The Simulation of Ship Capsizing Course and the Calculation of Safe Basin in Random Beam Seas*

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Abstract

The random and nonlinear differential equation of a ship's rolling motion in random beam seas is established with considering the nonlinear damping and nonlinear restoring moment. The narrowband wave energy spectrum and different significant wave height are taken into account. The random differential equation is solved by the harmonic acceleration method. The numerical simulation method of random capsizing course is brought forward. Integrating the instantaneous states and random waves, the process of calculating the safe basin of ships' random and nonlinear rolling motions is investigated. The computer program is worked out. The capsizing simulation of fishing vessel is calculated in different significant wave height. The safe basin in the initial value plane is constructed for the different initial conditions and random wave parameters. The probability of the vessel's survival is determined depending on significant wave height.

Key words: random beam waves; ship's capsizing; safe basin; probability of survival

1. Introduction

The stability criteria on the hydrostatics theories are used to estimate whether a ship in the heavy seas capsize or not. This method ignored the time domain characteristics of ship motions, frequency characteristics and the randomicity of waves. Therefore, there are capsizing accidents now and then although ships are designed according to stability criterion. Furthermore, reasonable explanations can't be given for capsizing ships based on stability criterion. Nowadays, based on nonlinear dynamics theories, many scholars have investigated the capsizing problems of ships in random waves to improve the safety and reveal the capsizing mechanism of ships considering the instantaneous states of ship motions and random waves [1~4].

Lots of studies on the ship motions in regular waves have been done. Non-linear phenomena of amplitude jumping, superharmonic and subharmonic response, symmetry breaking and period doubling have been found [1~4]. Considering the parameters of regular wave, Ou yang etc. calculated the stability of rolling motion by numerical integration [2]. Jin Xianding etc. investigated the capsizing mechanism of ship in the regular wave by non-linear dynamics theories, and stabilities of roll motion were analyzed by the theories of bifurcation and Floquet theory [3]. Zhang Weikang etc. calculated the stability of ships with the theory of safe basin, the results show that the safe basins break in definite conditions when a ship capsizes [4]. The above mentioned studies all considered the ship's motion in regular waves. But, the dynamics characteristics of ship motions in random waves are short of enough investigation. The investigation in the early days is basically done with equivalent linearization method so the essential non-linearity of rolling process can't be described. Francescutto primarily investigated the bifurcation and chaos phenomena of ship rolling motion when the wave is assumed as the narrowband spectrum [5]. Introducing phase flux rate, Troesch etc. investigated the capsizing of biased ships [6], but, the prediction of capsizing probability of ships can't be given numerically by all of these methods.

The random and nonlinear differential equation of a ship's rolling motion in random beam seas is established with considering the nonlinear damping, nonlinear restoring moment and random wave excitation here. The capsizing course of ship rolling is simulated by applying the harmonic acceratin method to solve the differential equation. The capsizing process is calculated and analyzed for fishing vessel and the safe basin in the initial value plane is constructed. The probability of the vessel's survival is obtained under the different wave parameters.

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2. Roll equation

The rolling motion is larger than the motion of other five components for the ships in the beam seas, the coupled motions of a ship can be ignored. So the rolling differential equation is written as following

$$I\ddot{\varphi} + D(\dot{\varphi}) + R(\varphi) = M(t) \tag{1}$$

where φ is the roll angle; I is the virtual moment of inertia including the ship's mass moment and the added mass moment of the surrounding water; $D(\dot{\varphi})$ is the damping moment, it may be approximated by a cubic polynomial as an analytical function

$$D(\dot{\varphi}) = D_0\dot{\varphi}\,|\,\dot{\varphi}\,| = D_1\dot{\varphi} \ + D_3\dot{\varphi}^3 \tag{2}$$

$R(\varphi)$ is restoring moment, it is hydrostatic and given by a nonlinear anti-symmetric function. It may be represented by a fifth polynomial

$$R(\varphi) = K_1\varphi + K_3\varphi^3 + K_5\varphi^5 \tag{3}$$

The rolling excitation moment of a regular wave of beam seas may be presented in the form [7]

$$M(t) = I\alpha_0\omega_0^2\pi\frac{h}{\lambda}\cos\omega t \tag{4}$$

Where α_0 is the effective wave slope coefficient, the large amplitude of ships' rolling near to capsizing states is investigated in this thesis, so α_0 can be taken as a constant; h is the wave height; λ is the wavelength of beam sea; ω_0 is the initial roll natural frequency and ω is the wave frequency. Random wave excitation moment is presented as the sum of harmonic wave excitation

$$M(t) = I\alpha_0\omega_0^2\pi\sum_{n=1}^{N}\frac{h_n}{\lambda_n}\cos(\omega_n t + \varepsilon_n) \tag{5}$$

