Some Remarks on Theoretical Modelling of Intact Stability

N. Umeda, Y. Ohkura, S. Urano, M. Hori and H. Hashimoto
Dept. of Naval Architecture and Ocean Engineering, Osaka University,
2-1, Yamadaoka, Suita, Osaka, 565-0871, Japan

Abstract

It is essential for realising risk-based design or performance-based criteria to establish theoretical modelling of extreme behaviours of an intact ship in waves up to capsizing. For this purpose, this paper summarises latest progresses in the researches conducted by the authors at Osaka University for improving theoretical modelling of intact stability in waves.

Firstly, the methodology for calculating capsizing probability in beam wind and waves with piece-wise linear assumption is examined. Here a new formula covering both capsizing towards windward and leeward directions is provided.

Secondly, for calculating restoring moment in longitudinal waves with sufficient accuracy, a hydrodynamic prediction method with wave-making and lifting components taken into account is proposed, and is reasonably well compared with captive model tests of a post Panamax container ship in head and following waves. This can be used in place of the Froude-Krylov prediction or captive tests for assessing the danger of parametric rolling for a practical purpose.

Thirdly, for more efficiently identifying dangerous conditions in following and quartering waves, a numerical method for determining a heteroclinic bifurcation is proposed with its successful example to applied to surf-riding thresholds with nonlinear factors taken into account.

Keywords

capsizing probability, dead ship condition, parametric rolling, restoring arm in waves, surf-riding, heteroclinic bifurcation.

Introduction

At the International Maritime Organisation (IMO), the review of Intact Stability Code started in 2002 and could open a door to use direct stability assessment with physical and theoretical modelling as an alternatives to prescriptive rules in the near future. Here quantitative accuracy of these modelling is essential. The benchmark testing of the 23rd International Towing Tank Conference (ITTC), however, revealed that existing theoretical models for predicting capsizing do have only qualitative accuracy. This means that the existing best models can predict whether a ship capsizes or not and how she does but they can not predict exactly when she does after the specified initial conditions. (Vassalos et al., 2002) Therefore, improvement of theoretical modelling is indispensable for its practical use.

After the 23rd ITTC, the research team of Osaka University continues to improve its own theoretical models from various aspects. (Umeda, Hashimoto et al., 2003, Umeda et al., 2004, Hashimoto et al., 2004A, Hashimoto et al., 2004B) In this paper, some of their latest progress are presented to initiate the discussion at the stability workshop. These theoretical modelling to be investigated at Osaka University covers the dead ship condition, parametric rolling and broaching. The dead ship condition is related to the revision of the weather criterion and the others were the capsizing scenarios that the 23rd ITTC benchmark testing specified.

Capsizing probability in beam wind and waves

Although a ship could capsize in following and quartering seas with much smaller wave steepness than in beam seas, it is still important to assess stability in beam seas. This is because dangerous situations in following and quartering seas can be avoided by ship operation but those in beam seas cannot be. For example, if all operational means such as propeller thrust and rudder control are lost, a ship that is almost longitudinally symmetric suffers bean wind and waves. Here, since a ship cannot escape from a severe sea state by herself, she should survive for sufficiently long duration. For guaranteeing survivability of such dead ship condition (IMO, 2001), the weather criterion was adopted by several administration (Yamagata, 1959) and then by IMO (IMO, 2002).

Since this criterion utilises some empirical methods to predict aero- and hydrodynamic coefficients, applicability of this criterion to new ship types such as a large passenger ship is rather questionable. However, because the probabilistic safety level of the weather criterion was adjusted by selecting the average wind velocity with casualty statistics, (Yamagata 1959) it is necessary to evaluate the safety level that the current weather criterion implicitly guarantees for conventional ships. (Umeda et al. 1992, Umeda & Yoshinari 2003) For this purpose, analytical methods to calculate capsizing probability in beam wind and waves are indispensable. This is because numerical or experimental method requires prohibitively many realisations to obtain the reliable value of capsizing probability for a practical ship, which should have very small capsizing probability. Among some existing analytical methods, a piece-wise linear approach proposed by Belenky (1993) seems to be the most promising because it utilises analytically-obtained exact bur simple solutions of linear equations only.

