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Beyond the Wall

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ABSTRACT

Inaccuracy in evaluation of inclining experiments by the application of the wall-sided concept was discussed by Dunworth (2014). KG can be significantly underestimated for V-bottomed hull forms when relying on GM to evaluate inclining experiments. A solution was proposed which derived KG and TCG to align heeling and righting moments without reference to the metacentre.

Looking beyond the theory, this paper describes practical model testing to explore the accuracy of the new method, reviews experiences in its use by the Australian Department of Defence and offers guidance in its application.

Keywords: *Stability; Inclining; Metacentre; Wall-sided*

NOMENCLATURE

Δ	Displacement of the system (ship plus inclining masses) (t)
d	Distance of inclining mass shift (m)
φ	Angle of heel (degrees)
GG'	Shift of centre of gravity (m)
GM_0	Transverse metacentric height when upright (m)
GZ	Righting arm (m)
GZ'	Righting arm corrected for TCG_I (m)
HZ	Heeling arm (m)
HZ_0	Heeling arm when upright (m)
KG	Height of vertical centre of gravity above baseline (m)
KG_I	Estimated height of vertical centre of gravity above the origin, in global coordinates, with inclining masses in their initial position (m)
KG_L	Estimated height of vertical centre of gravity above baseline, in local (ship) coordinates, with inclining masses in their initial position (m)
KM_0	Height of transverse metacentre above baseline (m)
KN	Righting arm about the origin (m)
KN_0	Righting arm about the origin when upright (m)
TCB_0	Transverse centre of buoyancy when upright (m)
TCG	Transverse centre of gravity (m)
TCG_0	Estimated transverse centre of gravity when in upright equilibrium (m)
TCG_I	Estimated transverse centre of gravity of the system with inclining masses in their initial position (m)
w	Inclining mass (t)

1. INTRODUCTION

The concept of an inclining experiment was first proposed by Hoste (1697), a professor of mathematics at the Royal Naval College in Toulon, France. However it was nearly fifty years before a practical method of conducting an inclining experiment was described by Bouguer (1746).

The traditional calculation associated with an inclining experiment led directly to a value of GM_0 and, as this was the primary measure of stability, it was not necessary to know *KG* itself until the development of the concept of *GZ*. By that time, Bouguer's calculation method was well established and continues to be used to this day.

Growth in displacement and *KG* is common on naval ships. Communication, navigation and armament equipment increase with time and tend to be placed high up. Conversely, when heavy machinery low down in the ship is upgraded, it is often replaced with more efficient, lighter equipment. Without compensation, these effects almost guarantee that *KG* will rise over time. Growth must be captured and updated regularly in the stability information provided to ships.

For RAN ships, stability is managed by comparison of a load condition's *KG* with a curve of limiting *KG*. If *KG* has been over-estimated, then unnecessary operational restrictions may result but, if *KG* has been under-estimated, then the vessel may be at risk if it encounters the environment and/or damage which underlie the curve of limiting *KG*.

The lightship characteristics of RAN ships are regularly checked by inclining experiments with the interval determined by the expected time before any standard load condition will exceed the limiting *KG*, due to growth. Over the whole fleet, there is about one ship checked every three or four months.

Although there is ample guidance available on the conduct of an inclining, Administrations rarely, if ever, prescribe the method of deriving *KG* from the recorded data.

2. THE CLASSIC METHOD

2.1 Relying on the Metacentre

It has previously been assumed that the metacentre does not move significantly at small angles of heel. On this premise, the wall-sided concept has been used to derive GM from inclining experiment results using the relationship:

$$GM = \frac{w\ d}{\Delta\ \tan\varphi} \tag{1}$$

and *KG* is then calculated as:

$$KG = KM - GM \tag{2}$$

The derivation of GM is most commonly performed by fitting a line of best fit (trendline) through the plot of $w\ d$ against $\Delta\ \tan\varphi$. GM is then equal to the slope of that trendline.

2.2 The Moving Metacentre

Even for a wall-sided ship, the metacentre moves at small angles of heel and, for some hull forms, the movement is significant. Where a hull has a relatively shallow V-bottom over a significant proportion of its length, reliance on *GM* for determining *KG* is unsafe.

3. THE NEW METHOD

3.1 Balancing Heeling and Righting Arms

The new method recognises that, after each weight move, the vessel is in equilibrium and that there must be a righting arm, GZ, equal and opposite to the heeling arm HZ developed by the shift of inclining weights. With the vessel's trim and displacement known from draught readings, it is possible to calculate the corresponding value of KN for each weight move. GZ is derived from KN by the relationship:

$$GZ = KN - KG\ \sin\varphi - TCG\ \cos\varphi \tag{3}$$

In the absence of experimental error, there is a pair of unique values for TCG and VCG which will result in values of GZ for each heel which exactly match the corresponding HZ

values. Similarly, even when there are experimental errors present, there will be a pair of unique values for TCG and KG which will result in the trendlines through HZ and GZ against heel being coincident. When close to upright, TCG can be considered to raise or lower the trendline, while VCG skews the trendline about zero heel.

