On Uncertainty in Numerical Analysis of Parametric Roll

Y. Kim, D.M. Park, T. Kim,
Seoul National Univeristy

K.H. Song
Korean Register of Shipping

ABSTRACT
In this study, a systematic study is carried out for the parametric roll of two ships, particularly aiming the observation on the sensitivity of computational results to some parameters which can affect the analysis of parametric roll. The parameters to be considered are metacentric height(GM), simulaton time-window, and the descretization of wave spectrum. To this end, a series of numerical simulation based on impulse response function approach is carried out. Based on the statistical properties of parametric roll, numerical uncertainty in computational approche is discussed.

KEYWORDS
Parametric roll; Numerical uncertainty; Sensitivty study; Nonlinear seakeeping

INTRODUCTION

The prediction of parametric rolling is one of the important tasks in ship stability. When parametric roll occurs, the roll angle is rapidly increased, and consequently this can threaten ship stability. Due to such danger, IMO recently tries to establish a regulation for the dynamic stability of ships, including parametric roll.

Many researches have proved the possibility of this large roll angle in head or following sea. Paulling and Rosenberg (1959), Nayfeh (1988) and Dunwoody (1989) introduced fluctuation of metacentric height, GM, mentioning the possibility of instabilty. Among recent nonlinear numerical computations, France *et al*. (2003) and Shin *et al*. (2004) effectively applied Rankine panel methods, showing favorable results. In other way, Spanos and Papanikolaou (2007) have introduced the impulse response function (IRF) approach.

Although numerical compuation shows favarable prediction in the occurrence of parametric roll, validation work, especially for the roll angle, still remains as a weak point. Highly nonlinear and inherent non-ergodic properties raise a doubt in reliability.

This study aims to verify the possible uncertainties which cause reliabiltiy problem in numerical computation. The selected factors include the uncertainty of GM, simulaton time-window, and the discreted frequency components in wave spectra. Numerical simulations based on impulse response function approach are carried out for two container ships, focusing on the effect of uncertatiny parameters.

COMPUTATIONAL METHOD

Recently, Seoul National University (SNU) developed two programs for nonlinear seakeeping analysis: WISH and SNU-PARAROLL. WISH has been developed under the support of several industries, aiming the development of general purpose computer program for many different variations of linear and nonlinear seakeeping problems (Kim *et a*l., 2007). On the other hand, SNU-PARAROLL has been developed mainly targeting the analysis of parametric roll. The both programs can be applied for the analysis of parametric roll (e.g. Kim and Kim, 2010), but the latter is chosen in this study because it has much less computational time for very long simulations.

The impulse response function approach, introduced by Cummins (1961) in ship motion analysis, can be expanded into nonlinear analysis by calculating nonlinear Froude-Krylov force and nonlinear restoring force on an instantaneous wetted surface (Ballard *et al*., 2003). SNU-PARAROLL is based on a hybrid method which adopts nonlinear forces for Froude-Krylov force and restoring force, while the other hydrodynamic forces are linear.

This method is derived from conversion of the frequency domain solution to the time domain. When the frequency-domain solution is known, the radiation force $F_{Rad}(t)$ can be calculated from the convolution integration of retardation function, $R(t)$, as follows:

$$F_{Rad}(t) = -M_{\infty}\ddot{\xi}(t) - \int_0^t R(t-\tau)\dot{\xi}(\tau)d\tau \tag{1}$$

The infinite-frequency added mass, M_{∞}, and the retardation function can be obtained by using pre-computed hydrodynamic coefficients. Either added mass or damping coefficient can be used to obtain the retardation function. In the present study, the function is obtained by the damping coefficient. The formulation can be written as follows:

$$M_{\infty} = M(\omega) + \frac{1}{\omega}\int_0^{\infty} R(t)\sin(\omega t)dt \tag{2}$$

$$R(t) = \frac{2}{\pi}\int_0^{\infty} b(\omega)\cos(\omega t)d\omega \tag{3}$$

Including the nonlinear Froude-Krylov and restoring forces, the equation of motion based on the IRF formulation can be written as follows:

