Onboard Analysis of Ship Stability Based on Time-Varying Autoregressive Modeling Procedure

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ABSTRACT

In this study, it is clarified that dynamical system of roll motion can be approximated by time varying autoregressive models which is a kind of statistical model firstly. As the result, when it is possible to measure the time series of roll motion, the ship stability can be judged based on time series analysis by using the time varying autoregressive modeling procedure. As to the verification of this, results of the model experiment for parametric roll resonance are used. It is confirmed that the judgment of ship safety is possible based on the proposed procedure.

KEYWORDS

Time varying autoregressive model; Kalman filter; Time varying characteristic root

INTRODUCTION

It is very important for ship operators to understand ship stability of an operating ship. Fundamentally, before a stage of ship construction, ship stability is decided by various roles of IMO [Umeda, 2007]. However, the ship stability varies in the actual voyage, because the loading condition, seasons and navigation routine etc. vary. Therefore, from the view point of the safety operation of ships, we should consider ship stability as stochastic dynamical system and should actually measure the ship motion.

When we measure the ship motion actually, a time series of the ship motion can be obtained. As the results, it is possible to analyze the ship motion onboard based on appropriate time series modeling procedure [Kitagawa, 2010] and to judge ship stability onboard, since time series modeling procedure, stochastic dynamical process and physical process are indirectly related [Ozaki, 1986].

In this study, we firstly make it appear that physical process of ship stability is related to stochastic dynamical process. And then, we make it appear that stochastic dynamical process is related to time-varying autoregressive modeling procedure. After that, based on results of model experiments with respect to a parametric roll resonance [Hashimoto et al., 2005] , the validity of proposed method is verified. The obtained findings are reported

TIME VARYING AUTOREGRESSIVE MODELING PROCEDURE

Discretization of Second Order Nonlinear Dynamical Model

In this study, we focus on the nonlinear roll motion as the Second order Nonlinear statistical Dynamical (SNSD) system. As for the model of the nonlinear roll motion, by assuming that external disturbance is a random process, we consider the following parametoric roll resonance model [Umeda and Taguchi, 2003] as the SNSD model in meaning of Stratonovich [Ozaki, 1986]:

$$\begin{aligned}&\ddot{x}(t)+c_1\dot{x}(t)+c_2|\dot{x}(t)|\dot{x}(t)\\&+c_3(1+c_4\cos c_5 t)x(t)=u(t)\end{aligned} \tag{1}$$

where, $x(t)$ is a roll angle, the notation ($\ddot{}$) and ($\dot{}$) over $x(t)$ mean a differential operator with respect to time, c_1 is a coefficient of roll damping, c_2 is a coefficient of nonlinear roll damping, c_3 is a power of a natural angular frequency, the c_4 is a ratio of amplitude of parametric roll resonance and the transverse metacentric height (GM), c_5 is an encounter frequency with respect to waves and $u(t)$ is external disturbance treating as the stochastic process. Here, as the statistical characteristic of the $u(t)$, it has variance σ^2, however an assumption of a white noise sequence does not require. Eq. 1 can be also written by the following vector representation:

$$\dot{\mathbf{x}}_t = f(\mathbf{x}_t) + \mathbf{u}_t \tag{2}$$

where,

$\mathbf{x}_t = (x_1, x_2)^{\mathrm{T}} = (\dot{x}(t), x(t))^{\mathrm{T}}$,

$f(\mathbf{x}_t) = (f_1(\mathbf{x}_t), f_2(\mathbf{x}_t))^{\mathrm{T}}$,

$f_1(\mathbf{x}_t) = -c_1\dot{x}(t) - c_2|\dot{x}(t)|\dot{x}(t) - c_3(1 + c_4\cos c_5 t)x(t)$,

