Fig. 9. JONSWAP Spectrum (γ=1.39, Wind velocity 30 m/s, Fetch 100 nm)

To create a discrete wave model, 200 wave components were created with equal frequency intervals over a range from 0.245 s^{-1} to 1.04 s^{-1}; providing 26 minutes of simulation time before the second peak of the correlation function. The incident wave elevation is defined by the well-known form of a Fourier series:

$$\zeta_w(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t + \varphi_i) \qquad (2)$$

where ω_i is the frequency set, amplitudes a_i are defined from the spectrum, and phase shift φ_i are random numbers with uniform distribution. Each realization of waves is generated with a new set of random phase shift. A total of 50 wave realizations were created and analyzed for the present study.

5.2 Predicted Roll Response

For each of the wave realizations, a 26 minute LAMP simulation was made to evaluate the ship's response while running into the waves at 10 knots. Fig 10 shows the predicted roll response for the first realization.

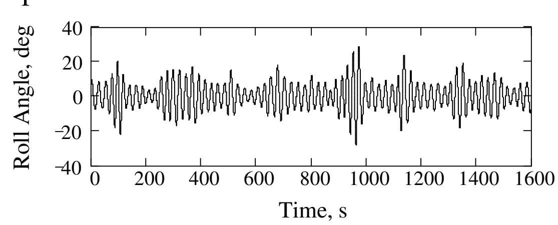

Fig. 10. Roll response in irregular long-crested waves (Realization 1)

The above result is similar to those published in [1] and show a highly pronounced group structure. The roll response to the 2nd realization (Fig. 11) shows a change in the sequence of groups and an interval with low roll angle, where parametric roll is not observed. After some time, however, parametric roll it is again excited.

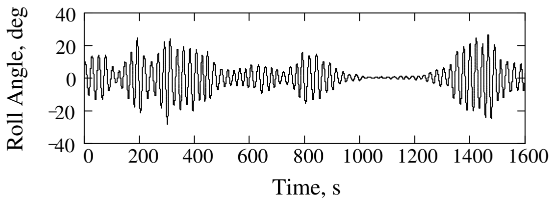

Fig. 11. Roll Response in irregular long-crested waves (Realization 2)

5.3 Stationarity

The statistical characteristics of waves at sea change with time. Formally, this means that waves cannot be considered as a stationary stochastic process. The changes of these characteristics, however, are typically slow in comparison with the wave period. This allows a hypothesis of quasi-stationarity to be introduced, assuming that the waves could be considered as a stationary process within a certain time, during which the changes of the statistics can be neglected. This time, commonly referred as "period of quasi-stationarity," lasts from half an hour to several hours. Extreme values observed during this time are defined in seakeeping analysis as "short term extremes."

Considering parametric roll within the period of quasi-stationarity, we have good reason to assume that it is a stationary process, as long as the speed, heading, and loading conditions of the ship are not altered.

5.4 Ergodicity: Visual Check

Ergodicity is only applicable to stationary stochastic processes. If the process is ergodic, its statistical characteristics could be estimated from one sufficiently long realization, rather than the whole set of realizations required for non-ergodic processes. This means that for an ergodic process, the following equality of mean values takes place:

$$m_x = \int_{-\infty}^{\infty} x f(x)dx = \int_{0}^{\infty} x(t)dt \qquad (3)$$

where $f(x)$ is probability density. The same could be written for variance estimates:

$$\begin{aligned} V_x &= \int_{-\infty}^{\infty} (x - m_x)^2 f(x)dx \\ &= \int_{0}^{\infty} (x(t) - m_x)^2 dt \end{aligned} \qquad (4)$$

Consider a number of realizations for the same stochastic process. As a consequence of equations (3) and (4), statistical characteristics estimated for different realizations of the ergodic process must be essentially the same. So, if the statistical characteristics are evaluated cumulatively over time, they would form a set of converging curves. Such a set is shown in Fig. 12, which plots the variance estimate of wave elevation at the fixed origin for a number of different realizations.

