Criteria for parametric rolling?

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Introduction

Whilst well-known as a phenomenon for at least half a century (Grim 1952; Kerwin 1995; Arndt & Roden 1958; Pauling & Rosenbersg 1959), no specific design requirements referring to parametric rolling have yet found their way into the IMO stability regulations. A possible explanation is that, whilst it is often the cause of intensive rolling, it is rarely documented to lurk behind a specific capsize accident. Yet, the Market is becoming increasingly alert to this problem because even 'non capsizal' instabilities can incur tremendous effects in terms of loss or damage of property and business interruption (Gray 2001; Tinslay 2003). This has led a classification society recently to take the lead and publish a guide for the parametric rolling of containerships (ABS 2004).

By-and-large, the dynamics of the parametric rolling of ships are nowadays well understood (e.g Spyrou 2000, Neves 2002, Bulian et al 2003; Shin et al 2004). Nonetheless, some ships like modern post panamax containerships and probably some of the new large passenger ships that are characterised by their heavily flared bow and flat stern with wide transom, may be sailing without having examined their tendency to display parametric rolling in a longitudinal seaway. Besides issues of ship design and/or operation, there is also a mounting discussion about the effectiveness of our physical model testing techniques for verifying that a hull form does not display tendency for parametric rolling (Belenky et al 2003). Whilst these issues are not separate, here we focus only on the question of design criteria. Specifically, we put forward a new concept for assessing, at an early design stage, the tendency of a ship for parametric rolling, which combines the deterministic and probabilistic sides of the problem and encompasses the following three principles:

<u>Probability of occurrence of critical behaviour:</u>

- The probability of exhibiting parametric rolling due to the encounter of a dangerous wave group should be kept lower than the acceptable level.

<u>Assessment of post-critical behaviour</u>

- Under no circumstances the amplitude of parametric rolling should exceed a limit of safe operation.
- Abrupt growth of roll within a small number of critical wave encounters should not be allowed.

We shall expand on these three principles, taking the last one first:

A transient criterion for initial parametric roll growth

It can be shown that for a Mathieu system the unstable motion in the first region of instability should build-up according to the following approximate general law (Hayashi 1985):

$$\varphi(t) \approx c_1 e^{\mu \omega_0 t} \sin(\omega_0 t - \sigma) + c_2 e^{-\mu \omega_0 t} \sin(\omega_0 t + \sigma) \tag{1}$$

where $a = 4\omega_0^2 / \omega_e^2$ (ω_0 is the natural roll frequency and ω_e is the wave encounter frequency) and h is the scaled amplitude of *GM* variation. Also, μ, σ are functions of a, h determined from the expressions (only first-order terms are kept):

$$\cos 2\sigma \approx \frac{2(a-1)}{ah} \quad (-\frac{\pi}{2} \le \sigma \le 0), \qquad \mu \approx -\frac{\sqrt{a^2 h^2 - 4(a-1)^2}}{4} \tag{2}$$

At $a = 1$ the coefficient μ obtains its maximum value $\mu_{\max} = -h/4$ where $\sigma = -\pi/4$.

From (1) and after substitution of the initial conditions $\varphi(0) = \varphi_0$ and $\dot{\varphi}(0) = 0$ we can extract the growth of amplitude after *p* roll cycles:

$$\frac{\varphi(pT_0)}{\varphi_0}=\frac{e^{\frac{p\pi h}{2}}+e^{-\frac{p\pi h}{2}}}{2} \tag{3}$$

In the presence of linear damping the growth rate is reduced:

$$\varphi(T_0)=w(T_0)e^{-\frac{2\pi k}{\omega_0}}=\frac{e^{\frac{\pi h}{2}}+e^{-\frac{\pi h}{2}}}{2}e^{-\frac{2\pi k}{\omega_0}}w_0=\frac{e^{\frac{\pi}{2}\left(h-\frac{4k}{\omega_0}\right)}+e^{-\frac{\pi}{2}\left(h+\frac{4k}{\omega_0}\right)}}{2}\varphi_0 \tag{4}$$

From the exponential term $e^{\frac{\pi}{2}\left(h-\frac{4k}{\omega_0}\right)}$ we can extract the well-known condition of stability (that rules out the possibility of growth):

$$h_{crit}=\frac{4k}{\omega_0} \tag{5}$$

The above is a condition of asymptotic stability; i.e. in principle a very long sequence of tuned waves having the right height is required for instability. Apparently, there is an advantage in exploiting equation (4), which corresponds to transient rolling, rather than (5), which refers to asymptotic behaviour and may lead to a stringent requirement.