Where ε_n is the random phase angle in the range $(0,2\pi)$, whose probability of occurrence $1/2\pi$ is uniformly distributed. The wave height is obtained from the wave spectrum $S(\omega)$ by [8]

$$h_n = 2\sqrt{2\tilde{\omega}S(n\tilde{\omega})} \tag{6}$$

Where $\tilde{\omega}$ is the interval of wave frequency, i.e. $\omega_n = n\tilde{\omega}$. The wavelength is given by

$$\lambda_n = \frac{2\pi g}{(n\tilde{\omega})^2} \tag{7}$$

Where g is gravity acceleration. Substituting (7) and (8) into (6), it is obtained

$$M(t) = I\alpha_0\omega_0^2\frac{\sqrt{2\tilde{\omega}}}{g}\sum_{n=1}^{N}(n\tilde{\omega})^2\sqrt{S(n\tilde{\omega})}\cos(n\tilde{\omega}t + \varepsilon_n) \tag{8}$$

Taking into account the above definition of rolling parameters, the differential equation (1) is divided by the moment of inertia I, and presented in the following form

$$\ddot{\varphi} + d_1\dot{\varphi} + d_3\dot{\varphi}^3 + k_1\varphi + k_3\varphi^3 + k_5\varphi^5 = m(t) \tag{9}$$

where

$$d_i = \frac{D_i}{I}, i = 1,3; k_i = \frac{K_i}{I}, i = 1,3,5; m(t) = \frac{M(t)}{I} \tag{10}$$

When the wave spectrum is given, wave moment is formed from the equation (8). The rolling motion response is obtained by integrating the equation (9).

3. Solved by harmonic acceration method

Ship rolling with large amplitudes is a non-linear problem and has to be analyzed by non-linear dynamics. Time domain calculation is preferable since only in that way can the complete response be obtained, especially for a random excitation. The governing differential equation of rolling motion (9) is integrated in the time domain by the harmonic acceleration method [9].

The application of the harmonic acceleration method, which shows some advantages in linear transient vibration analysis compared to the other commonly used methods, is extended to nonlinear problems employing the mode superposition technique and an iteration procedure. Stability analysis shows that the method is unconditionally stable.

For this purpose, the equation (9) is transformed into a pseudo-linear form

$$\ddot{\varphi} + 2\xi\overline{\omega}\dot{\varphi} + \overline{\omega}^2\varphi = \psi(t) \tag{11}$$

where

$$\psi(t) = m(t) + (2\xi\overline{\omega} - d_1)\dot{\varphi} - d_3\dot{\varphi}^3 + (\overline{\omega}^2 - k_1)\varphi - k_3\varphi^3 - k_5\varphi^5 \tag{12}$$

is pseudo-excitation, ξ is assumed the integration damping coefficient, $\overline{\omega}$ is the integration frequency, it is equal to the peak frequency of wave spectrum.

Equation (11) may be integrated in the time domain using a numerical step-by-step procedure. Employing the harmonic acceleration method, due to its above-mentioned advantages, the following recurrent formula expresses the relationship between responses at two close instants t_{i+1} and t_i.

$$\{Y\}_{i+1}^{k+1} = [T]\{Y\}_i + \{L\}\psi_{i+1}^k \tag{13}$$

Where $\{Y\}$ is response vector, which includes φ, $\dot{\varphi}$ and $\ddot{\varphi}$. $[T]$ is the transfer matrix, $\{L\}$ is the load vector, *i* is time step, and *k* is iteration step. $[T]$ and $\{L\}$ are related to ξ, $\overline{\omega}$ and Δt, which are specified in reference [9].

The initial displacement vector and velocity vector in the iteration may be specified as

$$\langle\varphi\rangle_{i+1}^0 = \langle\varphi\rangle_i, \qquad \langle\dot{\varphi}\rangle_{i+1}^0 = \langle\varphi\rangle_i \tag{14}$$

The iteration procedure continues until convergence is achieved. For example, a possible convergence criterion is

$$\left[\frac{\sum(\varphi_{i+1}^{k+1} - \varphi_{i+1}^k)^2}{\sum(\varphi_{i+1}^k)^2}\right]^{1/2} \le \varepsilon \tag{15}$$

Where ε is a pre-assigned error tolerance (a small positive number).

According to the iteration step (13), the rolling motion is numerically simulated.