The authors investigate Belenky's method by executing numerical calculation with his exact and simplified methods and extend his methods to the case in beam wind and waves with both capsizing in windward and leeward directions taken into account. (Paroka, Ohkura and Umeda, 2004) As a result, some problems and their measures are found. Among them this paper shows one problem that capsizing probability could exceed 1 when a sea state is extremely severe. According to Belenky (1993), the formula of capsizing probability for the duration T without beam wind is as follows:

$$P(T) = 2P_T(\phi > \phi_{mo})P_A(A > 0; \phi > \phi_{mo}) \tag{1}$$

where

$$A = \frac{1}{\lambda_1 - \lambda_2}\left\{\left(\dot{\phi}_1 - \dot{p}_1\right) + \lambda_2(p_1 - \phi_1)\right\} \tag{2}$$

ϕ: apparent roll angle, ϕ_{m0}: border of two roll angle regions, λ_1, λ_2: eigenvalues of the roll system in the second range, ϕ_1. $\dot{\phi}_1$: initial values of roll angle and roll angular velocity in the second range, p_1, $\dot{p}_1$: initial values of forced roll angle and roll angular velocity in the second range, P_T: probability of at least one up-crossing and P_A: probability of positive value of the coefficient A when the up-crossing of ϕ occurs.

On the other hand, the authors propose the following formula for the case of wind and waves.

$$\begin{aligned} P(T) = {} & P_l P_T(\phi > \phi_{mo} or \phi < -\phi_{mo})P_A(A > 0; \phi > \phi_{mo}) \\ & + P_w P_T(\phi > \phi_{mo} or \phi < -\phi_{mo})P_A(A < 0; \phi < -\phi_{mo}) \end{aligned} \tag{3}$$

where

$$P_l = \frac{u_l}{u_l + u_w}; \quad P_w = \frac{u_w}{u_l + u_w}, \tag{4}$$

u_l: expected number of up-crossing at ϕ_{mo} for a unit time and u_w: expected number of down-crossing at $-\phi_{mo}$ for a unit time. If we ignore wind here, the following formula can be obtained:

$$P(T) = P_T(\phi > \phi_{mo})P_A(A > 0; \phi > \phi_{mo}). \tag{5}$$

Therefore, P(T) defined here cannot be greater than 1.

The authors applied the above new formula to a car carrier, whose principal particulars are shown in Table 1. Numerical results are presented in Fig. 1. Here the righting arm is approximated with two lines by keeping the metacentric height, the angle of vanishing stability and the dynamic stability from upright to the angle of vanishing stability because a time-varying external force induces capsizing in this scenario. The wind velocity, U_T, is assumed to be constant and waves are done to be fully developed with this wind velocity and to be modelled with ITTC spectrum. P_A are calculated with both exact and simplified methods. The former is derived from 3-dimensional Gaussian probabilistic density function; the latter is based on the assumption that no resonance occurs in the second region. The results demonstrate that capsizing probability tends to 1 when the sea state becomes extremely severe and Belenky's simplified method slightly overestimates capsizing probability obtained by the exact method. Further discussion will be published in a separate paper. (Paroka, Ohkura and Umeda, 2004) Moreover, the effect of fluctuating wind has been incorporated for the capsizing probability calculation by the authors. (Francescutto, Umeda et al. 2004)

Table 1 Principal dimensions of the car carrier

Items	*Car carrier*
Length overall: Loa	190 m
Length between perpendicular: Lpp	180 m
Breadth: B	32.20 m
Draft: T	8.925 m
Vertical centre of gravity: KG	14.105 m
Metacentric height: GM	1.300 m
Lateral projected area: A_L	4327.860 m^2
Height to centre of lateral projected area: H_C	11.827 m

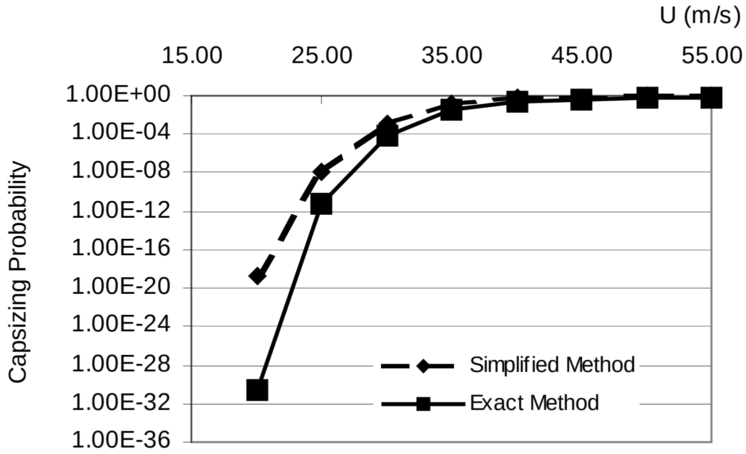

Fig. 1 Capsizing probability of the car carrier in beam wind and waves by using the simplified and exact methods of the piece-wise linear approach. (Paroka Ohkura & Umeda 2004)

Hydrodynamic effect on the change in GM due to waves

At the 23rd ITTC benchmark testing even the most reliable mathematical model can provide only qualitative agreements with free-running model experiments for capsizing due to parametric rolling. As well known, parametric rolling is induced by the change in GM due to waves. Thus the accurate prediction of the change in GM due to waves is essential but all mathematical model in the benchmark testing predict the change in GM due to waves with the Froude-Krylov assumption.