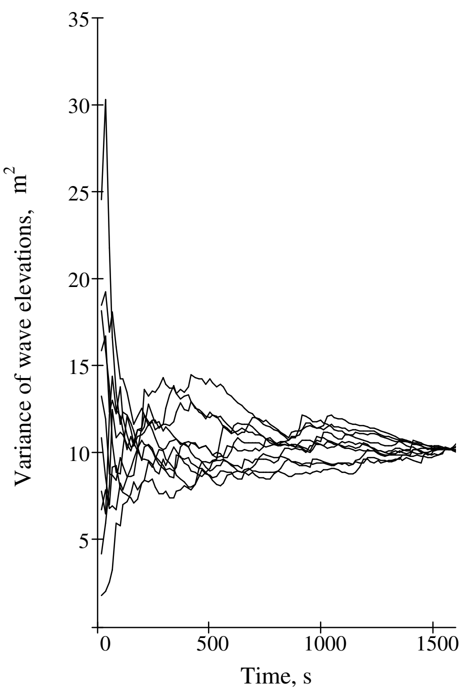

Fig. 12. Variance estimate for wave at fixed origin

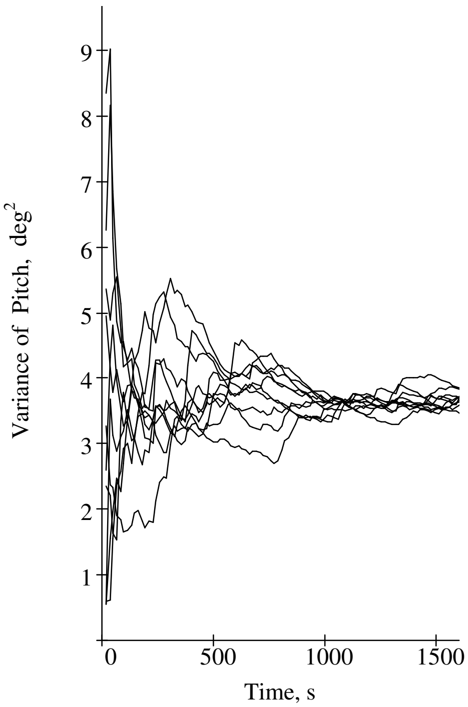

Fig. 13. Variance estimates for pitch

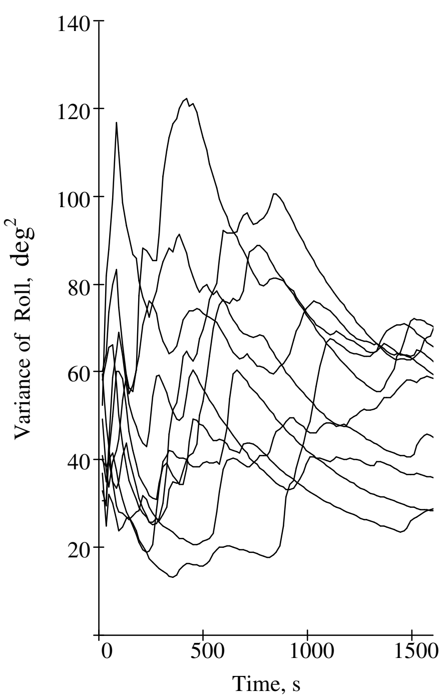

Fig. 14. Variance estimates for roll in parametric regime

Fig. 12 clearly shows the tendency of convergence. Analogous behavior was observed for the encounter waves, heave and pitch motions, and heave and pitch velocities. For example, Fig 13 shows the behavior of variance estimates for pitch.

A very strong tendency to converge has been found for mean values of all studied processes, including roll angle and roll velocity in the regime of parametric resonance

Estimates of variances in roll angle and velocity, however did not converge (see Fig. 14), which might be considered as sites of non-ergodicity.