$f_2(\mathbf{x}_t) = \dot{x}(t)$,

$\mathbf{u}_t = (u(t), 0)^{\mathrm{T}}$,

and the notation T is the transpose of the vector or the matrix. According to Ozaki (1986), Eq. 2 can be discretized as follows:

$$\mathbf{x}_{n+1} = \mathrm{EXP}(\mathbf{K}_n \Delta t)\mathbf{x}_n + \mathbf{B}_n \mathbf{u}_{n+1} \tag{3}$$

where,

$\mathbf{x}_n = (x_1, x_2)^{\mathrm{T}} = (\dot{x}_n, x_n)^{\mathrm{T}}$,

$\mathbf{K}_n = \dfrac{1}{\Delta t}\mathrm{LOG}(\mathbf{A}_n)$,

$\mathbf{A}_n = \mathbf{I} + \mathbf{J}_n^{-1}\{\mathrm{EXP}(\mathbf{J}_n \Delta t) - \mathbf{I}\}\mathbf{F}_n$,

$\mathrm{LOG}(\mathbf{A}_n) = \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k}(\mathbf{A}_n - \mathbf{I})^k$,

$$\mathrm{EXP}(\mathbf{J}_n \Delta t) = \frac{1}{\mu_1 - \mu_2}\begin{pmatrix} \mu_1\exp(\mu_1\Delta t) - \mu_2\exp(\mu_2\Delta t) & \mu_1\mu_2[\exp(\mu_2\Delta t) - \exp(\mu_1\Delta t)] \\ \exp(\mu_1\Delta t) - \exp(\mu_2\Delta t) & \mu_1\exp(\mu_2\Delta t) - \mu_2\exp(\mu_1\Delta t) \end{pmatrix},$$

$$\mathbf{J}_n = \begin{pmatrix} -c_1 - c_2|\dot{x}_n| - c_2\dfrac{\dot{x}_n}{|\dot{x}_n|}\dot{x}_n & -c_3(1 + c_4\cos c_5 n\Delta t) \\ 1 & 0 \end{pmatrix},$$

$$\mathbf{F}_n = \begin{pmatrix} -c_1 - c_2|\dot{x}_n| & -c_3(1 + c_4\cos c_5 n\Delta t) \\ 1 & 0 \end{pmatrix},$$

$$\mathbf{B}_n = \mathbf{U}\begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{pmatrix},$$

here, $\mathbf{u}_n$ is a bi-variate colored noise with mean zero and variance $\mathbf{I}\sigma^2$, μ_1 and μ_2 are eigenvalues of matrix $\mathbf{J}_n$ and $\mathbf{U}$ is a unitary matrix which consists of the following elements:

$u_{11} = \sigma_{12} \Big/ \sqrt{\{\sigma_{12}^2 + (\lambda_1 - \sigma_{11})^2\}}$,

$u_{22} = \sigma_{12} \Big/ \sqrt{\{\sigma_{12}^2 + (\lambda_2 - \sigma_{22})^2\}}$,

$u_{12} = (\lambda_1 - \sigma_{11}) \Big/ \sqrt{\{\sigma_{12}^2 + (\lambda_1 - \sigma_{11})^2\}}$,

$u_{21} = -u_{12}$.

Where, λ_1 and λ_2 is calculated by solving the following equation:

$$\lambda^2 - (\sigma_{11} + \sigma_{22})\lambda + \sigma_{11}\sigma_{22} - \sigma_{12}^2 \tag{4}$$

here,

$$\sigma_{11} = \frac{1}{(\alpha_1 - \alpha_2)^2}\left[\frac{\alpha_1}{2}\{\exp(2\alpha_1\Delta t) - 1\} + \frac{\alpha_2}{2}\{\exp(2\alpha_2\Delta t) - 1\} - \frac{2\alpha_1\alpha_2}{(\alpha_1 + \alpha_2)}\{\exp((\alpha_1 + \alpha_2)\Delta t) - 1\}\right],$$

$$\sigma_{22} = \frac{1}{(\alpha_1 - \alpha_2)^2}\left[\frac{1}{2\alpha_1}\{\exp(2\alpha_1\Delta t) - 1\} + \frac{1}{2\alpha_2}\{\exp(2\alpha_2\Delta t) - 1\} - \frac{2}{(\alpha_1 + \alpha_2)}\{\exp((\alpha_1 + \alpha_2)\Delta t) - 1\}\right],$$