The captive model experiment for the ITTC A-1 Ship, as a follow-up of the ITTC benchmark testing, revealed that the Froude-Krylov calculation significantly overestimates the change in GM due to waves. (Umeda et al., 2004) Possible reason of the difference between the experiment and the Froude-Krylov calculation are hydrodynamic forces such as a wave-making effect and a hydrodynamic lift due to a heeled hull. For modelling such hydrodynamic effects on the change in GM due to waves, Boroday (1990) attempted to use a strip theory with the heel angle taken into account. He reported that good agreement with model experiment is obtained by ignoring wave-making damping components but the wave-making damping deteriorates the agreement.

For finding a final conclusion on this issue, the authors reformulate a strip theory to calculate the change in GM due to waves for a heeled ship in longitudinal waves. Here all radiation components in the roll moment due to heave and pitch motions and the heel effect of the roll diffraction moment are consistently taken into account; the end terms are also included to explain hydrodynamic lift components. Two-dimensional hydrodynamic forces are estimated with an integral equation method with the Green function. The subject ship used here is a 6600TEU post-Panamax container ship, whose principal particulars are shown in Table 2.

Table 2 Principal dimensions of the post Panamax container ship

Items	*Containert Ship*
length : L_{pp}	283.8m
breadth : B	42.8m
depth : D	24.0m
draught at FP : T_f	14.0m
mean draught : T	14.0m
draught at AP : T_a	14.0m
block coefficient : C_b	0.630
pitch radius of gyration : κ_{yy}/L_{pp}	0.239
longitudinal position of centre of gravity from the midship : x_{CG}	5.74m aft
metacentric height : GM	1.08m
natural roll period : T_{ϕ}	30.3 s.

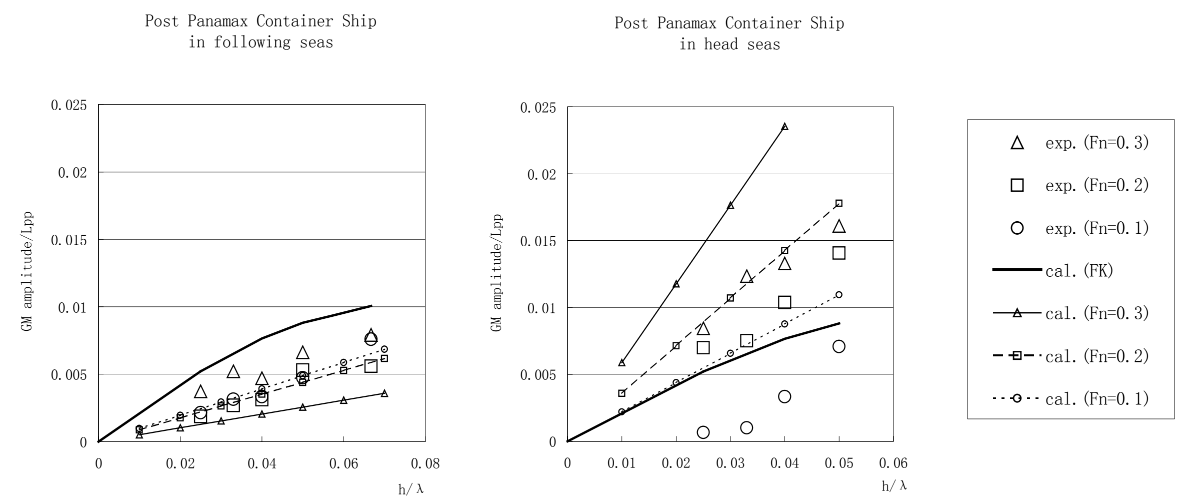

Fig. 2 Amplitude of GM variation in following and head seas for the post Panamax container ship with the wave length to ship length ratio of 1.0. Here h/λ and Fn indicates the wave steepness and the Froude number, respectively. And "cal. (FK)" means the Froude-Krylov calculation.