5.5 Ergodicity: Confidence Intervals

The mean values and variances calculated with any finite data volume are only estimates. These estimates actually are random numbers, because deviation from the theoretical value is random. Therefore, there is always a possibility that a difference between roll variances estimated on different realizations is caused by finiteness of available data. To evaluate the likelihood that observed behavior is (or is not) caused by statistical uncertainty, confidence intervals were used, following a similar analysis applied in [10] and [11].

The confidence interval is a range that contains the true value of mean value, variance, or any other probabilistic characteristic, with a given confidence probability β. Here β=0.9973 is used.

Following standard statistical procedure, the distribution of the random deviation of the estimated value from the true value is assumed to be Gaussian (this does *not* imply Gaussian distribution for the analyzed stochastic process). Then the confidence interval half-width for estimate Z (mean value or variance) can be calculated as:

$$\Delta Z = P_{inv}(\beta, m[Z], V[Z]) \qquad (5)$$

where P_{inv} is the inverse Gaussian cumulative probability, m[Z] is the mean value of the estimate, and V[V] is the variance of the estimate.

As can be seen from equation (5), in order to calculate the width of the confidence intervals, it is necessary to evaluate the mean value and variance of the statistical characteristics, which are the mean values and variances of each realization. To avoid confusion, all estimates of realizations bear a subscript r and all estimates of estimates are marked with a tilde (~) above.

It is proven in mathematical statistics that the mean value is the estimate of mean value for itself. The variance of the mean value is related to the estimated variance of the realization and the number of points:

$$\begin{aligned} \tilde{m}[m_r] &= m_r \\ \tilde{V}[m_r] &= V_r \cdot N_r^{-1} \end{aligned} \qquad (6)$$

where m_r and V_r are mean value and variance estimated for one realization and N_r is the number of points in the realization.

The mean value and variance of the variance estimate can be calculated as follows:

$$\begin{aligned} \tilde{m}[V_r] &= V_r \\ \tilde{V}[V_r] &= \frac{M_{4r}}{N_r} - \frac{(N_r - 3)\cdot V_r^2}{N_r(N_r - 1)} \end{aligned} \qquad (7)$$

where M_{4r} is the fourth central statistical moment of the realization. This value can be estimated directly from the realization; provided a relatively large number of data points are available (the accuracy of the estimation of the 4th moment on a limited amount of statistical material is low):

$$M_{4r} = \frac{1}{N_r}\sum_{i=1}^{N}(X_i - m_r)^2 \qquad (8)$$

where X_i are wave elevations or ship motion displacements obtained from the simulation. Each realization was simulated with 10,000 points, an amount that can be considered large enough for reasonable evaluation of the 4th moment.

The confidence intervals for each realization are shown in Figs. 15-18. Each figure contains two diagrams: one for mean value and one for variance. Each diagram shows an estimate for each realization with corresponding confidence interval. When the confidence intervals for different realizations have an overlap, the difference between realization estimates might well be treated as statistical error. When such overlap does not exist, the likelihood of the above hypothesis can be rejected.

As clearly seen in Figs. 15-18, the confidence intervals overlap for all mean values but do not overlap for the estimates of roll variances. These results give the visual impression that, in the regime of parametric rolling in head seas, roll is not ergodic, despite the fact that the ship motions related to parametric excitation (heave and pitch) seem to be ergodic.