$$\sigma_{12} = \frac{1}{(\alpha_1 - \alpha_2)^2} \left[\frac{1}{2}\{\exp(2\alpha_1 \Delta t) - 1\} + \frac{1}{2}\{\exp(2\alpha_2 \Delta t) - 1\} - \{\exp((\alpha_1 + \alpha_2)\Delta t) - 1\} \right],$$

$$\sigma_{12} = \sigma_{21}$$

and α_1 and α_2 are eigenvalues of matrix $\mathbf{K}_n$. Note that the above discussion is basically satisfied in how nonlinear models, although $\mathbf{F}_n$ and $\mathbf{J}_n$ change with the considering model.

Bi-variate Time Varying AutoRegressive model

In the discrete model introduced previous section, the statistical characteristic of the external force treated as the stochastic process is the colored noise sequence. Therefore, we have to be transformed it into the white noise sequence. In this study, according to Yamanouchi (1956), the colored noise sequence is approximated by a Discrete AutoRegressive (DAR) process in order to implement the whiting of the noise sequence. That is, in Eq. 3 we consider that $\boldsymbol{\varepsilon}_n \equiv \mathbf{B}_{n-1}\mathbf{u}_n$, and this team can be approximated by the m-th order DAR process. Then the colored noise sequence is represented as follows:

$$\boldsymbol{\varepsilon}_n = \sum_{i=1}^{m} \mathbf{D}_i \boldsymbol{\varepsilon}_{n-i} + \mathbf{w}_n, \ \boldsymbol{\varepsilon}_n = \mathbf{w}_n \quad for\ i = 0 \quad (5)$$

where, $\mathbf{w}_n$ is the Gaussian white noise sequence with mean zero and variance-covariance matrix $\sigma^2\mathbf{I}$ and $\mathbf{I}$ is the 2 × 2 dimensional identity matrix. On the other hand, since the following relation,

$$\begin{aligned} \boldsymbol{\varepsilon}_n &= \mathbf{x}_n - \mathbf{A}_{n-1}\mathbf{x}_{n-1} \\ \boldsymbol{\varepsilon}_{n-1} &= \mathbf{x}_{n-1} - \mathbf{A}_{n-2}\mathbf{x}_{n-2} \\ &\vdots \\ \boldsymbol{\varepsilon}_{n-m} &= \mathbf{x}_{n-m} - \mathbf{A}_{n-m-1}\mathbf{x}_{n-m-1} \end{aligned} \quad (6)$$

is clear, from Eq. 5 and 6 we can obtain the following bi-variate TVAR (BTVAR) model.

$$\mathbf{x}_n = \sum_{i=1}^{m+1} \mathbf{C}_i \mathbf{x}_{n-i} + \mathbf{w}_n \quad (7)$$

here, $\mathbf{C}_i$ (i=1, …, m+1) is the TVAR coefficients that are represented as follows:

$\mathbf{C}_1 = \mathbf{D}_1 + \mathbf{A}_{n-1}$,

$\mathbf{C}_2 = \mathbf{D}_2 - \mathbf{D}_1\mathbf{A}_{n-2}$, …,

$\mathbf{C}_m = \mathbf{D}_m - \mathbf{D}_{m-1}\mathbf{A}_{n-m}$,

$\mathbf{C}_{m+1} = -\mathbf{D}_m\mathbf{A}_{n-m-1}$.

As this, the SNSD system can be approximated by the Bi-variate TVAR (BTVAR) model. Therefore, when we apply the idea of stationary AR modeling procedure [Kitagawa & Gersch, 1996] into this problem, we can judge the stability of the system in time step n by using characteristic roots of the following characteristic equation associated with the TVAR operator.

$$\mathbf{C}_n(B) = \mathbf{I} - \sum_{l=1}^{m} \mathbf{C}_l(n) \cdot B^l = 0 \quad (8)$$

here B is the "time shift operator" that defined by

$B^l \cdot x(n) = x(n-l)$,

where, note that the order of the TVAR model is replaced from m+1 to m. In this case, if characteristic roots in time step n calculated from Eq. 8 lie outside the unit circle, then the system is stable.

