Viscosity in Ship's Dynamics

Maciej Pawłowski
School of Ocean Engineering and Ship Technology, TU Gdansk, Poland

ABSTRACT

The application of strip theory for the prediction of ship response in waves relies on the sectional hydrodynamic coefficients; i.e. added mass and damping coefficient. These coefficients apply to linearized problems and are normally computed for inviscid fluids. It is possible to account for viscosity but this cannot be done by the RANS equations, as in linear problems there is no room for turbulence. The hydrodynamic coefficients can include the effect of viscosity but this can be done rightly through the Navier–Stokes equations only. For solving these equations commercial RANS software can be used assuming no turbulence stresses.

KEYWORDS

Hydrodynamics, strip theory, hydrodynamic coefficients (added mass and damping coefficient), viscosity.

INTRODUCTION

A question arises if viscosity should be accounted for in a ship's dynamics, particularly in seakeeping. When analysing a non-stationary ship's motion two hydrodynamic coefficients are used, comprising the added mass and damping coefficient. Strictly speaking, they are fully applicable only to linearized equations of motion in inviscid flows. In such a case, the field velocity around the body has a potential, fulfilling the Laplace equation $\Delta\phi = 0$ along with the boundary conditions. This is a linear equation, leaving no space for turbulence. In other words, such a velocity field is always smooth, irrespective of the fact whether fluid is viscous or inviscid, clearly observed outside the boundary layer, where flow is potential

For rotational flows, the potential of velocity does not exist. Vorticity is due to viscosity, which comes to action in the close vicinity of the boundary surface. In such a case the governing equation of flow motion is the Navier–Stokes equation:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = \frac{1}{\rho}\,\mathrm{grad}\,p + \mu\,\Delta\mathbf{v} \tag{1}$$

The two terms on the left hand-side represent the acceleration of a particle $d\mathbf{v}/dt$. This is a non-linear equation of motion. The only non-linear term – the second one on the left hand-side of equation (1) – is the acceleration related to the convection of the particle, known as the ‘convection term’, which is the source of turbulence. For linearized equations, in which the convection term $(\mathbf{v}\cdot\nabla)\mathbf{v}$ is neglected, the averaging process introduces no Reynolds stresses and therefore the equations of motion remain unchanged. In other words, solutions of linearized N–S equations, despite viscosity, are still smooth. Such equations are closed and need no turbulence models. For that reason, employing a turbulence model for linear problems, as for instance done by Querard *et al*, is conceptually wrong.

LINEAR SEAKEEPING

Linearized seakeeping analysis is normally based on the assumption of a potential flow, applicable for an inviscid fluid. The governing equation for flow is the Laplace equation for the velocity potential $\Delta\phi = 0$ along with the boundary conditions. Such a model leaves no room for turbulence. Furthermore, in a linear approach, despite oscillations the body is assumed to be stationary. Motion of the fluid is induced by the boundary conditions, crucial for the problem. That is, the normal component of fluid velocity on the outer surface of the body equals the normal component of the outer surface, which is such as if the body was in motion, completed by the boundary conditions on the free surface. Fluid particles move along the surface of the body but this does not create any vorticity, as by assumption the fluid is inviscid.

The hydrodynamic forces acting on the body are found by integration of the dynamic pressure p, given by the Cauchy–Lagrange equation:

$$p = -\rho \frac{\partial \phi}{\partial t}, \tag{2}$$

which results in the added mass m and damping coefficient N, both dependent on the circular frequency of oscillations ω.

LINEARIZED EQUATION OF MOTION

In linear problems the convection acceleration $(\mathbf{v} \cdot \nabla)\mathbf{v}$ is neglected without bothering about its magnitude. We are entitled to neglect it, if it is small in relation to the remaining terms in the N–S equation (1).

