Approximation of Roll Damping by a Second Order Oscillator for Experiments at Different Scales and Numerical Calculations

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ABSTRACT
In this paper, a classic model for roll damping is described. Starting from an experimental signal, we develop an algorithm to extract linear and quadratic damping coefficients, with information regarding the uncertainty of results. This model proves to be relevant for large scale trials, where simulated roll damping fits experimental data. Smaller-scale trials show unpredictable but reproducible phenomena and thus, a physical boundary to Froude extrapolation law. This methodology is used on four different scales (including full scale) of French frigate and also on numerical calculations.

KEYWORDS

Roll ; damping ; scale effect

INTRODUCTION

A long term study about roll damping estimations was decided when testing on a large scale model (1/5) of a French frigate was launched (Leguen 2007). Two other scales were used (1/23 and 1/69). Systematic roll extinction tests were performed. The experimental methodology and signal treatments were improved, in particular for the small model. In parallel to those experiments, numerical calculations were performed with RANS code. To complete this study full scale tests are expected and numerical calculations will follow up.

ROLL DAMPING ESTIMATION

Roll Equation

The efficiency of roll stabilization devices requires trials or experiments. In order to quantitatively compare different hull configurations, one must extract relevant coefficients from roll signals.

Provided the initial heel angle is zero, a free roll motion is described by the following equation:

$$(I + A)\ddot{\varphi} + D(\dot{\varphi}) + C\varphi = 0$$

where I is the inertia of the ship in x-axis rotation, A its added inertia, C the stiffness and $D(\dot{\varphi})$ the damping function.

Around the initial heel angle we have, $C = \Delta g GM_t$ where Δ is the displacement, GM_t the transverse metacentric height, g the gravity acceleration.

One of the classical descriptions for $D(\dot{\varphi})$ is based on a quadratic approximation:

$$(A + I)\ddot{\varphi} + B_1\dot{\varphi} + B_2\dot{\varphi}|\dot{\varphi}| + C\varphi = 0 \quad (1)$$

Although it shows a singularity at $\dot{\varphi} = 0$, it has a physical meaning. $B_1\dot{\varphi}$ corresponds to wave-making energy loss (among others), and $B_2\dot{\varphi}|\dot{\varphi}|$ to viscous damping mechanisms.

Calculation of Coefficients from a Roll Signal

A general solution for (1) would be:

$$\varphi(t) = f(t)\cos(\omega t)$$

where $f(t)$ is a concave function which decreases to zero, and which we call envelope. In the case of roll damping, the oscillator is pseudo-periodic and weakly damped in comparison to an aperiodic system. In other words, $f(t)$ decreases slowly. This simple assumption is crucial to our algorithm.

The first step consists in finding the local extrema of the roll signal and the associated timestamps. Notations used in this procedure are summed up in Figure 1: $\Phi_{2n} = \varphi(t_{2n})$ is the n^{th} maximum and $\Phi_{2n+1} = \varphi(t_{2n+1})$ is the n^{th} minimum.

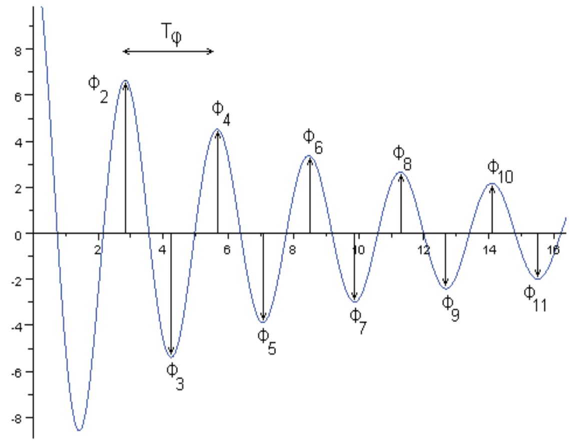

Fig. 1: Typical roll signal, definitions

First, some results concerning linear oscillators are still useable. With this kind of oscillators, the analytical solution yields:

$$\omega = \frac{\sqrt{4(A+I)C - B_1^2}}{2(A+I)} \tag{2}$$

which becomes, knowing that damping is weak:

$$\omega \approx \sqrt{\frac{C}{A+I}}$$

Hence:

$$A + I \approx C\left(\frac{T_\varphi}{2\pi}\right)^2$$

From the maxima we calculate:

$$\begin{cases} \delta\phi_{2n} = \phi_{2n} - \phi_{2n+2} \\ \bar{\phi}_{2n} = \dfrac{\phi_{2n} + \phi_{2n+2}}{2} \end{cases}$$

