# A Study on Estimation Method of Bilge-keel component of Roll Damping for Time Domain Simulation

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**ABSTRACT**
Ikeda's estimation method is well-known as an estimation method of roll damping. This gives an equivalent linear roll damping coefficient for a periodic rolling with one frequency. Therefore, an approximate transformation is necessary in order to apply it to non-periodic or irregular rolling in a time-domain simulation. In this study, an estimation method of bilge-keel component of roll damping for time-domain simulation is proposed, and its estimated result is compared with a measured result of roll damping in irregular forced rolling.

**KEYWORDS**
bilge-keel component, Keulegan-Carpenter number, drag coefficient, transitional motion, time-domain simulation

## INTRODUCTION

It is important to evaluate stability (especially roll motion) in order to secure its safety. For accurate prediction of stability, it is significant to estimate hydrodynamic forces acting on ship with accuracy. However, it is not easy to estimate roll damping, which includes significant viscous effects.

It is well-known that there is a prediction method of roll damping proposed by Ikeda's et al., (1976, 1977, 1978). This gives an equivalent linear roll damping coefficient in frequency domain for periodic rolling. Therefore, an approximate transformation is necessary in order to apply it to non-periodic rolling in time domain simulation.

The purpose of this study is to propose an estimation method of bilge-keel component of roll damping for time domain simulation. Roll damping consists of other components, however, because the bilge-keel component accounts for most of the roll damping. First, in order to investigate the characteristics of the normal force component of bilge-keel roll damping in time domain, based on the measured result of a flat plate, an empirical formula of drag coefficient of flat plate in time domain is proposed. Moreover, according to the basic idea of Ikeda's bilge-keel roll damping in frequency domain, based on the proposed empirical formula of drag coefficient for flat plate, an estimation method of bilge-keel component of roll damping in time domain is proposed and its estimated results are compared with measured results of roll damping in irregular forced rolling.

## DRAG COEFFICIENT OF FLAT PLATE FOR TIME DOMAIN SIMULATION

In the previous paper (Katayama et al., 2010 ), the effects of transitional motion were investigated. The characteristics of the drag coefficients in uniform flow, steady oscillatory flow and under one-way acceleration are compared. The results show that in the region at Keulegan-Carpenter number smaller than 250, the drag coefficient of a flat plate under one-way acceleration is larger than that in uniform flow and smaller than that in steady oscillatory flow. Moreover, using the forced oscillating device, measurements of forces acting on a flat plate in each swing (from stop to stop) of steady oscillation from rest is also carried out. The results show that the drag coefficients from 1st swing to 3rd swing are smaller than that in steady oscillatory flow.

Based on the previous results, in this section, the drag coefficients in time domain for steady oscillation from rest is proposed.

***Empirical Formula of Drag Coefficient of Flat Plate in Steady Oscillation***

In the previous paper (Katayama et al., 2010), an empirical fomula of drag coefficient of flat plate under steady oscillation is proposed as following

$$\frac{C_{Dperi}}{C_{D0}} = (20.0e^{-1.23Kc} + 2.86e^{-0.174Kc} + 1) \times \left(0.908 + \frac{1.2}{1+1.01^{Kc}}\right) \quad (1)$$

$$(0 < Kc \le 250)$$

where $C_{Dperi}$ and $C_{D0}$ are the drag coeficient under steady oscillation and uniform flow. Keulegan-Carpenter number of steady oscillation is expressed as following

$$Kc = \frac{2\pi\, y_A}{D_P} \quad (2)$$

where $y_A$ and $D_P$ are the amplitude of oscillation and breadth of flat plate.

It is noted that the original drag coefficients $C_D$, which are used to make the empirical formula, are obtained by using the following equation

$$\frac{1}{2}\rho S C_D \int_0^T \dot{y}(t)^3\, dt = \int_0^t F(t)\dot{y}(t)dt\,, \quad (3)$$

where ρ is a density of fluid, $S$ is area of flat plate, $T$ and $2\pi/\omega$ are period of oscillation, $F(t)$ and $\dot{y}(t)$ are drag component of measured force and velocity of oscillation.