Scolor Time Varying AutoRegressive model

In previous subsection, we introduced the relationship between the dynamics of nonlinear roll motion and the BTVAR model. When it is possible to measure the roll angle and the roll rate, the BTVAR modeling procedure is the most effective for the judgment of the stability of roll motion. However, it could be difficult to measure these two sequence at the same time because of the measurement device. In this case, since the model order adjusts the good fit to the phenomenon of the model in the time series modeling procedure, we can regard the following scolor Time Varying AutoRegressive (TVAR) model with respect to the roll angle or the roll rate as the approximation of the dynamics model of nonlinear roll motion.

$$y(n) = \sum_{j=1}^{m} c_j(n) \cdot y(n-j) + w(n) \quad (9)$$

Estimation of TVAR coefficients

Since these are $m \times N$ AR coefficients in Eq. 9, an attempt to fit the parameters by the least squares method or any other ordinary means to the N observations $y(1)$, …, $y(N)$, will yield poor parameter estimates. To solve this difficulty, it is assumed that the unknown TVAR coefficients are random variables and a Gaussian distribution. That is, it is introduced that the following stochastically perturbed difference equation constraint model

$$\Delta^k c_j(n) = v_j(n) \tag{10}$$

where $v_j(n) = [v_1(n),\ldots,v_m(n)]^{\mathrm{T}}$ denotes the m-th order Gaussian white noise sequence with mean zero and covariance matrix $\mathbf{Q}$, which is supposed here to be a diagonal matrix with diagonal values τ^2, Δ is the difference operator at time step n and defined by

$$\begin{aligned}\Delta c_j(n) &= c_j(n) - c_j(n-1)\\ \Delta^k c_j(n) &= \Delta^{k(\ )}\left(\Delta c_j(n)\right)\end{aligned} \tag{11}$$

and k is the order of difference operator and is 1 or 2 in this study. In order to estimate efficiently unknown TVAR coefficients, the following state space representation is introduced regarding Eq. 9 and Eq. 11 as a system model and an observation model.

$$\mathbf{x}(n) = \mathbf{F}\mathbf{x}(n-1) + \mathbf{G}\mathbf{v}(n) \tag{12}$$

$$y(n) = \mathbf{H}(n)\mathbf{x}(n) + w(n) \tag{13}$$

In Eq. 12 and Eq. 13, $\mathbf{F}$ and $\mathbf{G}$ is respectively the $km \times km$ and the $km \times m$ matrix, $\mathbf{H}(n)$ and $\mathbf{x}(n)$ is the km vector, and these notations can be written as follows:

$$\mathbf{x}(n) = \begin{cases}\left[c_1(n),\cdots,c_m(n)\right]^{\mathrm{T}} : k=1\\ \left[c_1(n),c_1(n-1),\cdots,c_m(n),c_m(n-1)\right]^{\mathrm{T}} : k=2\end{cases},$$

$$\mathbf{F} = \begin{bmatrix}\mathbf{F}^{(k)} & & \\ & \ddots & \\ & & \mathbf{F}^{(k)}\end{bmatrix} = \mathbf{I}_m \otimes \mathbf{F}^{(k)} ,$$

$$\mathbf{F}^{(1)} = 1,\quad \mathbf{F}^{(2)} = \begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix}\mathbf{G}^{(k)} & & \\ & \ddots & \\ & & \mathbf{G}^{(k)}\end{bmatrix} = \mathbf{I}_m \otimes \mathbf{G}^{(k)} ,$$

$$\mathbf{G}^{(1)} = 1,\quad \mathbf{G}^{(2)} = \begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\mathbf{H}(n) = \left(y(n-1),\cdots,y(n-m)\right) \otimes \mathbf{H}^{(k)}(n),$$