Variations of the envelope being small, at the maxima, we have $\cos(\omega t_{2n}) \approx 1$ and $\sin(\omega t_{2n}) \approx 0$. Therefore:

$$\begin{cases} \delta\phi_{2n} & \approx f(t_{2n}) - f(t_{2n+2}) \\ & \approx (t_{2n} - t_{2n+2})\dot{f}\left(\dfrac{t_{2n} - t_{2n+2}}{2}\right) \\ & \approx -T_\phi \dot{f}(t_{2n+1}) \\ \bar{\phi}_{2n} & \approx \dfrac{f(t_{2n}) + f(t_{2n+2})}{2} \approx f(t_{2n+1}) \end{cases} \tag{3}$$

Between t_{2n} and t_{2n+1}, $\varphi(t)$crosses the zero-threshold twice, at t_+ and t_-, where respectively :

$$\begin{cases} \cos(\omega t_+) = 0 \\ \sin(\omega t_+) = 1 \end{cases} \text{ and } \begin{cases} \cos(\omega t_-) = 0 \\ \sin(\omega t_-) = -1 \end{cases}$$

At those instants, (1) becomes:

$$2(A+I)\omega \dot{f}(t_\pm) + B_1 \omega f(t_\pm) + B_2 \omega^2 f^2(t_\pm) = 0$$

Provided $f(t)$ is sufficiently regular, this property is also true at

$$t_{2n+1} \approx \frac{t_+ + t_-}{2}.$$

Putting in (3) then gives:

$$\frac{\delta\phi_{2n}}{\bar{\phi}_{2n}} = \frac{\pi B_2}{(A+I)}\bar{\phi}_{2n} + \frac{T_\phi B_1}{2(A+I)}$$

The same procedure with minima leads to:

$$\frac{\delta\phi_{2n+1}}{\bar{\phi}_{2n+1}} = \frac{\pi B_2}{(A+I)}\left(-\bar{\phi}_{2n+1}\right) + \frac{T_\phi B_1}{2(A+I)} \tag{4}$$

By fitting a curve $y = ax+b$ on the data set

$$\left(\pm\bar{\phi}_n, \frac{\delta\phi_n}{\bar{\phi}_n}\right),$$

using the least squares method, we come to:

$$\begin{cases} B_1 = b\dfrac{CT_\phi}{2\pi^2} \\ B_2 = a\dfrac{CT_\phi^2}{4\pi^3} \end{cases}$$

To sum up, only with the assumption that the envelope $f(t)$ is regular enough, we are able to determine all the unknown coefficients in (1).

Mathematical Bias in the Extraction Algorithm

The first step into validating the algorithm consists of running it on theoretical signals strictly obeying (1). A database of 49 signals is generated by a 4th order Runge-Kutta solver using (1), starting with a roll angle of 12° and no initial speed, with following parameters (in arbitrary units):

$$\begin{cases} (A+I) = 25 \\ C = 100 \\ \mathrm{B}_1 \in \{0.1\,;2\,;4\,;6\,;8\,;10\,;12\} \\ \mathrm{B}_2 \in \{0.01\,;0.2\,;0.4\,;0.6\,;0.8\,;1\,;1.2\} \end{cases}$$

B_1 and B_2 were chosen as fractions of the critical damping, which ensures that the database covers a wide range of typical existing ships and even more (figure 2).