***Emprical Formula of Drag Coefficient of Flat Plate under One-Way Acceleration***

Under one-way acceleration, the drag coefficient of flat plate is also measured in the same previous paper (Katayama et al 2010). Here, the measured data are re-analyzed by the same method in the previous paper, and a new empirical formula is obtained as following

$$\frac{C_{Dacc}}{C_{D0}} = (14.3e^{-1.80Kc_d} + 4.41e^{-0.37Kc_d} - 10.4e^{-1.03Kc_d} - 0.30e^{-0.17Kc_d} + 1.0) \times \left(0.908 + \frac{1.2}{1+1.01^{Kc_d}}\right) \quad (4)$$

$$(0 < Kc_d \le 250)$$

where $C_{Dacc}$ is the drag coefficient under one-way acceleration. Moreover, in the equation, Keulegan-Carpenter number is obtained by the following equation

$$Kc_d = \frac{\pi\, y}{D_P} \quad (5)$$

where $y$ is a moving distance from the starting position where velocity of flat plate is zero. It is not that the original drag coefficients $C_D$, which is used to make the empirical formula, is obtained by using the following equation

$$\frac{1}{2}\rho S C_D \dot{y}(t)^2 = F(t)\,. \quad (6)$$

***Empirical Formula of Drag Coefficient of Flat Plate for 1st Swing in Steady Oscillation***

Using Eq.(4), the drag coefficient of flat plate for 1st swing in steady oscillation can be calculated according to the following equation

$$\int_0^{\frac{T}{4}} C_{D1} \left(\sin\omega t\right)^3 dt = \int_0^{\frac{T}{4}} C_{Dacc}(Kc_d)\left(\sin\omega t\right)^3 dt \quad (7)$$

Eq.(7) indicates energy integration for 1st swing in steady oscillation. The drag coefficient of flat plate for 1st swing in steady oscillation is obtained by following equation

$$\frac{C_{D1}}{C_{D0}} = \left\{\begin{matrix} 5.42e^{-0.23Kc} + 13.2e^{-1.25Kc} - \\ 1.96e^{-0.21Kc} - 8.72e^{-0.78Kc} + 1.0 \end{matrix}\right\} \times \left(0.908 + \frac{1.2}{1+1.01^{Kc}}\right) \quad (8)$$

$$(0 < Kc \le 250)$$

Comparison among the drag coefficients in steady oscillation, under one-way acceleration and in first swing of steady oscillation is shown in Fig.1. In this calculation, the drag coefficient in uniform flow $C_{D0}$=1.26, which is measured value by Katayama et al (2010) for the same flat plate, is used.

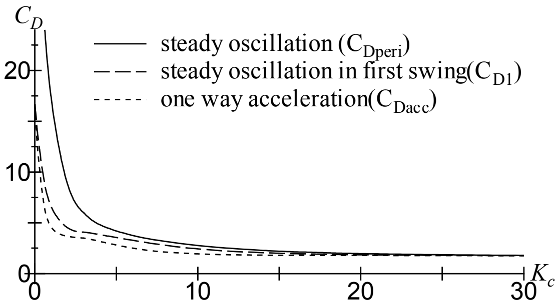


**Fig. 1: Comparison among drag coefficients in steady oscillation, in first swing of steady oscillation and under one-way acceleration vs. *Kc* number.**

### *Drag Coefficient under Transitional Condition in Periodic Oscillation*

In the previous paper (Katayama et al. 2010), it is confirmed that the drag coefficient in each swing of steady oscillation from rest is gradually increasing, and after the 4th swing the drag coefficient becomes constant. These characteristics are also expressed by an empirical formula in the paper. In order to apply it to time domain estimation of drag coefficient, the following equation is proposed:

$$\frac{C_{Dacc(n)}}{C_{D0}} = \frac{C_{Dacc}}{C_{D0}} \cdot \left[ 1 + \left( \frac{C_{Dperi}}{C_{D1}} - 1 \right) \cdot \frac{n-1}{3} \right] \qquad (9)$$

where $n$ is the number of swing ($n$ = 1,2,3 and 4). In Eq.(9), it is assumed that the drag coefficient in first swing is $C_{Dacc}$ and the drag coefficient $C_{Dacc(n)}$ is increased from the 1st swing to the 4th swing according to the ratio of $C_{Dperi}$ and $C_{D1}$.