$$\begin{aligned}(M+M_{\infty})\ddot{\xi} + \int_0^t R(t-\tau)\dot{\xi}(\tau)d\tau &= (F_{F.K.})_{nonlinear} \\ &+ (F_{Rest})_{nonlinear} + (F_{Diff}) + F_{external} + F_{viscous}\end{aligned} \tag{4}$$

where the force terms consist of nonlinear Froude-Krylov and restoring force, linear diffraction force, external force and viscous force. The diffraction force can be converted from the frequency-domain solution. The external force includes soft spring mechanism for non-restoring motions, however no such external force is needed for roll motion. Viscous force is added to the roll excitation component because the amplitude of roll angle is sensitive to the viscous effect. In the present study, an equivalent linear damping (Himeno, 1981) is applied as follows:

$$F_{viscous} = -b_{equi\_viscous}\dot{\xi} \tag{5}$$

Choosing the viscous damping coefficient would not be easy, because the coefficient depends on body shape, ship speed, wave frequency, and other hydrodynamic factors. For an easy numerical implementation, the linear damping coefficient is adopted.

$$b_{equi\_viscous} = 2\gamma\sqrt{(M+M_{\infty})C} \tag{6}$$

where γ means the ratio with respect to the critical damping coefficient, and is generally in the range of 0.05~0.1. C refers to the restoring coefficient of the considered motion, i.e. roll in this case.

NUMERICAL RESULTS AND DISCUSSION

Ship Model

Numerical simulation is carried out for two container ships, since this type of ship is sensitive to parametric roll. Figs. 1~2 show panel models and Table 1 shows the principal dimensions of ships. In this study, all the cases are for head sea condition.

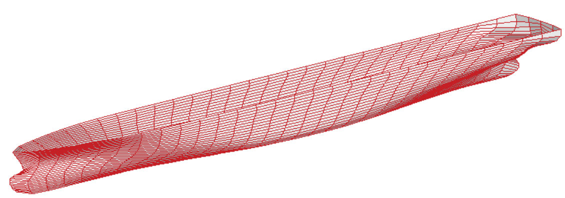


Fig. 1: Panel model of 6500 TEU container ship

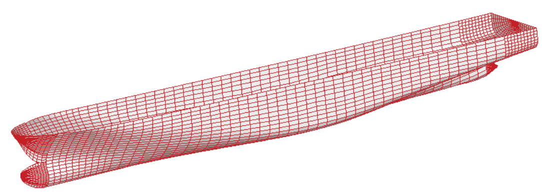


Fig. 2: Panel model of MARIN Model 8004-

Table 1: Principal dimensions of test ship models

Designation	6500 TEU container ship	MARIN Model 8004-2
LBP(m)	286.30	262.00
Beam(m)	40.30	40.00
Draft(m)	13.13	12.86
GM_T(m)	1.14	2.07
Natural period(sec)	30.86	25.17
Froude number	0.049	0.051

Uncertainty of GM

GM is one of the most important parameters in parametric roll motion. One knotty problem is that

GM contains uncertainty in real, experimental and numerical situations. In a real ship sailing, loading condition of cargo causes a slight different GM value. In towing tank experiment, there are many uncertainties in hull fabrication, mass distribution, and so on. The uncertainty in numerical computation contains not-converged hull modelling, lacking knowledge of mass distribution, and other human-made error.

GM is calculated as follow:

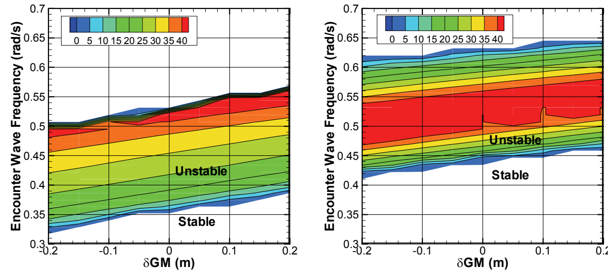
$$GM_{calculated} = BM - KG + VCB \tag{7}$$

$$GM = GM_{calculated} + \delta GM \tag{8}$$

where BM, KG, and VCB denote the metacentric radius, the distance from keel line to the center of gravity, and the center of buoyancy, respectively. δGM is introduced to give a variation in the mean value of GM. δGM is adjusted by modifying KG, which makes it possible to keep the same wetted surface under various simulation conditions.