On the basis of (4), roll growth after p roll cycles, at exact resonance, ($a=1$), should be:

$$\varphi(pT_0)\approx e^{-\frac{2p\pi k}{\omega_0}}\left(\frac{e^{\frac{p\pi h}{2}}+e^{-\frac{p\pi h}{2}}}{2}\right)\varphi_0 \tag{6}$$

A q-fold increase of roll amplitude from its initial value should entail p roll cycles:

$$\ln q\approx-\frac{2p\pi k}{\omega_0}+\ln\left(\frac{e^{\frac{p\pi h}{2}}+e^{-\frac{p\pi h}{2}}}{2}\right) \tag{7}$$

Since, after one or two roll cycles the exponential term of (7) with positive sign becomes dominant, the above may be written further, approximately, as:

$$\ln q\approx-\frac{2p\pi k}{\omega_0}+\frac{p\pi h}{2}-\ln 2=\frac{p\pi}{2}\left(h-\frac{4k}{\omega_0}\right)-0.693 \tag{8}$$

It accrues that the number of cycles p required for a q-fold increase of amplitude may be obtained from the expession:

$$h-\frac{4k}{\omega_0}=\frac{0.693+\ln q}{1.571p} \tag{9}$$

The corresponding time t_m should be p times the natural period T_0:

$$t_q=pT_0 \tag{10}$$

To demonstrate the value of (9), let us thing in terms of the following criterion: a 10-fold increase of roll amplitude should not come about in less than 4 roll cycles (which means 8 wave encounters – it is possible to link this to the probability of encountering a dangerous wave group). Let's consider waves with $\lambda/L=1.0,\ H/\lambda=1/20$. These translate into the following requirement for $h,\ k$:

$$h-\frac{4k}{\omega_0}\le\frac{0.693+\ln 10}{1.57\times 4}\approx\frac{3}{6.282}=0.477 \tag{11}$$

In summary, by focusing on transient response, we can determine the critical h, or equivalently the critical damping, for any wave group run length, thus achieving a meaningful and flexible interface with the probabilistic nature of ocean waves (we expand on this later). The condition of asymptotic stability is recovered from (9) if we set $p \to \infty$. The criterion should probably be supplemented by a time requirement based on (10). For example, the 4 roll cycles should take $25.7 \times 4 = 102.8$ s. For very low natural frequencies, the required time becomes excessive. This leaves time for reaction (i.e. change of speed or heading) if the beginning of the phenomenon is promptly recognised.

GROWTH ENDS (NONLINEARLY) ON THE STEADY ROLL OSCILLATION

As is well known, there is no reason for this growth to persist up to infinity and thus lead by necessity to capsize. The detuning due to the nonlinear character of the GZ curve together with the increased dissipation due to the mild nonlinearity of damping, create the prospect of steady oscillatory rolling with moderate amplitude. In effect, for a typical parametric growth with nonlinear restoring the boundary curves of stability discussed earlier represent loci of bifurcations giving birth to unstable and unstable oscillatory behaviour [see for example Skalak & Yarymovych 1960; Soliman & Thompson 1992].

The instability boundary curves of the upright state of a ship do not contain entirely the domain where parametric oscillations are realisable. The emerging stable roll oscillations need not be confined inside the "tongues" of the linear system and stable oscillations exist also well outside these regions [Scalak & Yarymovich 1961, Thompson & Soliman 1993, Francescutto & Dessi 2001]. Should we worry about these nonlinear oscillations that "lıve" outside the "tongues"? The answer is probably yes. In an idealised environment of a periodic seaway that is free from other external disturbances, the ship should find no reason to leave the upright state as long as the combination of frequency ratio and parametric amplitude corresponds to some point in the region of stability. However, should the stable upright condition be *sufficiently disturbed*, this oscillatory behaviour can be incurred *in an abrupt way*. We may say that the probability of occurrence of parametric rolling decreases as we move away from the condition of exact resonance but it is doubted whether it should be assumed as acceptably low. Perhaps we should place less emphasis on the necessity of fulfilling the condition of exact principal resonance for the occurrence of parametric rolling. To explain these points further, let us consider a Mathieu-type roll equation with a single, cubic nonlinear term:

$$\ddot{\varphi} + 2k\,\dot{\varphi} + \omega_0^2\left[1 - h\cos(\omega_e\, t)\right]\varphi - n\omega_0^2\,\varphi^3 = 0 \tag{12}$$

The constant n could be negative, in which case we are examining the oscillations corresponding to the initial part of the $GZ(\varphi)$ curve which is often of "hardening spring" type; or it could be positive in which case we may refer generically to the entire $GZ(\varphi)$ curve up to the angle of vanishing stability.