Fig.2 shows comparison among the measured and the two estimated drag forces in time domain under steady oscillation in trasitional condition. The first estimation method uses the drag coefficient, which changes in every time step, depending on $Kc_d$ number expressed by Eq.(5), and the second estimation method uses a constant drag coefficient depending on $Kc$ number expressed by Eq.(2). In the both estimation method drag forces are calculated by Eq.(6). In the measurerment, the drag component is obtained by the same way in the paper (Katayama et al. (2010)) and its detail is shown in apendix. The result by the first estimation method is better agreement with measured results. In the lower figure of Fig.2, the results of the first estimation method shows that the drag coefficient changes in time step and its value is maximum at the start position of a swing. As the results, the estimated result of the first estimation method become larger than one of the second estimation method.

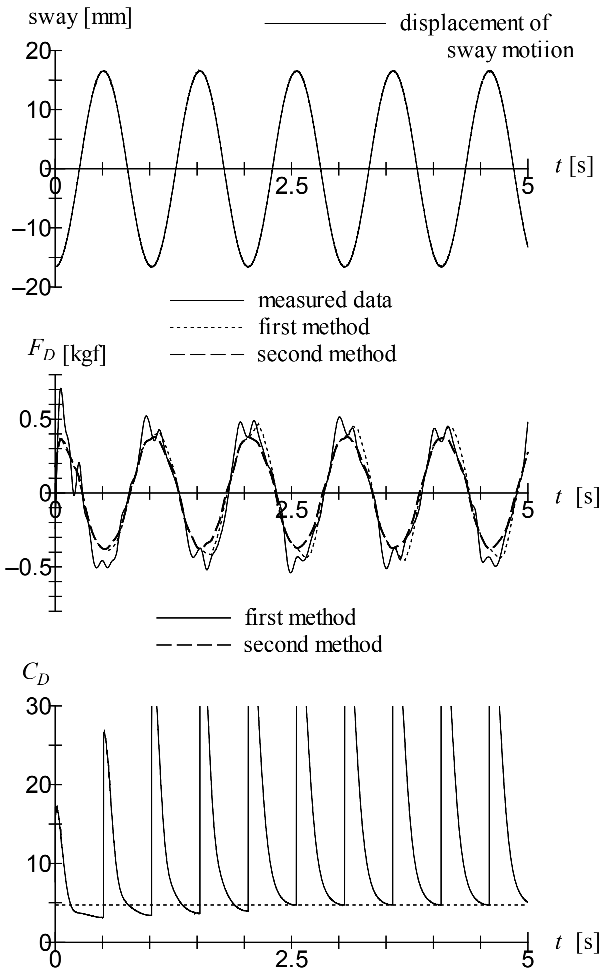


**Fig. 2: Comparison among drag forces under steady oscillation in transitional condition. (First estimation method uses the drag coefficient, which changes in every time step, depending on $Kc_d$ number expressed by Eq.(5). Second estimation method uses a constant drag coefficient depending on *Kc* number expressed by Eq.(2).)**

Fig.3 shows comparison of the drag forces under un-sinusoidal oscillation. In the figure, there are two estimated results as same as shown in Fig.2. It is noted that the drag coefficient of the second estimation method does not consider the memory effects, which is expressed by Eq.(9), and the drag coefficient is estimated at $n$=4 in Eq.(9). The result of the first method shows better agreement with measured result, and the amplitude of drag force of the first estimation method is almost same as measured one because of the change of drag coefficient. However, drag coefficient is estimated at $n$=4 in this case, then memory effects of drag coefficients are not clear, and more detailed measurement is desired.