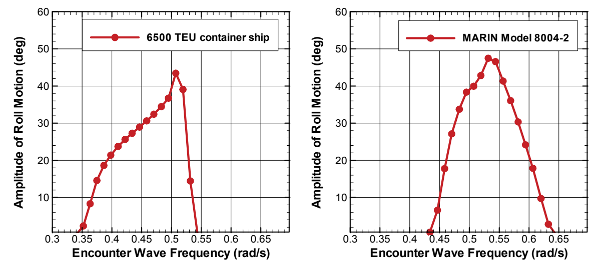
Fig. 3 shows the stability diagram of parametric roll and the contour of roll amplitude for two container ships when δGM is selected within the range -0.2 ~ 0.2 m. These diagram and contours are obtained from very heavy numerical simulations. Naturally, the change of δGM value influences on the location of unstable region and amplitude of roll motion for both ships, since the resonance frequency is dependent on GM. However, it is interesting that that the two ships show different trend of roll angle increase/ decrease.

The roll angle is also plotted in Fig. 4 for the unvaried GM. An interesting point is that maximum roll angle is observed near the top of occurrence band for 6500 TEU container ship. That means roll angle can rapidly decrease even for a small decline of δGM near these frequencies in upper limit. Compared to the 6500 TEU container ship, MARIN model shows less sensitive occurrence band. As well known, the sensitivity on δGM is highly related to the shape of ship, but such difference is very interesting. In the case of sensitive hull model, careful caution seems necessary on the tuning of GM value in numerical simulation. Furthermore, this kind of analysis can provide important information to avoid severe parametric roll.



(a) 6500 TEU container ship (b)MARIN Model 8004-2

Fig. 3: Stability diagram for GM variations on two containerships: A/L=0.01



(a) 6500 TEU container ship (b)MARIN Model 8004-2

Fig. 4: Amplitude of roll motion on two containerships: δGM=0, A/L=0.01

Uncertainty due to Simulation Time Window

To observe parametric roll in real ocean waves, experimental and/or numerical analyses in irregular waves are getting popular. In such analyses, it should be noticed that the analysis of parametric roll in irregular wave is not linear deterministic process since parametric roll is a nonlinear phenomenon. In stochastic process, the statistical properties may be sensitive to simulation parameters, such as simulation time window, the components of discrete wave spectrum, and a set of phase seed.

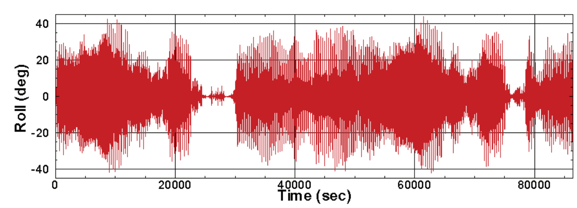
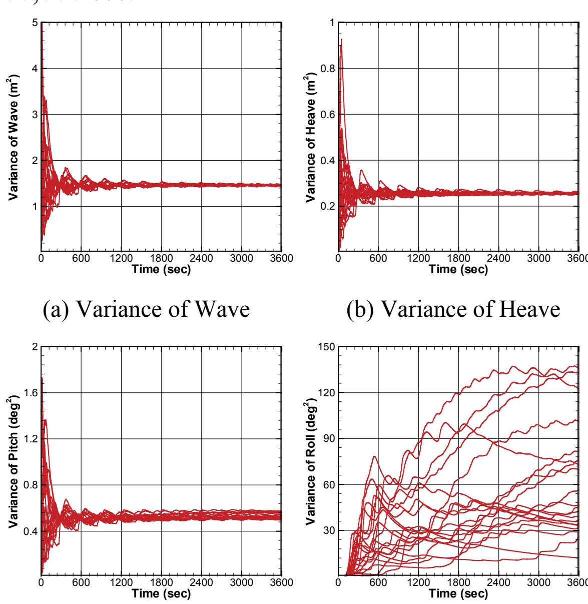