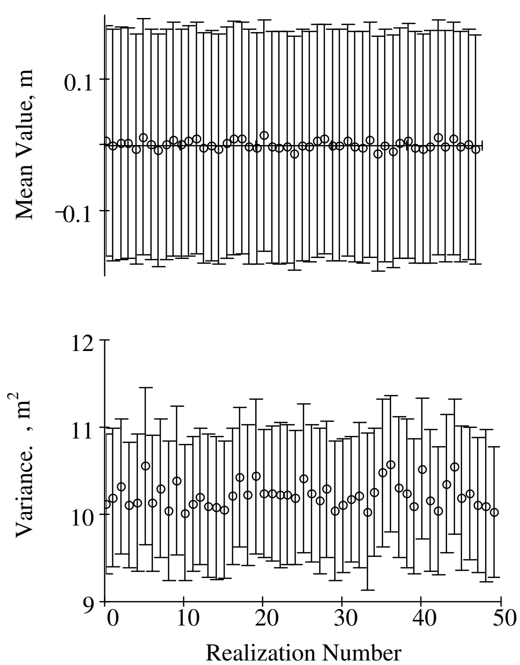

Fig. 15. Realization estimates and confidence intervals for wave at fixed origin

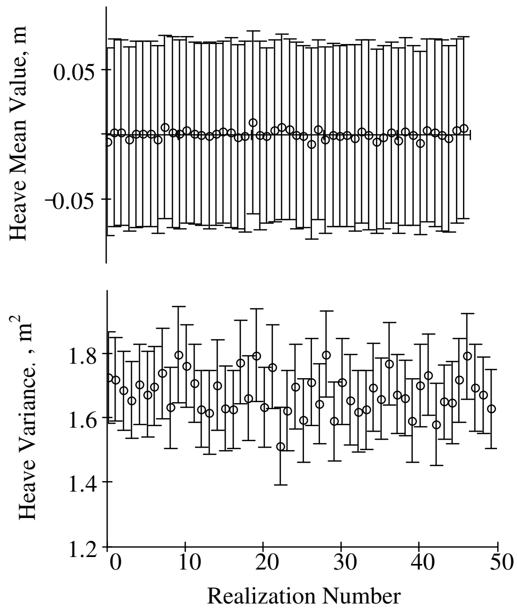

Fig. 16. Realization estimates and confidence intervals for heave motion

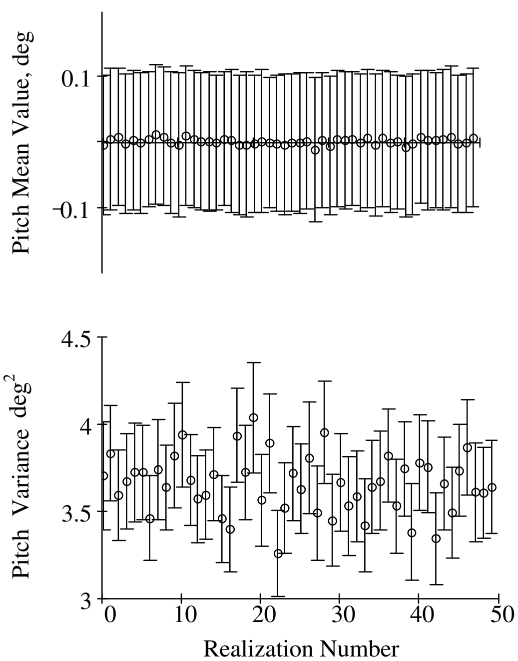

Fig. 17. Realization estimates and confidence intervals for pitch motion

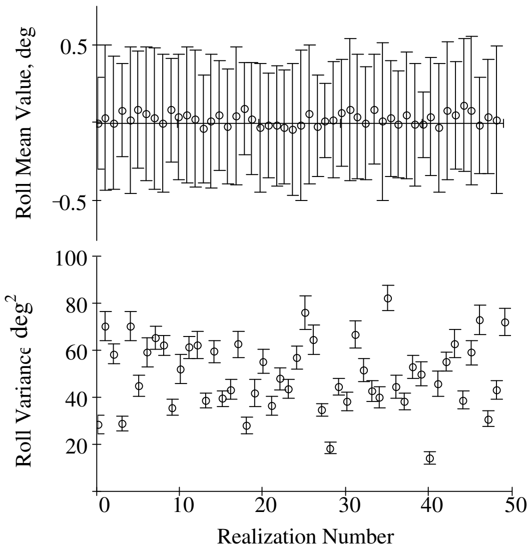

Fig. 18. Realization estimates and confidence intervals for roll motion

5.6 Quantification of Ergodicity

Comparing overlaps of variances of waves, heave, pitch, and roll in Figs. 15-18 or converging tendency of waves, pitch, and roll in Figs. 12-14, it is natural to observe that the wave elevations are "most ergodic", pitch and heave are "less ergodic", and roll is not ergodic at all. Following [10], we tried to quantify this observation.