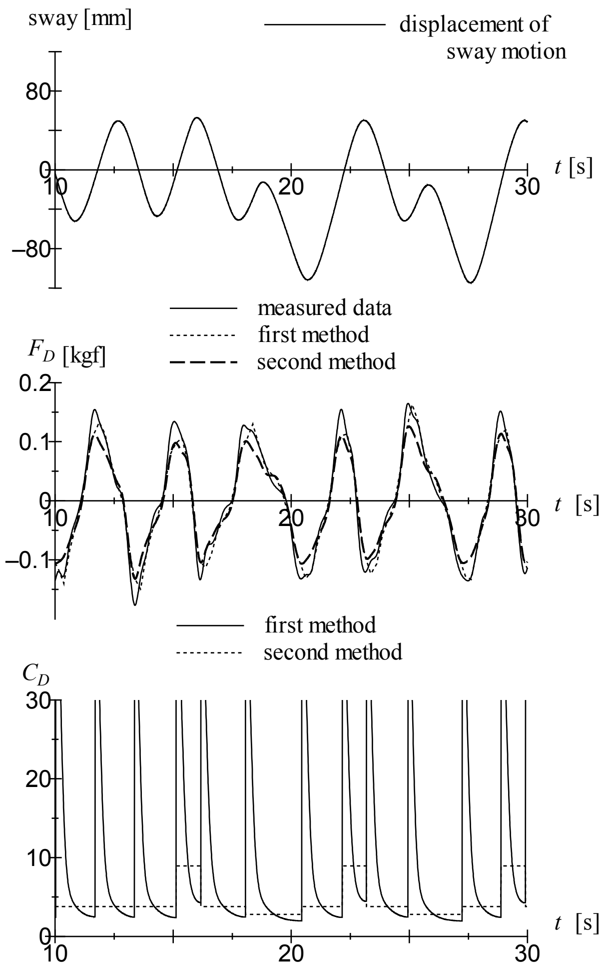


**Fig. 3: Comparison amang drag forces under un-sinusoidal oscillation. (First estimation method uses the drag coefficient, which changes in every time step, depending on $Kc_d$ number expressed by Eq.(5). Second estimation method uses a constant drag coefficient depending on $Kc$ number expressed by Eq.(2).)**

## BILGE KEEL ROLL DAMPING FOR TIME DOMAIN SIMULATION

Ikeda's estimation method is one of discrete type prediction method. It is composed of wave, lift, frictional, eddy and appendages contributions (bilge keel, skeg, rudder etc). In this section, an estimation method of the bilge-keel component in time domain is proposed to refer the basic concept of the bilge-keel component of Ikeda's estimation method (refer to Appendix). Moreover, the comparison between estimated and measured results is shown.

In periodic rolling, a drag coefficient acting on bilge-keels is expressed as Eq.(A3) in Ikeda's method. This formula is obtained by fitting the measured drag coefficients of a flat plate under steady oscillation, which are shown in Fig.4. In this figure, horizontal axis is Keulegan-Carpenter number and vertical axis is drag coefficient of flat plate under steady oscillation. Therefore Eq.(A3) can be replaced with Eq.(1). Moreover, if Eqs.(5) and (9) is applied, the normal force component of bilge-keels can be obtained by following equation

$$M_{BKN} = \frac{1}{2}\rho\left(2\, l_{BK}\; b_{BK}\right) C_{Dacc(n)}\; l^2\; \dot{\phi}\left|\dot{\phi}\right|\, r\, f \qquad (10)$$

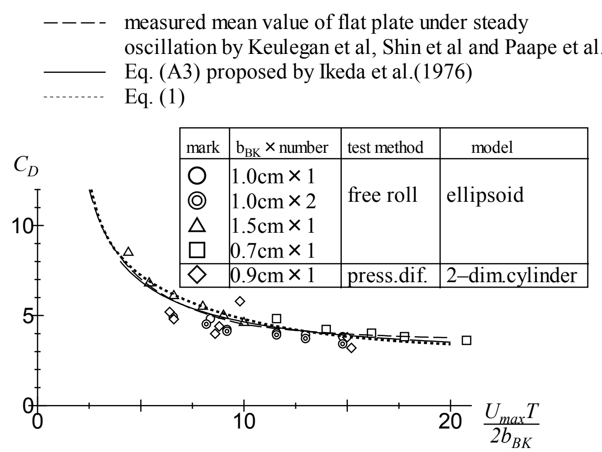


**Fig. 4 Comparisons of drag coefficients of a bilge-keel and the flat plate, those areas are same.**

Using the same manner as the normal force component of bilge-keels, the hull surface presser component of bilge-keel in time domain is obtained from Eq.(11) with replacing $C_D$ in Eq.(A8) to Eq.(9) and $K_C$ number in Eq.(A6) to Eq.(12).