Fig. 5: Roll motion in irregular waves: MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec

To observe the effect of time window, numerical simulations on irregular wave are carried out for 24 hours, i.e. 86,400 sec, in real time scale for each simulation. It should be mentioned that this time window is much longer than typical experimental or numerical tests for parametric roll analysis. Linear incident wave is given from wave spectrum whose significant wave height is 5.25m and modal period is 12.5 sec. Fig. 5 shows n example signal of roll motion obtained from the present simulation. Roll angle sometimes reaches over 40 deg, while it remains nearly rest in some stage.

Fig. 6 shows the cumulative variances of 20 times simulation result in the same wave condition but different phase sets for only 3,600 sec. Compared to the results of wave, heave and pitch, any kind of convergence is hardly observed for roll motion. As many researchers have mentioned (e.g. Belenky et al., 2003), the roll motion clearly shows non-ergodic property when only 3,600 sec is chosen for time window.

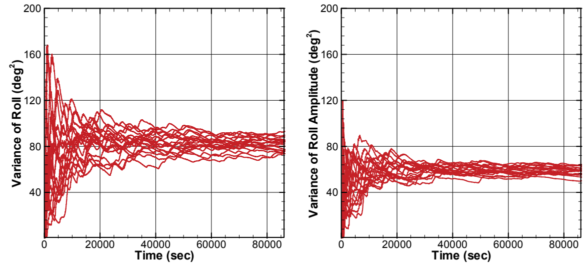
In contrast to Fig. 6 for 1 hour simulation, Fig. 7 shows the variances of roll angle and roll amplitude when the simulation time window is extended to 24 hours (86,400 sec). Here, roll amplitude means the half of selected zero-crossing roll height. Although the convergence band is still not that small, remarkably narrower band is obtained when the time window is larger than 60,000 sec.



(a) Variance of Wave (b) Variance of Heave

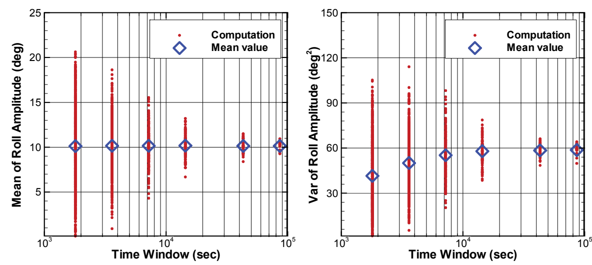
(c) Variance of Pitch (d) Variance of Roll

Fig. 6: Cumulative variances of wave, heave, pitch, and roll: MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec



(a) Variance of Roll (b) Variance of Roll Amplitude

Fig. 7: Cumulative variances for roll motion and amplitude: MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec



(a) Mean of roll amplitude (b) Variance of roll amplitude

Fig. 8: Mean and variance of roll amplitude with respect to time-window (Time Window axis is log scale): MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec

Fig. 8 shows the mean and variance of roll amplitude for different sets of time window. For these plots, 20 cases of simulation results in the same wave condition but different phase set are divided to different time intervals, i.e. 1800, 3600, 7200, 14400, 43200, and 86400 sec, and their means and variances are computed. In these plots, it is obvious that the selection of time window, i.e. starting and ending times, is also important although the lengths of time window are the same. This result shows that both the length of time window, and starting and ending times of time window can be crucial uncertainty parameters in parametric roll analysis. A favourable finding from this observation is that the band ranges of mean and variance are decreased for large time window. This means that we need a very long simulation to reduce numerical uncertainty due to time window. Otherwise, a lot of simulations should be carried out with different starting times and/or phase seeds, and averaging the mean values of all the cases can be considered. However, according to Fig. 8, it also should be noticed that the mean value of variance increases up to a certain value as time window becomes longer. Therefore it should be recognizded that expectation, even for the mean value of variance, in small simulation time can not be alike as in longer simulation.