Application of a perturbation method like harmonic balance or averaging leads to the following explicit formula for the amplitude:

$$A^2 = \frac{4}{3n}\left[\left(1 - \frac{1}{a}\right) \mp \sqrt{\frac{h^2}{4} - \frac{4k^2}{a\omega_0^2}}\right] \tag{13}$$

Setting $A \to 0$ we find the curve whereon the oscillations are created. It comes to no surprise that this curve is independent of the nonlinear coefficient n and it coincides with the boundary of linear stability. Also, the term inside the square root, as well as the whole expression of A^2, should be non-negative. For an initially hardening restoring ($n < 0$) these yield,

$$h \geq \frac{4k}{\omega_0\sqrt{a}} \qquad \text{and} \qquad a \leq 1 \tag{14}$$

In essence, (14) defines the locus of "saddle-node" (or so-called "fold") bifurcations. On this curve, the unstable periodic orbits that are shed from the left boundary of the instability region perform a U turn and become stable (this can be understood considering that, for negative n, at the 'lower a' boundary of the instability region a subrcritical bifurcation takes place thus creating unstable oscillations; whereas the boundary at $a > 1$ gives birth to a supercritical one i.e. stable oscillations).

Condition (14) determines the true boundary of periodic response. For a certain level of h, the region where stable oscillations are encountered is wider than the predicted from the linear analysis. Fig. 1 shows the development of amplitude as function of a and h ($A = A^*\sqrt{-4/3n}$) for a container with the dimensions of APL China and $\omega_0 = 0.2448 s^{-1}$, $k = 0.015 s$. The stability of the emerging steady roll oscillations is shown (dashed line represents unstable).

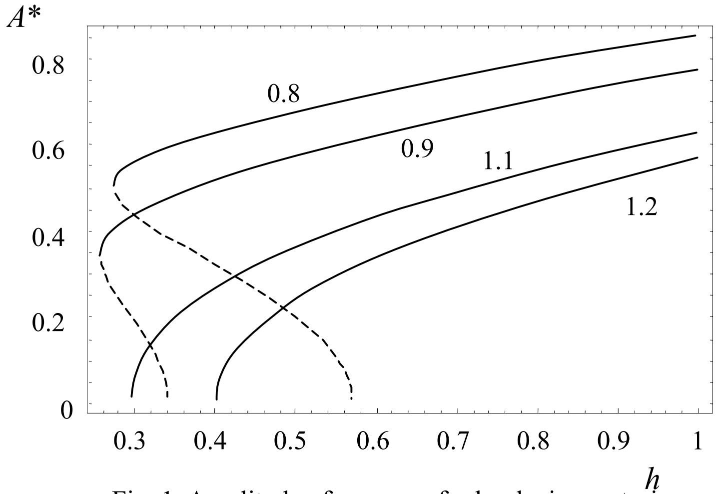

Fig. 1: Amplitude of response for hardening restoring

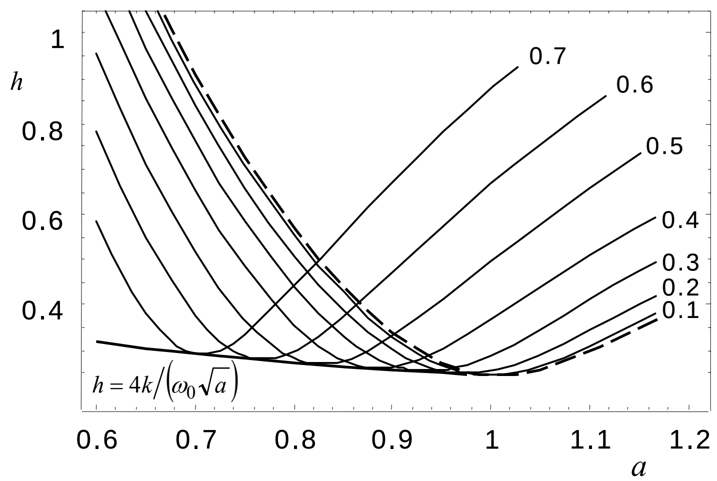

Fig. 2: Iso-A curves (from 0.1 to 0.7) for k=0.015, n=-2.35 (hardening), $\omega_0 = 0.2448\ s^{-1}$.