Plotting the data set (4) shows that points are indeed aligned. Only configurations with very strong linear damping show scattered points when $\overline{\phi}_n$ is small (figure 3). It is very probably due to numerical precision, since heel angle in this domain gets smaller than 0.5°. It can be easily prevented by cutting the signal as soon as roll amplitude is too weak (furthermore, it is not recommended to use experimental signals with such small amplitudes, due to noise). Linear regression is relevant despite approximations done in the second subsection. Correlation coefficient of the fitted curve is, in every case, excellent.

Despite this satisfactory result, further computations show that the procedure has an intrinsic bias. *A+I* is slightly over-evaluated, B_2 heavily under-evaluated and B_1 shows no clear trend.

Bias on *A+I* is obviously due to the approximation used in (1). Taking into account damping leads to smaller values for *A+I*. Errors on B_1 are difficult to explain but remain small, which is not the case with B_2.

Since calculations of the second subsection cannot give any analytical estimation of the bias, a computational solution is needed. Mapping the bias for exhaustive combinations of the four parameters in (1) is illusive, so we decided to add the following loop in the algorithm (described in figure 4), so that it estimates its own bias:

Applying this correction strongly reduces errors on B_2.. It can be applied to B_1. as well, where it brings a small but not systematic improvement. Effect on *A+I* is significant and of interest only when damping is strong.

Acceptance Criteria on Measured Roll Signals

Validating the procedure on non-simulated signals, for which coefficients are unknown, yields another question: how to evaluate errors made on B_1 and B_2.

Therefore, we introduced three new factors, which are automatically calculated. The most obvious one (and most useful) comes at the end of the procedure. Knowing the first extremum in the measured signal, where speed roll is zero, a 4th order Runge-Kutta solver allows to rebuild the signal, and then determine the correlation coefficient between the measured and the simulated data. We call it Q, for "quality factor (of 2nd-order oscillator)".

From our many experiments, we observe that a quality factor greater than 0.99 is often reached and means the 2nd-order model very well describes physical phenomena causing roll damping. Between 0.95 and 0.99, it is mainly a sign that the trial has been disturbed by environmental factors. Under 0.95, validity of the 2nd order model is doubtful, and one should consider developing equation (1) to higher orders. Fortunately enough, this only happens in peculiar cases which do not represent actual ships.

On figure 5a, we see that the 2nd order model perfectly fits experimental data when Q is almost 1. 5b shows an example where the model gives a good approximation (in amplitude as well as in time), but measured roll damping was disturbed by waves. On 5c, our model is clearly not valid.

The least square method used in (4) calculates the standard deviation of curve coefficients *a* and *b*, from which we deduce the error (in percentile) on B_1 and B_2.

NUMERICAL CALCULATIONS

RANS simulations

With the improvement of numerical methods or algorithms and the development of parallel computing capabilities, we used the RANSE (Reynolds Averaged Navier-Stokes Equations) free surface solver ICARE, to complete the experiments of roll decay by DGA Hydrodynamics.