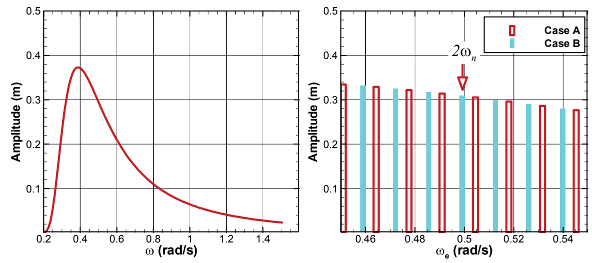
Currently many past researches and simulations are based on the time window less than or even not close to 1 hour. In such cases, significiant uncertainty in simulation or towing-tank test might be embedded implicitely due to the simulation or measurement time and phases

Components of Discrete Wave Spectrum

In numerical simulation and experiment, it is common to generate irregular waves by discretizing wave spectra. According to our numerical experience, the number of frequency components and their intervals for spectrum discretization can be other uncertainty parameters although the same wave energy is considered.

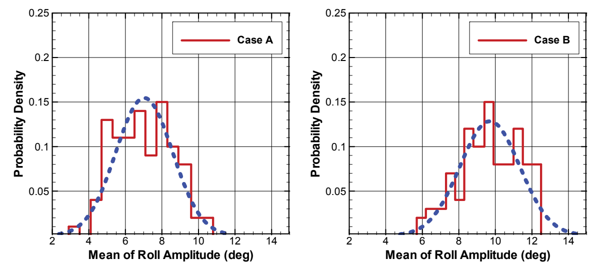
To observe such effect, the spectrum is discretized into two different 120 sets of frequency components for one wave spectrum. Fig. 9-(a) shows ISSC spectrum whose significant wave height is 5.25m and modal period is 12.5 sec, and Fig. 9-(b) shows the frequency components from 0.45 to 0.55 rad/sec in two different cases. The spectrum is uniformly discretized, but the starting points are slightly different. For the two wave sets, the 100 cases of simulation with random phase seeds are carried out for 86,400 sec. In fact, this amount of simulation is extremely heavy. However, as shown above, uncertainty due to the simulation time window should be minimized as much as we can. Therefore, a heavy computation is carried out for this observation.

Fig. 10 shows the probability density function of mean value for the two cases. The histograms are computation result and the dot line is the approximated Gaussian distribution. Based on this heavy computation, the mean values of roll amplitude are predicted to be 7.0 deg for Case A and 9.7 deg for Case B in Fig.9. This difference is very large and surprising. To explain the source of such difference, twice of roll natural frequency, which is the primary factor on the parametric roll, is marked in Fig. 9-(b). In Case B, wave components include one which is close to the twice of roll natural frequency, while less related components are included in Case A. This indicates that, although the same spectrum is used, the larger roll angle will be predicted if the wave components include twice that of roll natural frequency. This implies that the discretization of spectrum should be also considered as a crucial factor of uncertainty in parametric roll analysis.



(a) Wave spectrum (b) Wave components

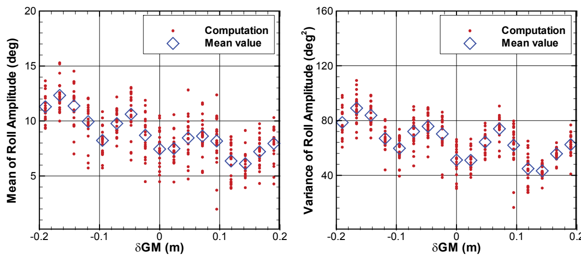
Fig. 9: Wave spectrum and wave components: wave components = 120, Hs = 5.25m, Tp=12.5 sec



(a) Case A (b) Case B

Fig. 10: Probability density functions of roll motion in irregular wave: MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec (histogram: computation, dot line: Gaussian distribution)