The absence or presence of ergodicity stated above is based on the difference in variances estimated on different realizations. Considering these estimates as random values, differences between them can be characterized with their variance, which is actually a measure of dispersion.

Considering variance estimated on different realizations as random values, it is possible to express their variance as:

$$V[V_r] = \frac{1}{n-1}\sum_{j=1}^{n}(V_{rj} - V_e)^2 \qquad (9)$$

where V_{rj} is the variance of j-th realization, n is the number of realizations, and V_e is the mean value of realization variance estimates

$$V_e = \frac{1}{n}\sum_{j=1}^{n} V_{rj} \qquad (10)$$

As is obvious from equation (10), V_e has a meaning of variance estimated over the entire set of realization and does not depend on whether the process is ergodic or not.

Variance of the variance (equation (9)), however, is not exactly the same as that defined with equation (7). The latter one characterizes the dispersion of the estimate due to the finite number of points in realization, while the value defined by equation (9) is related to all realizations and presumes formal treatment of each estimate as a separate random number not related to the amount of statistical data in each realization.

The relative measure of the dispersion due to non-ergodicity could therefore be expressed as:

$$k = \frac{\sqrt{V[V_r]}}{V_e} \cdot 100\% \qquad (11)$$

This value characterizes dispersion of the realization estimates around the ensemble estimate and can be used as the criterion for practical non-ergodicity. Table 1 shows the computed values of this criterion for the results of the 50 numerical simulations.

The nearly ten-fold increase in the value of practical non-ergodicity criterion for parametric roll and roll velocity in comparison to the values for pitch and heave (which are directly related to the parametric excitation) gives sufficient background to reject the hypothesis of ergodicity for roll and roll velocity.

At the same time, pitch and heave still can be considered as ergodic processes. The practical non-ergodicity criterion (equation (11)) can also be perceived as a relative error; 3-4 % of error can be considered as acceptable accuracy.

Table 1

Process	*Value of Criterion, %*
Wave elevation at fixed origin	1.4
Wave elevation at center of gravity of the ship (encounter wave elevation)	3.3
Heave displacement	3.8
Heave velocity	3.2
Pitch angle	4.6
Pitch velocity	4.6
Roll angle	30.5
Roll velocity	31.4

5.7 Probability Distribution

The next issue to address is probability distribution. It is widely accepted that wave elevation has normal distribution; it is attributed to the fact that many different factors influence wave generation. The Fourier series model of irregular wave (Equation 2) reproduces normal distribution very well. Fig. 19 shows a histogram of wave elevation at the global origin, calculated from all 50 realizations used in this study. On top of this is plotted the theoretical Gaussian distribution calculated from the mean value and variance estimate for the whole set of realizations. As shown in Fig. 19, the theoretical and statistical distributions are so close that the curve can barely be seen.

It is well known from probability theory that a linear dynamical system being excited by a normal process produces a normal response. So, the character of the probability distribution is known for linear ship motion analysis, but it is a problem for large-amplitude motions.

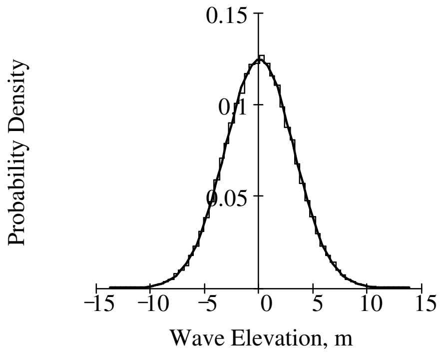

Fig. 19. Probability distribution for wave elevation at fixed origin

Previous studies of large-amplitude roll motion in beam seas [7, 8] (see also review [9]) have shown that the nonlinear roll response might be normal or not normal, depending on the shape of the nonlinear term in the most statistically significant range.