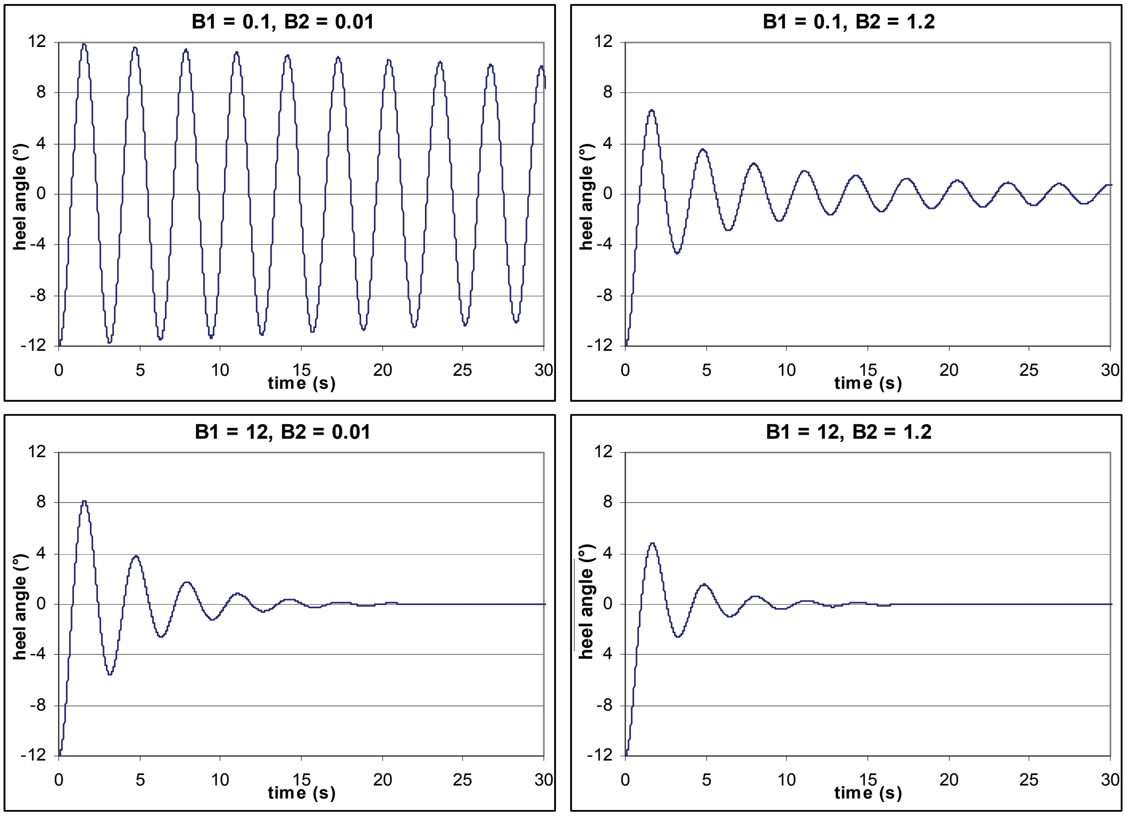

Fig. 2: Theoretical signals for different sets of parameters

RANSE free surface solver, ICARE

ICARE solves the viscous, turbulent and free surface flow around a ship with a forward speed by the Reynolds Averaged Navier-Stokes Equations. These equations are written in an unsteady curvilinear convective form and solved on a computational space grid fitted to the hull and to the free surface at each iteration using a free surface tracking method. It is based on 2nd order finite difference schemes for discretization of equations. A fully-coupled linear system in velocity, pressure and free surface elevation is solved by a bi-CGSTAB algorithm. Free surface elevation, the 3 Cartesian velocity components, pressure including gravitational effects and turbulent kinetic energy are the dependant unknowns. Mass conservation is expressed as the classical continuity equation. Finally to take into account the turbulence effects and to close the equations set, a classical k-ω turbulence model proposed by (Wilcox, 1988) is used. Free surface boundary conditions are the kinematic condition (coming from the continuity assumption, expresses that the fluid particles of free surface stay on it), the two tangential dynamic conditions (given by a linear combination of first order velocities derivatives) and the normal dynamic condition (pressure is assumed to be constant above free surface). These dynamic conditions are given by the continuity of strains at the free surface. Capabilities for use of multiblock grids and parallel computing have been implemented.

Roll decay on a DTMB 5512 model

We have started with evaluating the abilities of the RANS solver, ICARE, to simulate a case of roll decay with a DTMB 5512 model (with bilge keel and forward speed). This case is a part of the Gothenburg workshop (case 3.6). The following table presents the configuration of the case 3.6.

Not only have we studied the case where the roll was free, but we have also investigated the sway and heave influence on the rolling behavior.