It is relatively easy to assess magnitude of $(\mathbf{v} \cdot \nabla)\mathbf{v}$. The operator $(\mathbf{v} \cdot \nabla)$ means differentiation along the direction of velocity. In close vicinity of the body the velocity is basically parallel to its surface. Therefore $(\mathbf{v} \cdot \nabla)\mathbf{v} \sim v^2/\mathrm{l}$, where l is a characteristic dimension of the body. In an oscillatory motion the velocity $v \sim \omega a$, where a is the amplitude of oscillations. Hence,

$$(\mathbf{v} \cdot \nabla)\mathbf{v} \sim (\omega a)^2/l.$$

On the other hand, the following holds for the local acceleration

$$\partial \mathbf{v}/\partial t \sim \omega v \sim \omega^2 a.$$

Comparing the two expressions, we get that $(\mathbf{v} \cdot \nabla)\mathbf{v} \ll \partial \mathbf{v}/\partial t$, if $a \ll l$, i.e. if the amplitude of oscillations is small in relation to the size of the body. It is easy to show in addition that the terms $\partial \mathbf{v}/\partial t$ and $\nu \Delta \mathbf{v}$ are of the same magnitude.

Assuming that the convection acceleration is small relative to the other terms, the non-linear equation of motion (1) reduces to the linear equation

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{1}{\rho} \operatorname{grad} p + \mu \Delta \mathbf{v} \tag{3}$$

valid for small amplitudes of oscillations. As a linear equation, it leaves no room for turbulence. Hence, its solutions are smooth, though not necessarily of laminar type, due to the complexity of flow induced by vorticity, in particular – by trailing vortices. Complex flows are not the same as turbulent flows.

Regarding the Reynolds number, it is normally defined for stationary flows, when a body moves at a constant speed. For an oscillatory motion this number is defined as $\mathrm{Re} = \omega a l/\nu$. In linear problems amplitudes of motion are assumed to be small, as we say – infinitely small. The Reynolds numbers are, therefore, small by definition.

THE AVERAGED NAVIER–STOKES EQUATIONS

As we can see, the linearized Navier–Stokes equations (3) are capable of providing realistic solutions for viscous flows. Therefore, there is no need at all to resort to the regular N–S equations (1).

Due to the non-linear term, associated with the convection acceleration, it opens room for turbulence stresses. After averaging, equation (1) takes the form:

$$\frac{d\overline{\mathbf{v}}}{dt} = -\frac{1}{\rho} \operatorname{grad} \overline{p} + \nu \Delta \overline{\mathbf{v}} - \operatorname{Div} \mathbf{A}, \tag{4}$$

known as the Reynolds Averaged Navier–Stokes Equations (RANSE), where the bar above the notation denotes a smoothed (averaged) quantity. The two first terms on the right-hand side of (4) equals

$$\frac{1}{\rho} \operatorname{Div} \overline{\mathbf{P}},$$

where $\overline{\mathbf{P}}$ is a stress tensor for a smoothed motion. With this notation, the above equation can be written as follows:

$$\frac{d\overline{\mathbf{v}}}{dt} = \frac{1}{\rho} \operatorname{Div}\left(\overline{\mathbf{P}} - \rho \mathbf{A}\right).$$

As can be seen, the term $-\rho \boldsymbol{A}$ plays the role of the additional stress tensor, called the turbulent stress tensor, or the tensor of Reynolds stresses:

$$-\rho\mathbf{A} = \begin{bmatrix} -\rho\overline{v_1'^2} & -\rho\overline{v_1'v_2'} & -\rho\overline{v_1'v_3'} \\ -\rho\overline{v_2'v_1'} & -\rho\overline{v_2'^2} & -\rho\overline{v_2'v_3'} \\ -\rho\overline{v_3'v_1'} & -\rho\overline{v_3'v_2'} & -\rho\overline{v_3'^2} \end{bmatrix}, \qquad (5)$$

where ' means a fluctuation of the velocity in a given direction. To define the Reynolds' stresses turbulent models are necessary that are of approximate nature.

Turbulence models were developed mainly for stationary flows within the boundary layer. I mean stationary in terms of smoothed quantities. Outside the body (in the wake) and behind the separation point there are doubts if turbulence exists at all. These doubts are due to the decay of the normal derivative of velocity at the separation point and, what goes with it, the vanishing of turbulent stresses just at this point. This follows immediately from Prandtl's mixing-length hypothesis.