The heave and pitch motions, despite being calculated with nonlinear hydrostatic and Froude-Krylov forces (LAMP-2 formulation), do not show any visible deviation from Gaussian distribution (see Figs. 20 and 21).

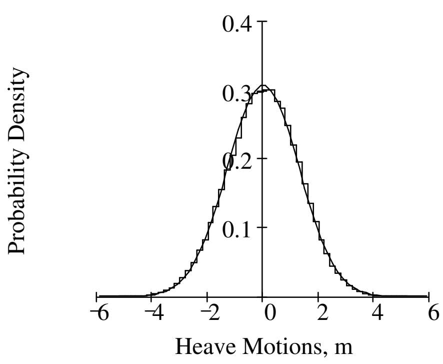

Fig. 20. Probability distribution of heave motions

The fact that pitch and heave in this case are nonlinear is beyond a doubt: the very reason for parametric roll is the changing the geometry of the hull's wetted portion, which can be reproduced only with nonlinear heave and pitch. However, this nonlinearity did not create significant non-ergodicity nor a deviation from Gaussian distribution.

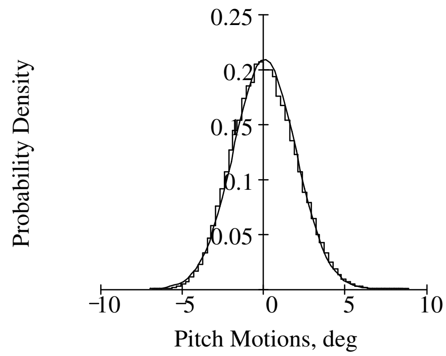

Fig. 21. Probability distribution of pitch motions

The distribution of the roll response from these realizations is shown in Fig. 22, and it is quite far from normal: the peak of the distribution is significantly sharper.

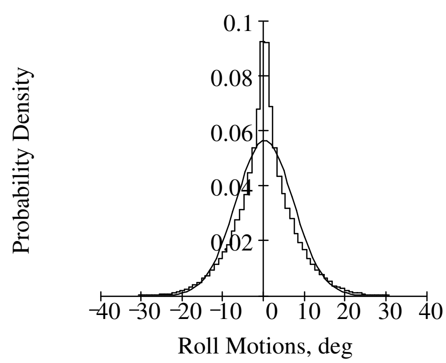

Fig. 22. Probability distribution of roll motions

The reason why rolling does not follow Gaussian distribution might be sought in two directions. First, it could be just inherent roll nonlinearity expressed in damping and *GZ* curve; a similar shape of roll distribution was observed in beam seas [7, 8]. Secondly, it could be the shape of parametric roll time history (see Fig. 3) and the stronger-than-usual group structure that contributes to the parametric roll distribution's deviation from Gaussian.

6. Probabilistic Qualities of Parametric Roll Response in Head Seas: Possible Applications

One of the principles of presenting scientific evidence suggests that any number of examples that agree with the theory would not prove it, but it's enough to present one that disagree to reject it. The study described above serves as such an example. It shows that any method developed to evaluate random values or probabilities of events related with parametric roll in head seas cannot use the assumption of ergodic or Gaussian character for this stochastic process.

One possible approach toward developing probabilistic methods for parametric roll would be to explore the Monte-Carlo extension of conventional frequency domain methods. To proceed that way, there must first be found a reliable way to calculate parametric excitation based on hull geometry and mass properties. Using the spectra of heave and pitch motions already known from frequency domain calculations, it would be possible to present parametric excitation in a form of Fourier series:

$$f(t) = \sum_{i=1}^{N} f_{ai} \cos(\omega_i t + \varphi_i + \varepsilon_i) \quad (12)$$

Here fai is an amplitude of irregular parametric excitation calculated from the pitch and heave spectra with an appropriate response amplitude operator (RAO), •i is phase shift between parametric excitation and the heave and pitch motion, and •i is a random phase with uniform distribution.