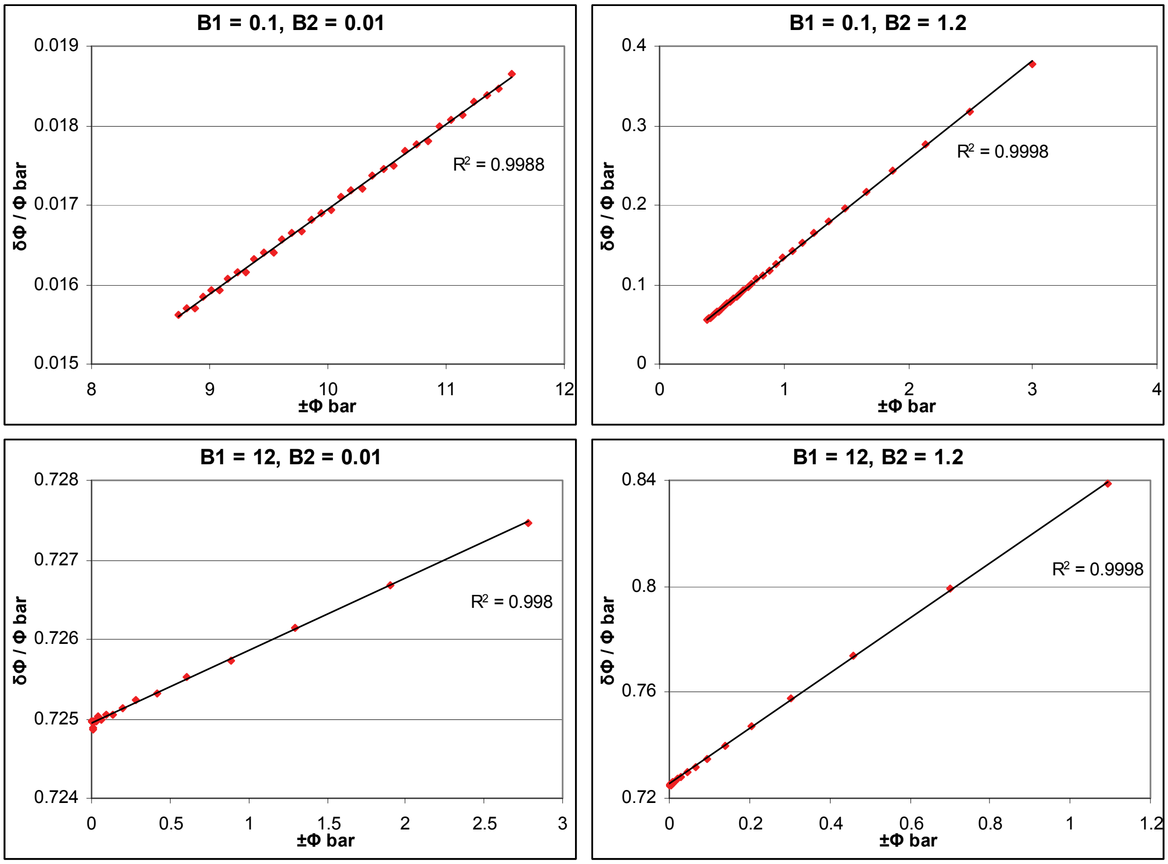

Figure 3 : Fitted curve from theoretical signals.