For non-stationary flows, for oscillatory motions in particular, no turbulence model exists. Further, these models would have to be time dependent, which is beyond feasibility. Presumably all the turbulence models have been developed and calibrated for stationary flows, not for oscillating bodies. Use of any turbulence model is therefore strongly speculative, and of little real merit.

The idea of the hydrodynamic coefficients, i.e. the added mass and damping coefficient is solely applicable to linearized problems, in which the body hardly moves, if at all. In such circumstances there is no room for developing turbulence. Hence, it can be assumed that there are no Reynolds stresses at all, which reduces the RANS equation (5) to the regular N–S equations (1). The same solver can be used for solving both the N–S equations and RANSE, e.g. a commercial RANS solver (ANSYSCFX10.0).

The main difficulty is to extract from the whole dynamic force that is time dependent, the harmonic part, which should be next resolved into the inertial and damping components.

HYDRODYNAMIC COEFFICIENTS

In non-linear harmonic oscillations of finite amplitudes the hydrodynamic coefficients are not constant in respect to time and, apart from that, they are dependent on the amplitude of oscillations. Consequently, they have to be averaged over the time.

In linear problems the body is stationary, whereas its motion is depicted by the kinematic boundary conditions. Though it is acceptable to assume that the body physically moves nevertheless it makes calculations cumbersome and reduces accuracy.

In case the body physically moves, equations for the hydrodynamic sectional forces are as follows:

$$\begin{aligned} &-A_{22}\ddot{y} - B_{22}\dot{y} = F_y(t) \quad , \\ &-A_{33}\ddot{z} - B_{33}\dot{z} - K_{33}z = F_z(t) \quad , \\ &-A_{44}\ddot{\alpha} - B_{44}\dot{\alpha} - K_{44}\alpha = M_x(t)\,, \end{aligned} \qquad (6)$$

where in general A is the added mass, B is the damping coefficient, and K is the coefficient of stiffness, all per unit length. The right hand-sides represent the hydrodynamic forces, obtained from integration of the pressure p and tangential stresses τ over the wetted surface of the body. For heave

$$K_{33} = B\rho g,$$

where B is breadth of the body at the waterline. For roll

$$K_{44} = \rho g \forall GM,$$

where $\forall$ is the sectional underwater area, and GM is the height of the metacenter above the waterline (the origin G is normally taken at the centerline of the waterline). The two coefficients of stiffness are known quantities.

The hydrodynamic forces on the right hand sides of equations (6): $F_y(t)$, $F_z(t)$, $M_x(t)$ are provided by numerical calculations per unit length as time histories. They are calculated for an imposed harmonic displacement for y, z, and α: $a\sin\omega t$, of amplitude a and circular frequency ω.

Applying Fourier analysis to equations (6) for hydrodynamic forces, after performing simple mathematics, we get in general the following expressions for the sectional added mass and damping coefficient

$$A = \frac{K}{\omega^2} + \frac{1}{\pi a \omega}\int_t^{t+T} F(t)\sin\omega t\,dt\,, \qquad (7)$$

$$B = -\frac{1}{\pi a}\int_{t}^{t+T} F(t)\cos\omega t\, dt, \tag{8}$$

where $T = 2\pi/\omega$ is a period of oscillations, and $F(t)$ stands for the time varying hydrodynamic force or moment, obtained from computations for given circular frequency. These forces deviate from harmonic runs, if the equations of motion are non-linear. The coefficient of stiffness in the equation (7) is treated as a known quantity.