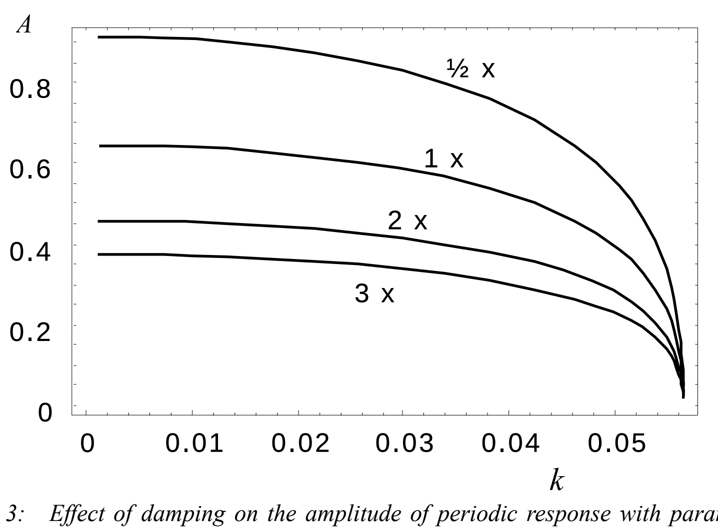

Fig. 3: Effect of damping on the amplitude of periodic response with parameter the coefficient of nonlinear stiffness ("hardening")

The domain of oscillatory behaviour (bounded by the thick continuous line) can easily be found with some manipulation:

$$h = \sqrt{4\left[\frac{3n}{4}A^2 - \left(1 - \frac{1}{a}\right)\right]^2 + \frac{16k^2}{\omega_0^2 a}} \tag{15}$$

The combinations of $(h,\, a)$ that give rise to oscillations of predefined A^* I s shown in Fig. 2. It is not difficult to prove that the descending part of each iso-A^* curve corresponds to stable rolling and the ascending to unstable. The boundary of stable rolling is reconfirmed. As we have multiple coexisting stable responses, the initial conditions and the availability of sufficiently strong external disturbances determine whether the ship can stay upright, or should adopt the one (desired) or the other (undesired and possibly dangerous) way of behaviour.

In Fig. 3 is shown the variation of the roll amplitude A for n=-2.35, as a function of the linear damping and the amplitude of parametric forcing. It should be noted that the amplitude of response is second order quantity in terms of damping. Hence, for small (yet realistic for many operating ships) damping, the effect on the response amplitude is small. The same applies for h. A more influential parameter is the coefficient of the cubic stiffness term n which, at first approximation, is linear to the response amplitude.

Effect of the fifth order term (initially hardening, then softening):
Consider again the roll equation, this time with a fifth-order polynomial for restoring which can take better into account the details of the GZ curve up to large inclinations:

$$\frac{d^2\varphi}{d\tau^2} + \frac{2k\sqrt{a}}{\omega_0}\frac{d\varphi}{d\tau} + a\left(1 - h\cos 2\tau\right)\varphi - na\,\varphi^3 - ma\varphi^5 = 0 \tag{16}$$

The approximate steady-state solution is:

$$A^2 = -\frac{3}{5}\frac{n}{m} \pm \sqrt{\left(\frac{3}{5}\frac{n}{m}\right)^2 - \frac{8}{5m}\left(-1 + \frac{1}{a} \pm \sqrt{\frac{h^2}{4} - \frac{4k^2}{\omega_0^2 a}}\right)} \tag{17}$$

We select a GZ curve (see Fig. 4) very close to that of the post-panamax containership of France et al (2001). The selected values for the coefficients n and m are, respectively, -0.14 and 0.25. The amplitudes as functions of the parametric term h, for the frequency ratios examined earlier, are shown in Fig. 5. Several changes of stability are taking place on each one of these curves. An interesting feature of this diagram is that it shows the behaviour at large angles where the fifth order term of the restoring function becomes influential. Contrast of Fig. 5 with Fig. 1 is enlightening in this respect.