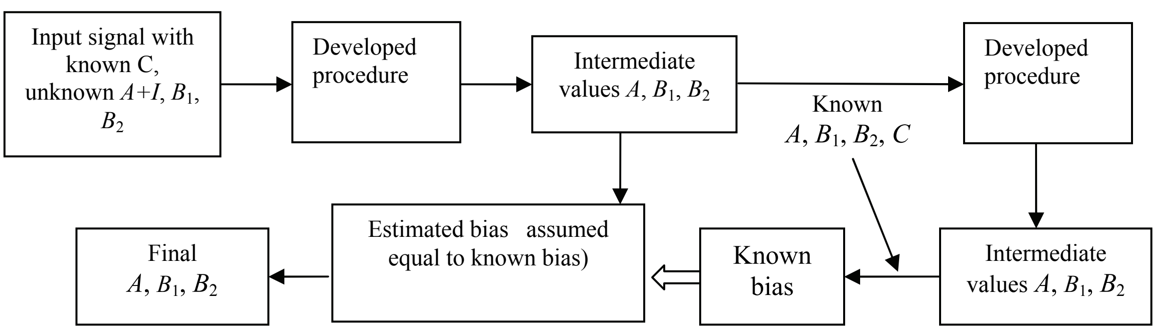

Fig. 4: Algorithm loop

The computations performed with ICARE in the figure 1 show a relatively good agreement with IIHR experiments data. This good agreement depends on which degrees of freedom are free. The computations show that the sway must be free to improve the correlation with the experiments. In fact, there is a strong coupling between roll and sway. The heave has less influence on the rolling behavior; nevertheless it seems more natural to let it free too. The discrepancies are small and can be assumed as numerical or experimental uncertainties. We are confident in using ICARE as a tool to evaluate the quality of our measurements or predict the roll at full scale.