$$M_{BKH} = \frac{1}{2}\rho\, l^2 f^2\; \dot{\phi}\left|\dot{\phi}\right| \int_G C_p \cdot l_p dG \qquad (11)$$

$$Kc = \frac{\pi\, l\, \phi}{2b_{BK}} \qquad (12)$$

Fig.5 shows the comparison between measured and estimated results. In the case, the results show that estimated results are good agreement with measured one.

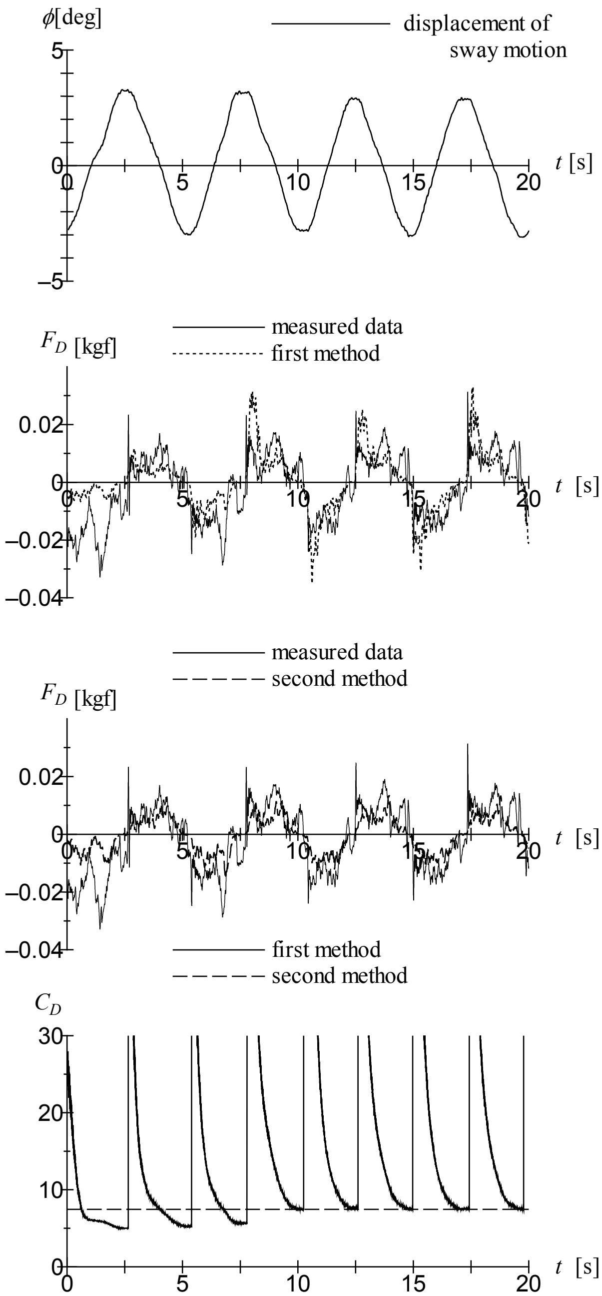


**Fig. 5: Comparison among drag forces under non-sinusoidal rolling. (First estimation method uses the drag coefficient, which changes in every time step, depending on $Kc_d$ number expressed by Eq.(5). Second estimation method uses a constant drag coefficient depending on *Kc* number expressed by Eq.(2).)**

## CONCLUSIONS

In this paper, some basic ideas on the estimation method of bilge-keel component of roll damping in time domain is poroposed. An estimated result is compared with a measuerd result and it shows good agreements depending on extremely large drag coefficient at low Keulegan-Carpenter number. However, until now, it is not necessarily clear how to consider the memory effects, which are the effects of the vortexes created by previous swings. In the future, more detailed measurement would be required.