$$\mathbf{H}^{(1)}(n) = 1,\quad \mathbf{H}^{(2)}(n) = \begin{bmatrix}1 & 0\end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix}\tau^2 & & \\ & \ddots & \\ & & \tau^2\end{bmatrix},\ R = \sigma^2 .$$

where $\mathbf{I}_m$ is the $m \times m$ identity matrix,and $\otimes$ denotes the Kronecker product. It is well known that Kalman filter is effective in the state estimation of the linear state space representation expressed by Eq. 12 and Eq. 13. When we give initial conditions $\mathbf{x}_{0|0}$ and $\mathbf{V}_{0|0}$, the state estimation can be implimented by the following formulas:

[Time update]

$$\begin{aligned}\mathbf{x}_{n|n-1} &= \mathbf{F}_n \mathbf{x}_{n-1|n-1}\\ \mathbf{V}_{n|n-1} &= \mathbf{F}_n \mathbf{V}_{n-1|n-1} \mathbf{F}_n^{\mathrm{T}} + \mathbf{G}_n \mathbf{Q}_{n-1|n-1} \mathbf{G}_n^{\mathrm{T}}\end{aligned} \tag{14}$$

[Measurement Update]

$$\begin{aligned}\mathbf{K}_n &= \mathbf{V}_{n|n-1} \mathbf{H}_n^{\mathrm{T}} \left(\mathbf{H}_n \mathbf{V}_{n|n-1} \mathbf{H}_n^{\mathrm{T}} + R_n\right)^{-1}\\ \mathbf{x}_{n|n} &= \mathbf{x}_{n|n-1} + \mathbf{K}_n \left(y(n) - \mathbf{H}_n \mathbf{x}_{n|n-1}\right)\\ \mathbf{V}_{n|n-1} &= \left(\mathbf{I} - \mathbf{K}_n \mathbf{H}_n\right) \mathbf{V}_{n|n-1}\end{aligned} \tag{15}$$

[Smoothing]

$$\begin{aligned}\mathbf{A}_n &= \mathbf{V}_{n|n} \mathbf{F}_n^{\mathrm{T}} \mathbf{V}_{n+1|n}^{-1}\\ \mathbf{x}_{n|N} &= \mathbf{x}_{n|n} + \mathbf{A}_n \left(\mathbf{x}_{n+1|N} - \mathbf{x}_{n+1|n}\right)\\ \mathbf{V}_{n|N} &= \mathbf{V}_{n|n} + \mathbf{A}_n \left(\mathbf{V}_{n+1|N} - \mathbf{V}_{n+1|n}\right) \mathbf{A}_n^{\mathrm{T}}\end{aligned} \tag{16}$$

In this case, suppose that the variance $\sigma^2(n)$ of observation noise $w(n)$ is constant it can be made the reduction of the dimension of parameters, and the state estimation can be efficiently implemented. And the optimum value of the model order m can be obtained by the minimizing the following Akaike Information Criterion (AIC) [Kitagawa & Gersch, 1996]:

$$\mathrm{AIC}(m) = N\left(\log 2\pi\hat{\sigma}_m^2 + 1\right) + \sum_{n=1}^{N} \log \tilde{d}_{n|n-1} + 2 \tag{17}$$

Where $\tilde{d}_{n|n-1}$ is the covariance matrix of the conditional distribution of $y(n)$ given the distribution at time stump n -1.

RESULTS AND DISCUSSION

In this section, we examined the characteristic of proposed procedure by using the data of free running model experiments conducted by Hashimoto et al. (2005) and shown the effectiveness of the proposed procedure. We analyzed the data which parametric rolling was observed and which parametric rolling was not observed in the experiment in irregular waves. Fig. 1 shows the analyzed results of the data which parametric rolling is observed. And Fig. 2 shows the analyzed results of the data which parametric rolling is not observed. Viewing downward, these figures respectively show the measured time series and the maximum values of the absolute values of the characteristic roots based on the TVAR modeling procedure. In Fig. 1 (a) the characteristic of measured time series, unlike the time series of regular waves, is to change the amplitude and the phase with time. And we can conform that in Fig. 1 (b) the maximum values of characteristic roots are moving to outside of the unit circle around from 50 to 80 second and from 100 to 160 second at which the covariance of measured time series becomes large. In this case, it is considered that the stable state and the unstable state are intermingled in the system. Therefore, we can judge that the stability in this case is wrong.