4. The calculation and analysis of example

According to the above method, the capsizing process of a fishing vessel is calculated and the safe basin is constructed. The parameters of fishing vessel are: length overall 30.27m; length between perpendiculars 25.00m; breadth 6.90m; depth 4.96m; draught 2.67m; displacement 195t. Virtual mass moment of inertia, *I*=1078tm^2; initial metacentric height; $\overline{GM} = 0.962m$; initial roll natural frequency, $\omega_0 = 1.32 rad/s$; Coefficients of damping moment: $d_1 = 0.0208s^{-1}, d_3 = 0.01648s^{-1}$; The effective coefficients of restoring moment is illustrated in the Fig. 1 and $k_1 = 1.7737s^{-2}, k_3 = -0.518291s^{-2}$, $k_5 = 0.0306808s^{-2}$. Considered the wave spectrum is

$$S(\omega) = 0.862\frac{0.0135g^2}{\omega^5}\exp\left[-\frac{5.186}{\omega^4 h_{1/3}^2}\right]1.63^p \tag{14}$$

Where $h_{1/3}$ is the significant wave height and

$$p = \exp\left[-\frac{(\omega - \omega_m)^2}{2\sigma^2\omega_m^2}\right],\ \omega_m = 0.32 + \frac{1.80}{h_{1/3} + 0.6},\ \sigma = \begin{cases} 0.08 & \omega < \omega_m \\ 0.10 & \omega > \omega_m \end{cases}.$$

The wave energy spectrum up to frequency $\omega_{\max} = 4.5 rad/s$, is taken into account. The frequency step $\widetilde{\omega} = 0.025 rad/s$ is chosen. The wave frequency for every harmonic wave is $n \times \widetilde{\omega}$, where *n* is the harmonic wave serial number. So the sum of 180 harmonic waves is instead of the random wave. The effective wave slope coefficient $\alpha_0 = 0.729$ and significant wave height 4 m are chosen respectively. The wave energy spectrum curve is illustrated in Fig. 2. The amplitude and time historical curves of relative random wave moment are illustrated in Figs. 3 and 4.

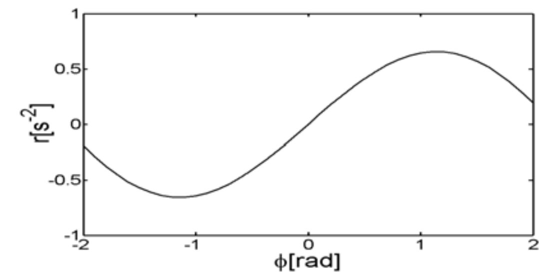

Fig.1Non-dimension restoring moment

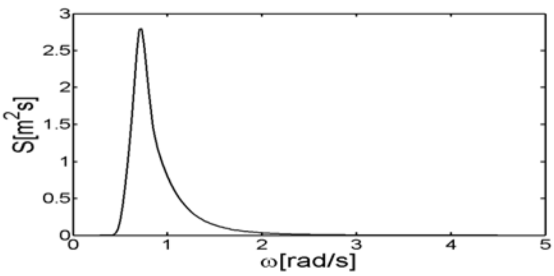

Fig.2 Wave energy spectrum

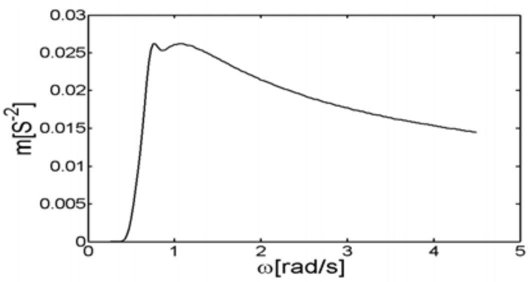

Fig. 3Amplitudes of relative excitation moment

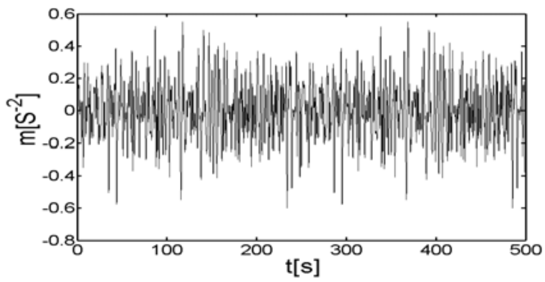

Fig.4 Relative wave moment

The equation (9) is integrated by the harmonic acceleration method. The ship rolling response is calculated in the case of $h_{1/3} = 4$ m within the time interval $t = 300s$ by time step $\Delta t = 0.0125s$, which includes 50s to achieve a stationary response and the period of the lowest harmonic excitation $\widetilde{T} = 2\pi / \widetilde{\omega} = 250s$. The simulation of rolling motion is illustrated in Fig. 5 and Fig.6.