The excitation in form (12) can be substituted into an ordinary nonlinear differential equation which approximately describes roll. This equation (which is a particular case of Hill equation) can be solved with Monte-Carlo method, producing statistics of roll extremes without any particular a priori assumption of distribution. The ergodic assumption also can be avoided here, as it always possible to generate as many realizations as are needed to achieve the required statistical accuracy of the parametric roll response. With the computer power available today, such calculations would be trivial, though more sophisticated simulations, like LAMP, will be necessary for validation.

Roll extremes with small probability might be extrapolated by fitting a theoretical distribution to the generated statistics.

The accuracy of such an approach will be somewhat dependent on the reliability of the roll damping terms, but it does seem to be possible to avoid this problem anyway.

At the same time, methods based on the group structure of waves and envelope presentation of parametric excitation, like [12, 13] might be very promising and deserve special attention.

7. PARAMETRIC ROLL AND ANTI-ROLL SYSTEMS

With the growing awareness of the parametric roll phenomenon, there is a corresponding growing interest in the use of roll control devices to mitigate parametric roll. Any technical capability to predict extreme values of parametric roll and evaluate corresponding risk will, for completeness, need to consider such systems. Similarly, the proper design of an anti-roll system will need to consider the system's impact on the extreme value responses. In the present work, we are working toward both of these goals by incorporating engineering level models of anti-roll systems into our nonlinear simulation tools.

Since parametric roll, like all parametric oscillations, has an excitation threshold that must be overcome in order for the phenomenon to exist, a natural way to mitigate parametric roll would be to decrease the excitation below the threshold by creating an opposing roll moment. The opposing roll moment could be generated by anti-rolling fins, water motion in tanks, moving mass systems, or rudder deflection and could be actively controlled or passive systems.

The present study considered a large, modern container ship fitted with a passive, U-tube anti-rolling tank. The ship in this study is similar to, but somewhat larger than, the C11 class container ship studied above.

7.1 Model of Anti-Rolling Tank

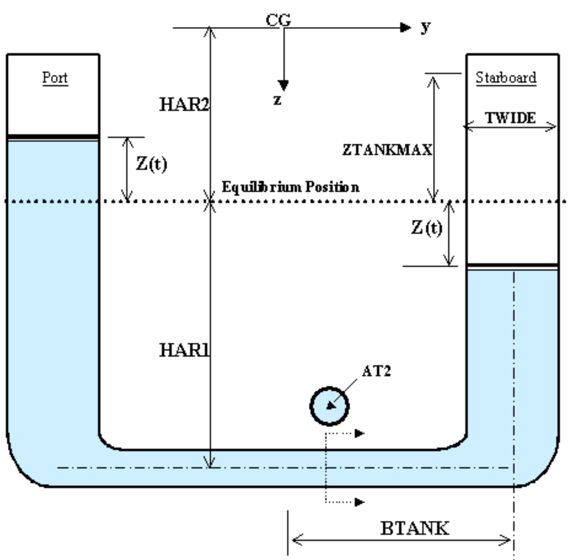

Fig. 23. Schematic of an anti-roll tank

A schematic of the anti-roll tank is shown in Fig. 23. In order to tank's effect on the roll motion, a model has been incorporated into LAMP that solves for the 1-DOF fluid motion in the U-tube tank and determines the coupled nonlinear 6-DOF forces acting on the ship. The tank model runs concurrently with the wave-body hydrodynamics solver in the time domain.

The tank model includes expressions for the shear stress on the tank walls and energy losses in the elbows, but does not account for "sloshing" in the vertical columns themselves. The theoretical description of the model can be found in [13].