Its numerical results are shown together with the captive model test results in Fig. 2. While the Froude-Krylov calculation does not depend on the Froude number, the present calculation depends on the Froude number because of hydrodynamic effects. In following seas the Froude-Krylov calculation significantly overestimates the amplitude of GM variation but the present calculation well explains the reduction of the amplitude. In head seas the Froude-Krylov calculation overestimates the amplitude of the model runs with the Froude number of 0.1 but underestimates those with the Froude numbers of 0.2 and 0.3. The present calculation predicts the amplitude in the order obtained from the experiment. This improvement indicates that the hydrodynamic effect cannot be ignored when we accurately estimate the change in GM due to waves.

Surf-riding threshold

Broaching associated with surf-riding was also the capsizing scenario used in the 23rd ITTC benchmark testing for intact stability and quantitative disagreement with free-running model experiments was pointed out. In particular, the surf-riding threshold estimated with the existing model usually underestimates that from the experiment. Then some of the authors continue their effort to improve theoretical modelling by examining various nonlinear factors that had been ignored. As a result, they found that nonlinearity in the wave-induced surge force acting on a hull and the wave effect on propeller thrust are essential to improve the prediction accuracy of surf-riding threshold. (Hashimoto et al, 2004A)

It is well established that the prediction of surf-riding threshold could depend on initial conditions if we simply apply numerical simulation in time domain but the surf-riding threshold for a self-propelled ship staring from sufficiently low propeller revolution does not depend on initial conditions. This is because onset of surf-riding or broaching can be regarded as a heteroclinic bifurcation. (Makov, 1969, Umeda, 1990, Spyrou, 1996, Umeda, 1999) That is, surf-riding or broaching occurs when an unstable invariant manifold of a saddle-type equilibrium in a phase space coincides with a stable invariant manifold of neighbouring saddle-type equilibrium. Although a perturbation method (Ananiev, 1966) and an exact method (Spyrou, 2001) are available for a simplest mathematical model, the introduction of nonlinear factors could prevent their application. A numerical technique to identify surf-riding threshold as a heteroclinic bifurcation can be applied to more complicated theoretical model. The technique reported so far, however, is not suitable for a computer algorithm. (Umeda, 1990, Umeda, 1999) Here it is necessary to calculate time series starting from a saddle-type equilibrium to other saddle-type one or not with the information from locally linearised systems. Thus, in this paper, a mathematical procedure suitable for a computer program is proposed with a successful example for following sea case and perspectives towards quartering sea case is added.

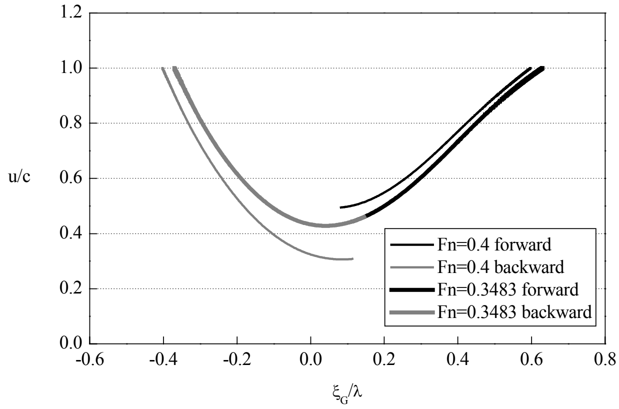

Fig.3 Time domain simulation from saddle-type equilibria with time forwards and backwards with the wave steepness of 1/15 and the wave length to ship length ratio of 1.5. Here c and λ are wave celerity and wave length, respectively: Fn indicates the nominal Froude number.

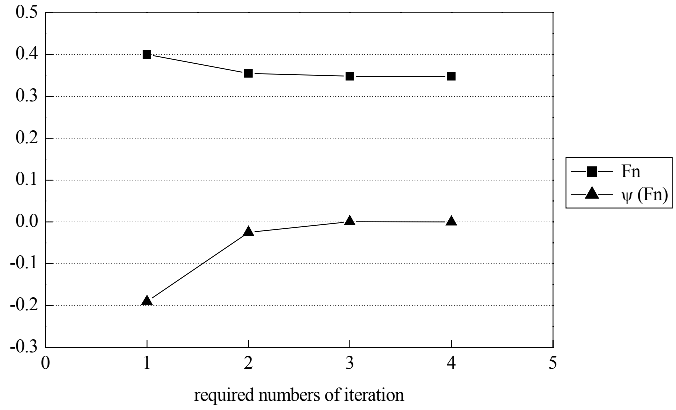

Fig.4 Convergence of the procedure for determining the surf-riding threshold. Here the wave steepness is 1/15 and the wave length to ship length ratio is 1.5.