From Fig. 2 (a) it can be seen that the maximum value of measured time series is over 10 degrees to one side, and so it can be considered that this data is the large amplitude rolling. However, from Fig. 2 (b), it can be seen that the maximum values of characteristic roots are constant with time. Therefore, it can be seen that the system is stable, and we can judge that the stability in this case is good.

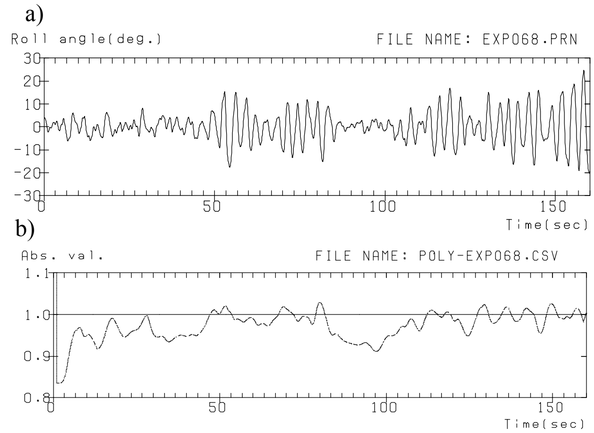

Fig. 1: Results in parametric roll resonance
(a) Measured time series (b) Maximum values of the absolute values of the characteristic roots

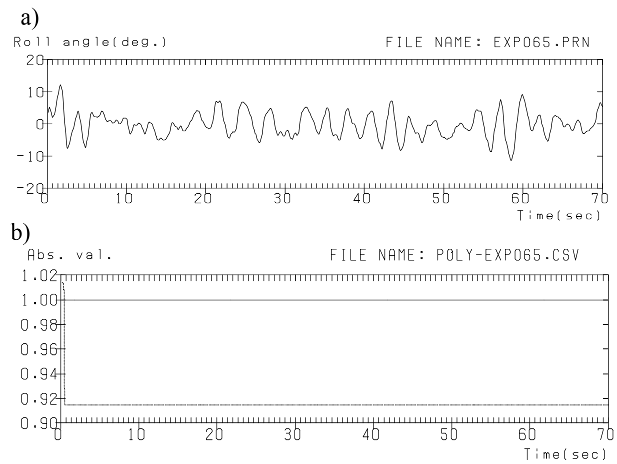

Fig. 2: Results in no parametric roll resonance
(a) Measured time series (b) Maximum values of the absolute values of the characteristic roots

Table 1: Principal perpendiculars of the post-Panamax container ship.

Items	Ship	Model
Lpp: L	238.8m	2.838m
Breadth: B	42.8m	0.428m
Depth: D	24.0m	0.24m
Mean draft: T	14.0m	0.14m
Block coefficient: C_b	0.630	0.630
Metacentric height: GM	1.08m	0.0106m
Natural roll period: T_φ	30.3 s	3.20 s

CONCLUSIONS

In this study, we focused on characteristic roots of TVAR process and attempted to judge the ship safety in large amplitude rolling from the measured ship motions data based on the estimates of characteristic roots with time varying. We examined the characteristic of proposed procedure by using the data of free running model experiments. The results may be summarized as follows:

(a) In experiment in irregular waves, in the case of parametric rolling and no parametoric rolling, the tendency of characteristic roots is different clearly.

(b) TVAR modeling procedure is effective to judge the stability in the large amplitude rolling.

Therefore, we conclude that the proposed procedure is powerful tool for the judgment of the stability.

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