7.2 Effectiveness of Anti-Rolling Tank

A series of calculations were performed to determine the effectiveness of the passive anti-roll tank system in reducing the ship's susceptibility to parametric roll. Fig. 24 shows the predicted maximum roll angles for this containership in regular head seas as a function of encounter frequency. The calculation for the roll response at each frequency is similar to that shown in Fig. 2 above, except that a large (5 degree) initial roll perturbation was used so that the steady state parametric rolling could be reached quickly.

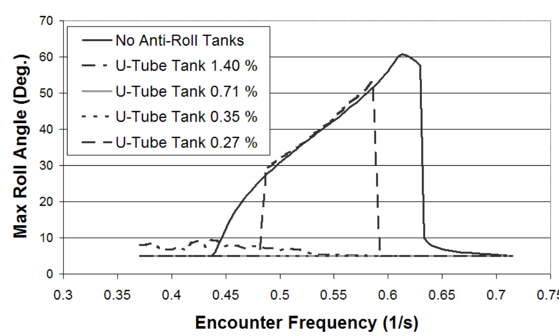

Fig. 24. Parametric Roll Response with different volume of anti-roll tank

The range of encounter frequency for which this particular ship might be susceptible to parametric roll is indicated by the curve for the case with no anti-roll tanks. The natural roll frequency for the ship in this case is 0.251 rad/sec, which corresponds to a 25 second period, so parametric rolling would be expected near an encounter frequency of 0.502 rad/sec, which is clearly shown in the results. This ship yielded a wide range of encounter frequencies where parametric roll is predicted, indicating that it is very susceptible to parametric roll.

Additional curves plotted in Fig. 24 show the response for the ship with a single passive anti-roll tank where the tank mass is equivalent to 0.27, 0.35, 0.71, and 1.4 percent of the ship's displacement. In each case, the natural period of the tank system is designed to be equal to the ship's natural roll period of 25 seconds. The 0.27% case shows a small reduction in the bandwidth where parametric roll occurs, but there is still a region of large roll angles. Both the 0.35% and 0.71% cases show virtually no roll larger than the initial 5-degree roll. The 1.4% case shows a region of slightly elevated roll motion (<10 deg.) for low encounter frequencies due to the inertia of the large mass of water in the tank system, which is not in equilibrium at the beginning of the simulation. The tank fluid continues to oscillate for a short period of time until it eventually damps out as the simulation progresses.

To check the performance of the U-tube tank in extreme irregular seas, simulations were made for the ship operating at various speed and headings in short-crested seaway with a significant wave height of 11.49 m, which corresponds to sea state 8. For head seas at 10 knots the 0.71% tank system reduced the maximum roll angle from 48 degrees to less than 10 degrees.

Overall, anti-rolling systems like the U-tube anti-roll tank appear to have a great deal or promise in the mitigation of the large roll motions caused by parametric roll. However the optimization of such a system for maximum benefit at minimum cost will likely require a fairly sophisticated simulation system coupled to advanced probabilistic methods.

8. CONCLUSIONS AND COMMENTS

While this and other efforts analyzing the parametric roll phenomena are far from being completed, some preliminary conclusions can be drawn.

With the recognition of parametric roll in head seas as a significant danger for the stability of large container ships and for the safety of the cargo and people aboard, reliable methods are needed to evaluate the risk of operation.

While model tests and nonlinear numerical simulations remain the best tools for evaluating parametric roll in particular situations, approximate methods should be developed for predicting the likelihood and magnitude of large roll events for everyday engineering analysis. Certain caution, however, has to be exercised, as far as modeling assumptions are concern, since parametric roll is not ergodic and not necessarily a Gaussian stochastic process.

It may well be that a not insignificant risk of parametric roll in head seas cannot be avoided, especially for large container carriers. To mitigate possible damage resulting from extreme roll motions and decrease danger of capsizing, installation of anti-rolling devices might be considered as a safety measure. Numerical simulation can be used as a primary analysis tool in the design of such systems.

9. ACKNOWLEDGEMENTS

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