To observe more systematically the effect of natural frequency on a given spectrum, the present computation is continued for different GM variations, i.e. δGM , in the range of -0.2 ~ 0.2 m. Fig. 11 shows the mean and variance of roll amplitude in the irregular wave condition which corresponds to the case A in Fig. 9. For one value of δGM , 20 simulations with different phase sets are carried out. 24 hour time scale is applied for all the simulations. As clearly shown, the mean and variance values at roll amplitude are fluctuating with respect to δGM . Even the mean of the two variables are also fluctuating. According to this results for MARIN model, more than 300% difference of mean value of roll amplitude can be observed when $\pm$10% error of GM is assumed. It should be mentioned that this result is for the same discretization of wave spectrum. If the effect of spectrum discretization is included, the fluctuation in Fig.11 can be more. Therefore, it is obvious that wave components of discrete wave spectrum are important parameter which causes uncertainty of parametric roll. Furthermore, it should be mentioned that the observation in this study is valid not only in numerical simulation but also experiment which more uncertainties due to test machinery and other parameters, e.g. GM, are involved.



(a) Mean (b) Variance

Fig. 11: Mean and variance of roll amplitude with respect to δGM : MARIN Model 8004-2, wave components = 120, Hs = 5.25m, Tp=12.5 sec

CONCLUSIONS

Based on a lot of present computational effort, numerical sensitivity and some degree of uncertainty are observed, and the following conclusions are suggested:

- The uncertainty of GM influences on the occurrence criteria of parametric roll and the maximum roll angle. Such effect is dependent on ship geometry.
- The statistical properties of parametric roll in irregular waves are highly sensitive on simulation time window. Long time simulation and the large number of tests are desirable to increase the reliability.
- The mean and variance of parametric roll amplitude is strongly sensitive to the discretization of wave spectrum in irregular wave and motion simulation. Particularly, the uncertainty level of roll amplitude is very high for the spectrum discretization and the error/variation of GM.

REFERENCES

Ballard E.J., Hudson D.A., Price W.G., and Temarel P. (2003), "Time domain simulation of symmetric ship motions in waves", Trans. RINA, Part A: International J of Maritime Engineering, 145, (A2), pp. 89-103.

Belenky V., Weems K.M., Lin W.M. and Paulling J.R. (2003), "Probabilistic Analysis of Roll Parametric Resonance in Head Seas", Proc. of the 8th Intl Conf on Stability of Ships and Ocean Vehicles (STAB2003), pp. 325-340.

Cummins W.E. (1962), "The impulse response function and ship motions", Schiffstechnik, Vol. 47, No. 9, pp. 101-109.

Dunwoody A.B. (1989), "Roll of a ship in astern Sea – Metacentric height spectra", J. of Ship Research, Vol 33, No 3, pp. 221-228.

France, W.N., Levadou, M., Treakle, T.W., Paulling, J.R., Michel, R.K. and Moore, C. (2003), "An Investigation of Head-Sea Parametric Rolling and its Influence on Container Lashing Systems", Marine Technology, Vol. 40, No. 1, pp. 1-19.

Himeno Y. (1981), "Prediction of ship roll damping – State of the Art", Report of NA & ME, The University of Michigan, No. 239, Ann Arbor, MI.

Kim K.H., Kim, Y., Kim, Y. (2007), Kim, K.H., Kim, Y., Kim, Y. WISH JIP project report and manual, Project Report, Seoul National University.

Kim T.Y. and Kim Y. (2010), "Multi-level Approach of Parametric Roll Analysis", Proc. ITTC Seakeeping Workshop, Seoul, Korea.

Nayfeh A.H. (1988), "On the undesirable roll characteristics of ships in regular seas", J. of Ship Research, Vol 32, No. 2, pp. 89-103.

Paulling J.R. and Rosenberg R.M. (1959), "On unstable ship motions resulting from nonlinear coupling", J. of Ship Research, Vol. 3, No. 1, pp. 36-46.

Shin Y.S., Belenky V.L., Weems K.M. and Lin W.M. (2004), "Criteria for parametric roll of large container ships in longitudinal seas", SNAME Trans., Vol. 112, pp. 14-47

Spanos D. and Papanikolaou A. (2007), "Numerical simulation of parametric roll in head seas", Intl. Shipbuilding Progress, 54, pp. 249-267.