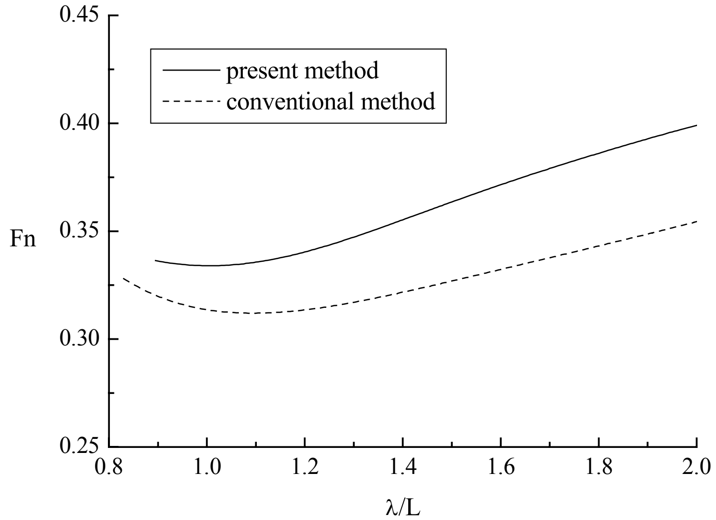

Fig. 5 Surf-riding threshold for the ITTC A-2 ship in following seas predicted by numerical procedure to identify a heteroclinic bifurcation with the wave steepness of 1/10. Here Fn and λ/L mean the nominal Froude number and the wave length to ship length ratio, respectively. The present method is one that takes nonlinearity in wave-induced surge force and the wave effect of propeller thrust into account and the conventional method ignores them.

For a simplicity sake, the uncoupled surge motion in following waves is investigated with the following mathematical model.

$$(m+m_x)\ddot{\xi}_G = T(u,\xi_G;\zeta_a,n) - R(u) + X_W(\xi_G;\zeta_a) \tag{6}$$

where ξ_G: horizontal displacement of the ship centre to a wave trough, m: ship mass, m_x: ship added mass, u: ship velocity ($\dot{\xi}_G = u - c$), n: propeller revolution number, ζ_a: wave amplitude, T: propeller thrust, R: ship resistance and X_w: wave-induced surge force acting on a hull.

The proposed procedure can be summarised as follows. First, by assuming a certain value of n, the equilibria from Equation (6) are identified and their eigenvalues and eigenvectors of the locally linearised systems at the equilibria are calculated. Here the equilibria correspond to surf-riding; one is stable and the other is unstable. Then we integrate Equation (6) with time from the point that is slightly shifted from an unstable equilibrium in the unstable direction of its eigenvector. And we also integrate Equation (6) with time backwards from the point slightly shifted from the neighbouring unstable equilibria in the stable direction of its eigenvector. Then we quantify the difference between two obtained values of u at a specified value of ξ_G as ψ(n). Next, we apply the Newton method to find the parameter n* that $\psi(n^*) = 0$. The obtained n* can be regarded as a heteroclinic bifurcation point. By using the relationship between the propeller revolution and ship speed in calm water, the nominal Froude number as the bifurcation point is determined.

The above numerical method is used to determine surf-riding threshold for the ITTC A-2 ship with and without the nonlinear surge force and the wave effect on propeller thrust taken into account. Figs. 3-4 illustrate rapid convergence of the Newton method applied to their problem. The results shown in Fig. 5 show that the nonlinear surge force and the wave effect on propeller thrust significantly increase critical velocity for surf-riding. This means that these two nonlinear factors are essential to improve prediction accuracy for capsizing associated with surf-riding.

For predicting the broaching threshold, it is necessary to use an coupled surge-sway-yaw-roll-rudder motion. As a result, the stable invariant manifold becomes 7-dimensional while the unstable invarioant manifold is 1-dimensional. (Umeda, 1999) Thus the additional condition to be satisfied is that the vector normal to the stable invariant manifold should be orthogonal to the unstable invariant manifold. Numerical works based on this procedure are now under way.

Conclusions

As follow-ups of the 23rd ITTC benchmark testing, the research team of Osaka University provides some improvements in theoretical modelling of intact stability from the viewpoints of probabilistic theory, hydrodynamics and nonlinear dynamics. The major outcomes are as follows:

1) The formula to calculate capsizing probability in a dead ship condition is presented as an extension of Belenky's piece-wise linear approach.
2) Hydrodynamic modelling of the change in GM due to waves could improve agreements with captive model experiments.
3) A numerical algorithm to detemine the surf-riding threshold is presented with some important nonlinear elements taken into account. The newly-obtained results provide much larger critical velocity for surf-riding than the conventional results.

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