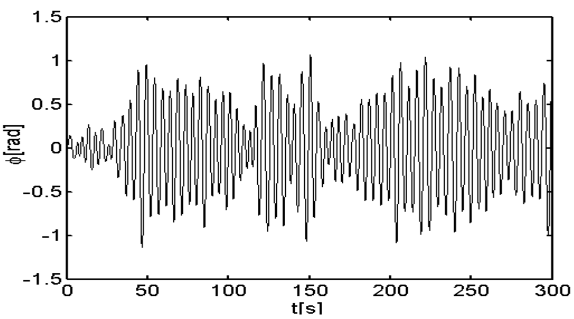

Fig. 5 simulation of rolling motion（$\phi_0 = 0, \dot{\phi}_0 = 0$）

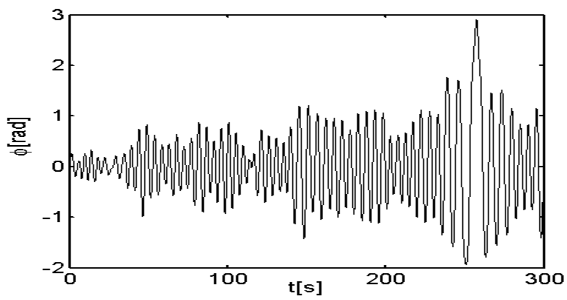

Fig. 6 simulation of rolling motion（$\phi_0 = 0, \dot{\phi}_0 = 0.24$）

The value of roll angle depends on random excitation and the instantaneous states of ship motions. Therefore, the safe basin is constructed with considering the initial conditions, instantaneous states and the random wave moment. Whether a ship in random capsize or not can be confirmed according to the safe basin. The computer programs are worked out and the safe basin is constructed.

The safe basin is constructed in the initial value plane. The initial values represent the instantaneous states in ships' navigation. The phase plane $\varphi \in$[-2，2]、$\dot{\varphi} \in$[-2，2](rad, rad/s) is divided into 200×200 grid points. Considering the grid nodes as initial conditions (instantaneously states), The equation (9) is integrated by applying the harmonic acceleration method. If $|\varphi| < \pi / 2$, the node is marked with ".". The equation (9) is integrated considering $h_{1/3} = 0, 2.5, 3, 4, 5$ and 6m respectively. The obtained results are shown in Fig. 6 ~11.

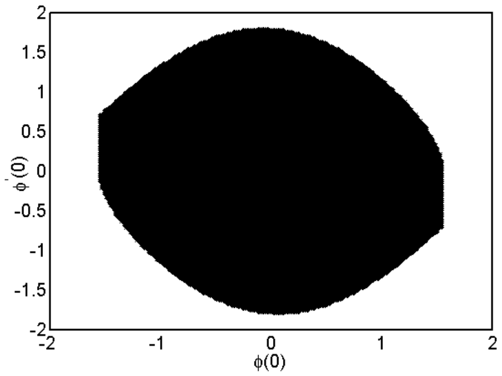

Fig.7 Safe basin ($h_{1/3}$=0m)

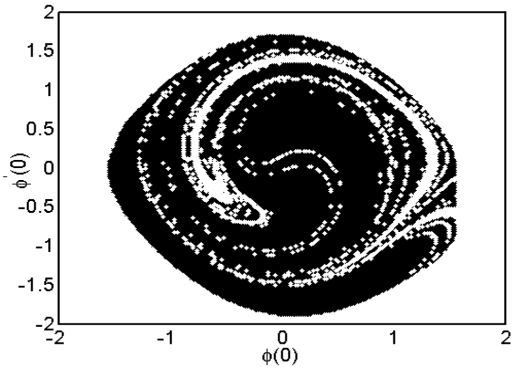

Fig.8 Safe basin ($h_{1/3}$=2.5m)

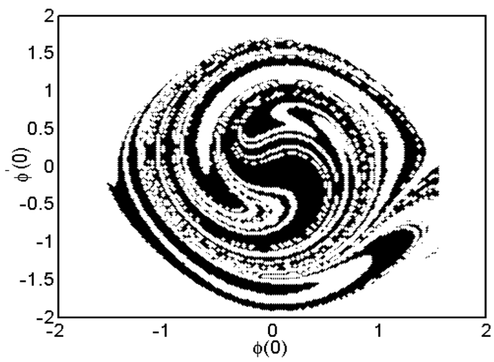

Fig.9 Safe basin ($h_{1/3}$=3m)