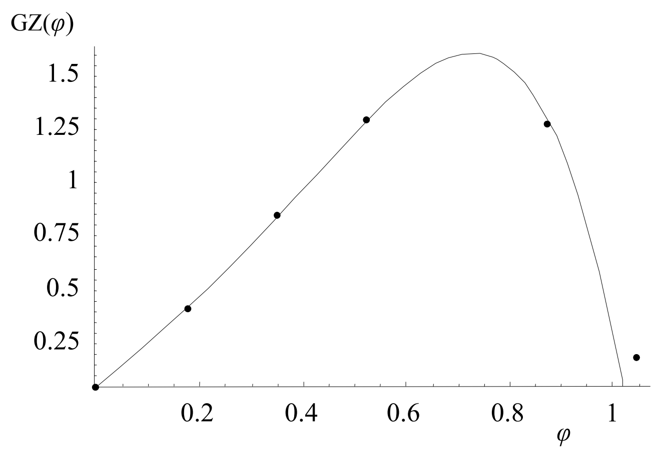

Fig. 4: The exact (dots) and the polynomial fit (line) of the considered GZ curve.

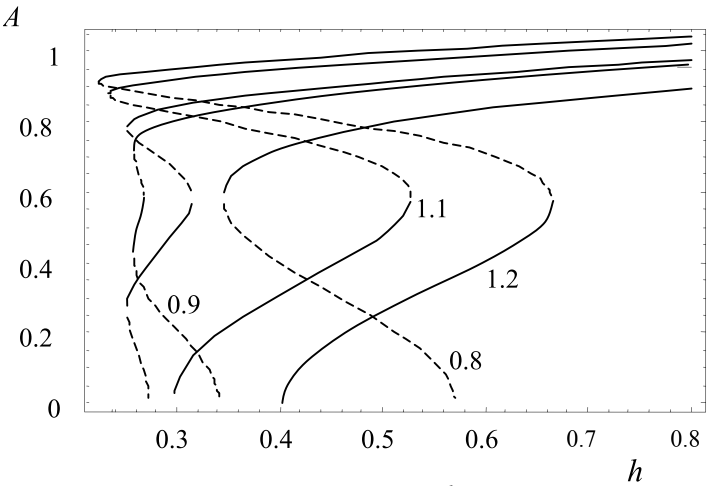

Fig. 5 Amplitude of roll oscillation for 5th order restoring, assuming time dependence only in the linear term. The parameter is a.

<u>Requirement of limited amplitude of steady parametric rolling</u>

A supplementary criterion based on the steady roll amplitude may thus be introduced at this stage as a second requirement concerning post-critical behaviour: For the critical wave, say with $\lambda/L = 1.0$, $H/\lambda = 1/20$, the max roll amplitude should not exceed, say, 15 deg (this value is proposed by ABS; perhaps this value can be vessel specific depending, e.g. on lashings' strength for a container). The combination of restoring and damping coefficients that satisfy the requirement of no exceedance of this limiting angle can be obtained with some manipulation of (13); or of (17) if judged that the later part of the GZ curve should also be considered (for a 15 deg limit it is unlikely to be necessary).

Interfacing with the probabilistic seaway: the key is in wave groupiness

It is well known that higher waves tend to arise in groups. As the nearly regular characteristics of waves in a group are essential for giving rise to fundamentally resonant motions like parametric rolling, there is a meaningful link between the probabilistic nature of ocean weaves and the deterministic analysis. The probability of occurrence of parametric rolling could be assumed to be equal to the probability of encountering longitudinally a wave group with sufficient run length and exceeding the threshold height (determined from the deterministic analysis), given that the frequency falls in the critical range (which however has to be very wide). One may use theoretical or parametric models for joint distributions of wave parameters but in general the required multivariate distributions are not available in the literature. As a matter of fact, in practical terms one has to make certain assumptions about the correlation of key parameters in a wave group and opt to use available bivariate distributions either of wave height and period, or of successive wave periods, or finally of successive wave heights.

Ocean wave statistics suggest that height H and period T are correlated (e.g. Longuet-Higgins 1975). The following bivariate distribution proposed by Longuet-Higgins (1983) is based on the spectral width parameter $\nu = \frac{m_0 m_2 - m_1^2}{m_1^2}$ (which has the advantage of depending only on the first three moments):

$$p(H,T) = \frac{\pi f(\nu)}{4}\left(\frac{H_*}{T_*}\right)^2 e^{-\frac{\pi H_*^2}{4}\left(1+\frac{1-\frac{\sqrt{1+\nu^2}}{T_*}}{\nu^2}\right)} \tag{18}$$

where $\quad H_* = \frac{H}{\bar{H}}, T_* = \frac{T}{\bar{T}_z}, f(\nu) \approx \frac{2\left(1+\nu^2\right)}{\nu + \frac{\nu}{\sqrt{1+\nu^2}}}$