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## APPENDIX

### *Bilge Keel Component of Original Ikeda's Estimation Method*

The bilge keel component $B_{44\mathrm{BK}}$ is composed of two components:

$$B_{44\mathrm{BK}} = B_{44\mathrm{BKN0}} + B_{44\mathrm{BKH0}} \qquad \text{(A1)}$$

The normal force component $B_{44\mathrm{BKN0}}$ can be deduced from the experimental results of oscillating flat plates (Ikeda *et al*, 1978d, 1979). The drag coefficient $C_D$ of an oscillating flat plate depends on the $K_C$ number.

$$Kc = \frac{\pi\, l\, \phi_A}{b_{BK}} \qquad \text{(A2)}$$

From the measurement of the drag coefficient, $C_D$, from free roll tests of an ellipsoid with and without bilge keels, the prediction formula for the drag coefficient of the normal force of a pair of the bilge keels can be expressed as follows:

$$C_D = 22.5\left(\frac{b_{\mathrm{BK}}}{\pi l \phi_{\mathrm{a}}}\right)\frac{1}{f} + 2.4 \qquad \text{(A3)}$$

where $b_{\mathrm{BK}}$ is the breadth of the bilge keel and $l$ is the distance from the roll axis to the tip of the bilge keel. The equivalent linear damping coefficient $B'_{44\mathrm{BKN0}}$ is:

$$B'_{44\mathrm{BKN0}} = \frac{8}{3\pi}\rho\, l^3 \omega_e \phi_{\mathrm{a}} b_{\mathrm{BK}} f\, C_D \qquad \text{(A4)}$$

where $f$ is a correction factor to take account of the increment of flow velocity at the bilge, determined from the experiments:

$$f = 1 + 0.3e^{\{-160\,(1-\sigma)\}} \qquad \text{(A5)}$$

From the measurement of the pressure on the hull surface created by the bilge keels, it was found that the coefficient $C_P^+$ of pressure on the front face of the bilge keels does not depend on the $K_C$ number. However, the coefficient $C_P^-$ of the pressure on the back face of a bilge keel and the length of negative-pressure region do depend on the $K_C$ number. From these results, the length of the negative-pressure region can be obtained as follows:

$$S_0 / b_{\mathrm{BK}} = 0.3\left(\frac{\pi l \phi_{\mathrm{a}}}{b_{\mathrm{BK}}}\right) f + 1.95 \qquad \text{(A6)}$$

assuming a pressure distribution on the hull as shown in Fig.A1.

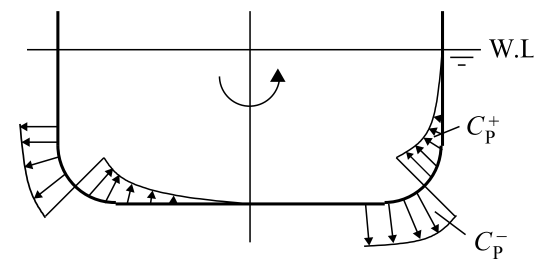


**Fig. A1: Assumed pressure distribution on the hull surface created by bilge keels (Ikeda et al 1976)**

The roll damping coefficient $B'_{\mathrm{BKH0}}$ can be expressed as follows (Ikeda *et al*, 1987):

$$B'_{44\mathrm{BKH0}} = \frac{4}{3\pi}\rho\, l^2 f^2 \omega_e \phi_{\mathrm{a}} \int_G C_{\mathrm{p}} \cdot l_{\mathrm{p}}\, dG \qquad \text{(A7)}$$

where $G$ is length along the girth and $l_{\mathrm{p}}$ is the moment lever.

The coefficient $C_{\mathrm{p}}^+$ can be taken approximately as 1.2 empirically. From the relation of $C_D = C_{\mathrm{p}}^+ - C_{\mathrm{p}}^-$, the coefficient $C_{\mathrm{p}}^-$ can be obtained as follows:

$$C_{\mathrm{p}}^- = 1.2 - C_D = -22.5\left(\frac{b_{\mathrm{BK}}}{\pi l \phi_{\mathrm{a}}}\right)\frac{1}{f} - 1.2 \qquad \text{(A8)}$$