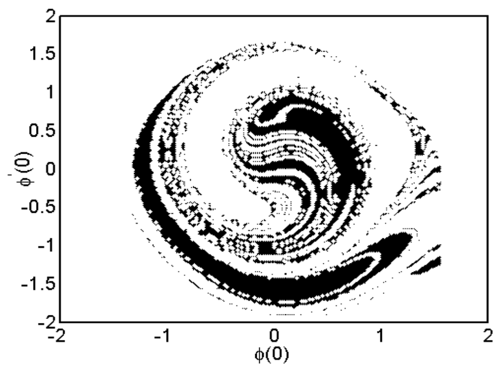

Fig.10 Safe basin ($h_{1/3}$=4m)

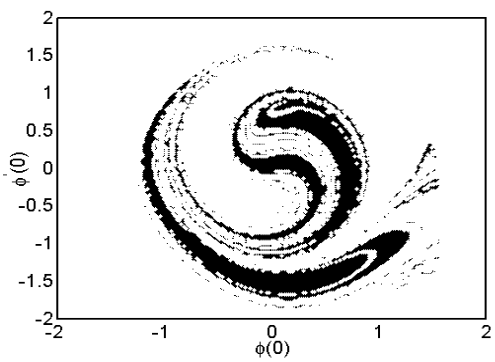

Fig.11 Safe basin ($h_{1/3}$=5m)

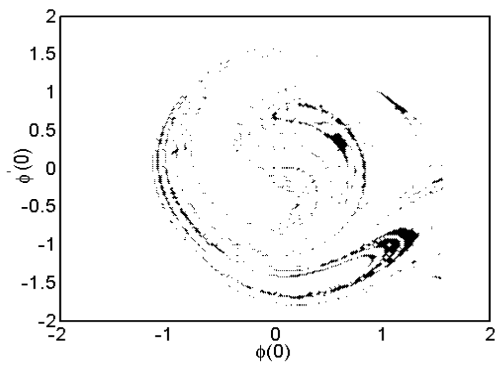

Fig.12 Safe basin ($h_{1/3}$=6m)

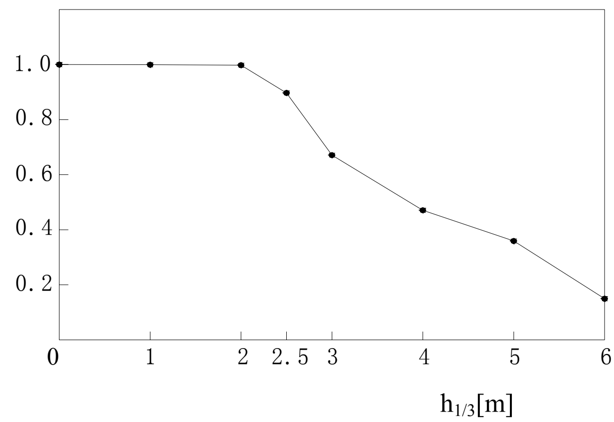

Fig.13 Probability of survival

The dark area is the safe basin in the Fig.7~12. The safety boundary is of fractal nature. When the wave height is smaller, the safe basin is a continuous area. As the wave height rises, the safe basin is not continuous any more. The safe area and the capsize area are interlaced from Fig.7~12. Otherwise, when the significant wave height is constant, different initial conditions are corresponding to different rolling states of capsizing or safety respectively. It shows that instantaneous states influence the safety of ships very much.

The ratio between the areas of the safe basin obtained for and without wave excitation represents the probability of survival. Such a diagram is shown in Fig. 13 depending on sea states given by the significant wave height. The probability of survival is considerably reduced for $h_{1/3}$=2.0m, which means to be a critical value.

5. Conclusion

In this paper, the ship motion is simulated numerically. The safe basin is constructed with considering the instantaneous states of ship motions and different sea states, which enabled the construction of a diagram of survival probability. The main conclusions are drawn as following:

1. The instantaneous states of ship's roll is closely relative to whether a ship capsize or not, the instantaneous states and sea states should be simultaneously considered to evaluate the capsizing of ships.
2. Both the sea states and the instantaneous states of ships should be simultaneously considered to estimate the safety of ships. It is not reasonable to estimate the stability of ships while neglecting the instantaneous states of ship motions.
3. The safe basin is primarily constructed when the ship navigates in random waves. The probability of vessel's survival is obtained for different wave parameters. Such a diagram can be used as the basis of evaluating ships' safety.
4. The method of analyzing the probability of ships' survival in time domain is pointed out in this paper. The advantage of this method is that the instantaneous states of ships' navigation and the random sea states are simultaneously considered.

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