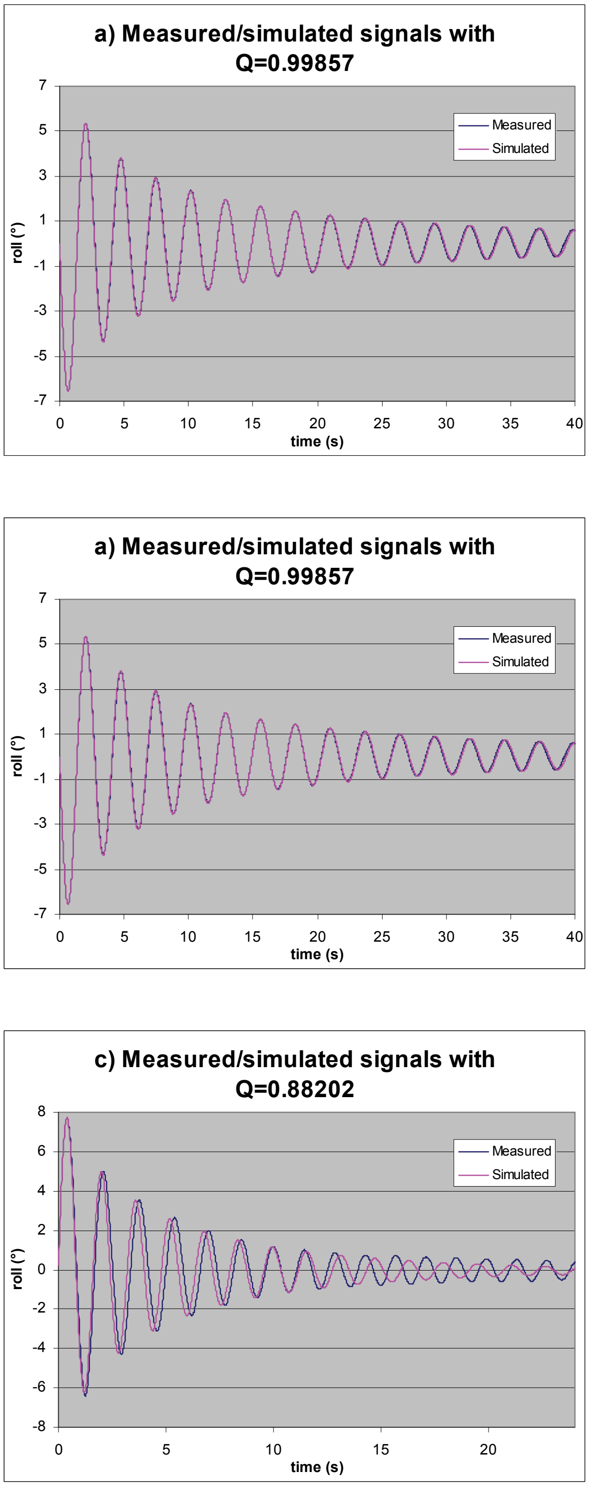

Fig. 5: Measured and simulated roll signals

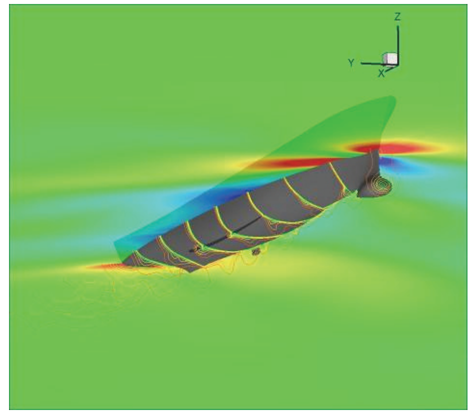

Fig. 6: Roll decay of a DTMB 5512 model with ICARE

Table 1 : Case 3.6 of the Gothenburg workshop

Requested Computations	
Case	3.6
Experiments from	IIHR
Type of data	Roll decay
References	3-4
Scale factor	1 :46.59
Lpp (m)	3.048
Speed (m/s)	0.7543
Froude number	0.138
Reynolds number	$2.56x10^{6}$
Bilge keel fitted	Yes
Sinkage s/Lpp	$2.93x10^{-4}$
Trim (degrees)	$-3.47x10^{-2}$
Wave direction	Calm water
Degrees of freedom	Free to roll
Initial roll angle (degrees)	10

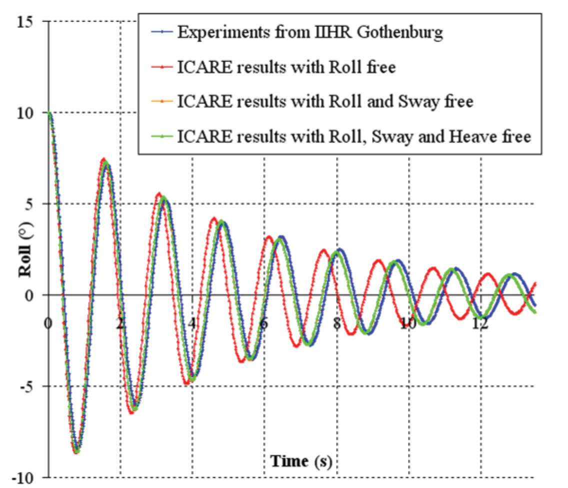

Fig. 7: The influence of the degrees of freedom in the roll decay of a DTMB model

Finally, those calculations allow us to compute seakeeping parameters such as roll decay coefficient, performing:

- natural roll decay with or without forward speed;
- forced roll simulations;
- trim, heave, sway can be fixed or free.

EXPERIMENTS

Experiments were performed on three different scales of a French frigate. Configurations tested include case with or without appendices, cases with different displacements and GM value, case with and without superstructure.

CONCLUSIONS

The first step of this study aimed at selecting a methodology to extract damping coefficients from standard extinction tests and choosing a numerical code. It has to be carried on with more results on scale effect, in particular with the help of results at full scale expected in May 2011.

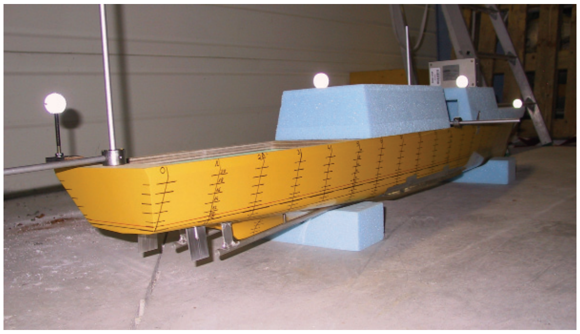

Fig. 8: Picture of the smallest model (1/69, about 1.5 meter and 10 kg).

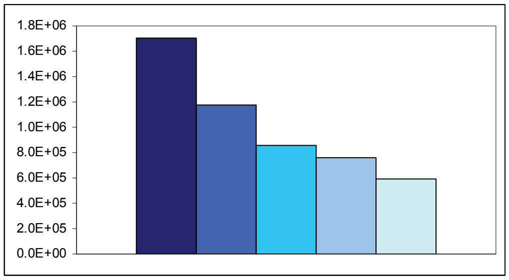

Fig. 9: Influence of appendages on linear roll damping on 1/23 scaled model (from bare hull to configuration with all appendages

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