is the spectral width parameter, m_0, m_1, m_2 are respectively zeroth, first and second moment of the wave spectrum, $\overline{H}$ is the mean wave height; and $\overline{T}_z$ is the mean zero-upcrossing period that is determined from spectral moments, $\overline{T}_z = 2\pi\sqrt{m_0 / m_0}$.
The use of joint distributions of successive wave periods for assessing probabilistically resonant ship rolling in beam seas was investigated by Myrhaug et al. (2000). However, unlike parametric rolling, the wave frequency where resonance occurs is well defined in a beam sea because the speed of the ship does not influence the encounter frequency. Moreover, to be initiated, it does not require a threshold wave height, like in the case of parametric rolling. Hence, an approach based on the statistics of successive wave heights, i.e. the condition of having a wave group with heights exceeding a known critical level, seems to be a more relevant statistic for the investigation of parametric rolling.
The probability to encounter a sequence of waves with height above the critical level H_c was considered by Blocki (1980) using the approach of Goda (1976). However he assumed, perhaps in the absence of data in those days, the occurrence of successive heights above H_c as independent events. This means that practically, the probability of encountering a certain run length was underestimated. The degree of correlation of successive wave weights depends on the sharpness of the spectral peak. For the effect of the spectral bandwidth on the distribution of wave height see for example Kimura (1980), Tayfun (1983) and Longuet-Higgins (1984). Stansell et al (2002) found that, as bandwidth increases, there is a rather slight reduction in the mean run and group length, up to a bandwidth $\nu = 0.6$ beyond which they become rather insensitive (to obtain a sense of magnitude we note that $\nu = 0.425$ for a Pierson Moskowitz and $\nu = 0.389$ for a JONSWAP spectrum).

According to Tayfun, the sharpness of the spectral peak reflects the variability of height between successive waves; and spectral peakedness is best represented by the correlation coefficient of the wave envelope R_{HH} which could be calculated as follows:

$$R_{HH} = \frac{E(\kappa) - (1-\kappa^2)\dfrac{K(\kappa)}{2} - \dfrac{\pi}{4}}{1 - \dfrac{\pi}{4}} \approx \frac{\pi}{16 - 4\pi}\left(\kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^6}{64}\right) \tag{19}$$

$E(\ )$, $K(\ )$ are complete elliptic integrals, respectively of the first and second kind. The correlation parameter κ could be calculated as follows (see Stansell et al. 2002 for an extensive discussion on alternative methods of calculation):

$$\kappa(\overline{T}) = \frac{1}{m_0}\sqrt{A^2 + B^2}\ , \quad A = \int_0^\infty E(f)\cos 2\pi f\,\overline{T}\,df\ , \quad B = \int_0^\infty E(f)\sin 2\pi f\,\overline{T}\,df \tag{20}$$

Goda (1976) has found that for swells the correlation coefficient R_{HH} is about 0.6 while for wind waves it is only about 0.2.
Assuming that successive wave heights follow a Rayleigh distribution, Kimura (1980) derived the following bivariate probability density function $p(H_1, H_2)$ for consecutive wave heights:

$$p_{HH}(H_1, H_2) = \frac{4H_1 H_2}{(1-\kappa^2)H_{rms}^2} e^{-\frac{(H_1^2 + H_2^2)}{(1-\kappa^2)H_{rms}^2}} I_0\left(-\frac{2\kappa H_1 H_2}{(1-\kappa^2)H_{rms}^2}\right) \tag{21}$$

where H_{rms} is the root mean square wave height and I_0 is the modified Bessel function of zeroth order. The probability of having two consecutive wave heights above the critical height H_c will be then:

$$P(H_{i+1} \geq H_c \mid H_i \geq H_c) = \frac{\int_{H_c}^\infty \int_{H_c}^\infty p_{HH}(H_1, H_2)\,dH_1\,dH_2}{\int_{H_c}^\infty p_H(H)\,dH} \tag{22}$$

where $p_H(H)$ is the marginal probability density which is Rayleigh type:

$$p_H(H)=\frac{2H}{H_{rms}^2}e^{-\frac{H^2}{H_{rms}^2}} \tag{23}$$

The assumption of Markov chain for successive wave heights leads to the following probability function for the occurrence of a group with length j and peaks higher than H_c which is in fact the probability of occurrence of parametric rolling:

$$P_{pr}=P^{j-1}(1-P) \tag{24}$$

The above value should be multiplied by a susceptibility factor indicating whether the speed range of the ship produces encounter frequencies that overlap with the frequencies of principal resonance.

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