A SIMPLE CASE STUDY

To shed some light on the effect of viscosity it is worth recalling a simple case study, known in literature. Unbounded surface at the *yz* plane, performs harmonic oscillation at the *z* direction with the velocity $v = v_0\cos\omega t$. Assuming that the field velocity $\mathbf{v} = v\mathbf{k}$ has only one component in the direction of the *z*-axis, where $v = v(x)$ is a function of *x* (distance from the plane), the N–S equation (1) reduces then to two scalar equations: $p = const$, and

$$\frac{\partial v}{\partial t} = \nu\frac{\partial^2 v}{\partial^2 x}, \tag{9}$$

known in mathematics as the *equation of diffusion*. Its solution is as follows:

$$v = v_0 e^{-kx}\cos(kx - \omega t), \tag{10}$$

where $k = (\omega/2\nu)^{1/2}$ is the wave number. The inverse of the wave number, denoted by $\delta \equiv 1/k = (2\nu/\omega)^{1/2}$ is the vanishing constant, known better as the *penetration distance*. At a distance $x = 3\delta$ the velocity drops to a value of $e^{-3} \approx 5\%$ of that at the oscillating surface. The depth of penetration increases with the kinematic viscosity ν and decreases with the circular frequency ω.

The tangential stress on the surface is given by $\tau = \mu\partial v/\partial x$. Substituting $x = 0$, the following is obtained:

$$\tau = -\mu k v_0(\cos\omega t - \sin\omega t),$$

where $\mu k = (\tfrac{1}{2}\rho\mu\omega)^{1/2}$. Since the acceleration of the surface $\dot{v} = -\omega v_0\sin\omega t$, the above can be written as

$$\begin{aligned}\tau &= -(\mu k/\omega)\,\dot{v} - \mu k v.\\ &\quad -(\tfrac{1}{2}\rho\mu/\omega)^{1/2}\,\dot{v} - (\tfrac{1}{2}\rho\mu\omega)^{1/2} v.\end{aligned} \tag{11}$$

As can be seen, despite the unbounded domain, the tangential stress has two components: one proportional to the acceleration and the other – proportional to the velocity. Both are opposite to the appropriate parameters of motion. The first square root in equation (11) has therefore the meaning of the added mass per unit area, whereas the other one – the meaning of the damping coefficient per unit area.

With the help of the above considerations, some properties of motion can be deduced from the linearized equation (3). The operator rotation (curl) can be applied to both its sides. As the rotation of gradient vanishes, we get the equation

$$\frac{\partial\,\mathrm{rot}\,\mathbf{v}}{\partial t} = \mathbf{v}\Delta\mathrm{rot}\,\mathbf{v}$$

i.e. $\mathrm{rot}\,\mathrm{v}$ fulfils the equation of diffusion (9). It follows from the foregoing that such an equation leads to an exponential decay of the quantity described by it, in this case the vorticity. In other words, motion of the fluid induced by an oscillating body is rotational in some layer around the body. Vorticity decays rapidly with the distance from the body, turning in some distance into a potential flow, despite viscosity. The depth of penetration of vorticity is of the order of δ.

The quantity δ can be large or small in relation to the body. The case of $\delta \gg l$ occurs, if $\omega l^2 \ll \nu$, i.e. when oscillations are extremely small, far below the range of interest. In such a case changes of velocity are very slow. Motion of the fluid is therefore quasi-stationary. That is to say, at each time instant fluid motion is such as it would be in the case of a uniform motion of the body with the speed it has at given time instant. The boundary layer as such stretches practically over the entire domain.

The opposite case $\delta \ll l$, i.e. of a thin boundary layer occurs, when $\omega l^2 \gg \nu$. As ν is small, this occurs practically at the entire range of frequency of oscillations that are of interest. As the boundary layer is thin the effect of viscosity on the hydrodynamic coefficient cannot be large.

CONCLUSIONS

Based on the results and arguments presented in this paper the following conclusions can be drawn:

- It is possible to account for viscosity in the hydrodynamic coefficients (added mass and damping coefficient) but this cannot be done by the RANS equations, as in linear problems there is no room for turbulence
- The hydrodynamic coefficients can include the effect of viscosity only through the Navier–Stokes equations that do not need any turbulence models
- Due to the thin boundary layer the effect of viscosity can only be of secondary meaning
- Viscosity opens room for the memory effect even in an unbounded domain
- Commercial RANS software can be used for solving the Navier–Stokes equations assuming no turbulence stresses

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