The value of the integral in Eq.(A7) can be obtained as follows:

$$\int_G C_{\mathrm{p}} \cdot l_{\mathrm{p}}\, dG = d^2\left(-A_0 C_{\mathrm{p}}^- + B_0 C_{\mathrm{p}}^+\right) \qquad \text{(A9)}$$

where:

$$A_0 = (m_3 + m_4)m_8 - m_7^2$$

$$B_0 = \frac{m_2^2}{3(H_0 - 0.215m_1)} + \frac{(1-m_1)^2(2m_3 - m_2)}{6(1-0.215m_1)} + m_1(m_3 m_5 + m_4 m_6)$$

$$m_1 = R / d$$

$$m_2 = \overline{OG} / d$$

$$m_3 = 1 - m_1 - m_2$$

$$m_4 = H_0 - m_1$$

$$m_5 = \frac{\begin{Bmatrix} 0.414H_0 + 0.0651m_1^2 - \\ (0.382H_0 + 0.0106)m_1 \end{Bmatrix}}{(H_0 - 0.215m_1)(1 - 0.215m_1)}$$

$$m_6 = \frac{\begin{Bmatrix} 0.414H_0 + 0.0651m_1^2 - \\ (0.382 + 0.0106H_0)m_1 \end{Bmatrix}}{(H_0 - 0.215m_1)(1 - 0.215m_1)}$$

$$m_7 = \begin{cases} S_0 / d - 0.25\pi m_1, & S_0 > 0.25\pi R \\ 0, & S_0 \le 0.25\pi R \end{cases}$$

$$m_8 = \begin{cases} m_7 + 0.414m_1, & S_0 > 0.25\pi R \\ m_7 + 1.414m_1\left(1 - \cos\left(\frac{S_0}{R}\right)\right), & S_0 \le 0.25\pi R \end{cases}$$

where $l$ is a distance from roll axis to the tip of bilge keels and $R$ is the bilge radius. These are calculated as follows:

$$l = d\left(\left\{H_0 - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\}^2 + \left\{1 - \frac{\overline{OG}}{d} - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\}^2\right)^{1/2} \qquad \text{(A10)}$$

$$R = \begin{cases} 2d\sqrt{\dfrac{H_0(\sigma - 1)}{\pi - 4}}, & R < d \cap R < \dfrac{B}{2} \\ d, & H_0 \ge 1 \cap \dfrac{R}{d} > 1 \\ \dfrac{B}{2}, & H_0 \le l \cap \dfrac{R}{d} > H_0 \end{cases} \qquad \text{(A11)}$$

### *Measurement and Analysis of Drag Force under Forced Oscillation*

In this study, a forced oscillation test is carried out. A strut and a flat plate are fixed by a load cell (shown in Fig.A2), and it is oscillated. And the same measurement is carried out for the strut without the flat plate.

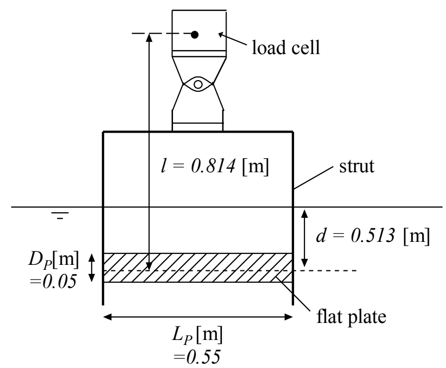


**Fig.A2: Schematic view of the experimental device.**

Drag force acting on the flat plate $D$ is obtained from subtracting the measured force of the strut without the flat plate and the inertia force (measured mass and estimated added mass) of the flat plate from the measured force of the strut with the flat plate.

$$D_P = F_{S+P} - F_S - a \cdot \left(m_P + \frac{\pi}{4}C_a \rho D_P S\right) \qquad \text{(A12)}$$

where $D_P$ is the drag force acting on the flat plate, $F_{S+P}$ and $F_S$ are the measured forces acting on strut + flat plate and only strut, $m_P$ is mass of flat plate, $C_a$ is coefficient of added mass for flat plate (in this case $C_a$=1.0), ρ is a density of fluid, $S$ is project area of strut in direction of oscillation and $a$ is acceleration determined by a time history of sway motion.