A method for deriving TCG and KG has been proposed (Dunworth, 2014) and is briefly described below.

3.2 The Solution for TCG_I

When $\varphi = 0$, $\sin\varphi = 0$ and $\cos\varphi = 1.0$, so Equation 3 reduces to:

$$HZ_0 = KN_0 - TCG_\mathrm{I} \tag{4}$$

Equation 4 can be re-arranged to give a solution for TCG_I:

$$TCG_\mathrm{I} = KN_0 - HZ_0 \tag{5}$$

KN_0 is identical to TCB_0 and could therefore be found from upright hydrostatics. However, it is more convenient to calculate KN_0 with the other KN values which will be required. KN_0 can be expected to be close to zero, but will only be exactly so if both hull and appendages are truly symmetric about the centreline. The actual value should generally be calculated.

HZ_0 can be found from the trendline through the HZ points when plotted against heel angle and is the value of HZ when heel = 0, i.e. the intercept of the trendline.

A third-order polynomial trendline should be used because it can closely match non-linear data sets which include a point of inflection - which generally occur near equilibrium in GZ plots.

When there is known to be a discontinuity in GZ within the range of heels at the inclining experiment, the points should be divided into two sets, either side of the discontinuity, and only the set which spans upright used to determine HZ_0. If the discontinuity is exactly at upright, both sets may be used and HZ_0 taken to be the mean of the two intersections.

3.3 The Solution for KG_I

Equation 3 can be re-arranged as:

$$KG_\mathrm{I}\ \sin\varphi\ = KN - HZ - TCG_\mathrm{I}\ \cos\varphi \tag{6}$$

and the solution for KG_1 is therefore:

$$KG_\mathrm{I} = \frac{KN - HZ - TCG_\mathrm{I}\ \cos\varphi}{\sin\varphi} \tag{7}$$

For each mass move, $KG_\mathrm{I}\ \sin\varphi$ (from Equation 6) is plotted against $\sin\varphi$. All points should lie on a straight line and the value of KG_I is then equal to the slope of the linear line of best fit through the points.

KG_I is a vertical measure and, to account for trim, G_I will need to be rotated about the ship's origin to give VCG in ship coordinates.

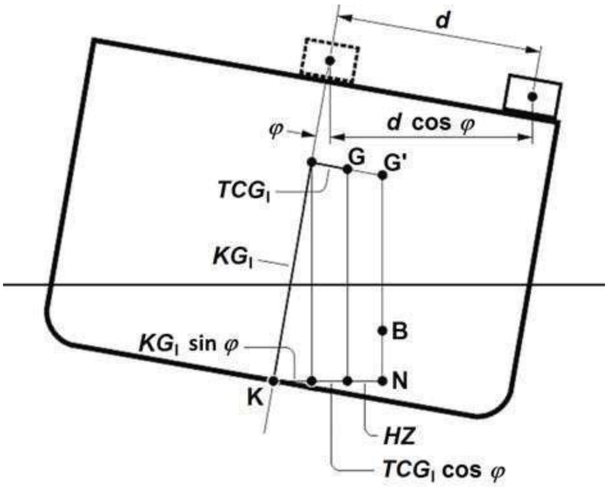

Figure 1 Illustration of Equation 6

4. VALIDATING THE NEW METHOD BY MODEL TESTING

4.1 Selection of the Hull Form

As part of the validation process for the new method, a scale model of a hull section was built and tested. The model was sized to fit into an existing trough used for teaching inclining experiments and was a practical size and weight to handle. As will be shown, there was sufficient difference between *KG*s calculated by the two methods for the result to be conclusive and not lost within variations caused by experimental error.

Being representative of a hull section only, the model is considerably wider than it is long with a beam of 1.2 m and length of only 0.3 m. Details are shown in Table 1 and Figure 2.

Table 1 Model particulars

Length	0.3	m
Beam	1.2	m
Displacement	17.560	kg
Inclining weight	3.098	kg
KM_0	1.073	m
Forward pendulum length	1086.35	mm
Aft pendulum length	1085.25	mm

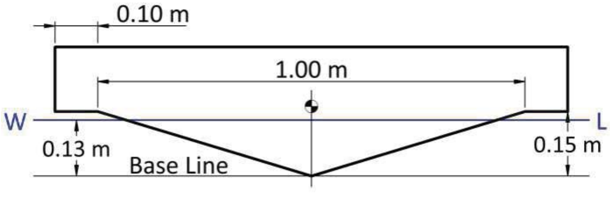

Figure 2 Model section

The model is not intended to accurately reflect any particular vessel, with the shape having been chosen specifically to demonstrate the difference in results between the classic and new methods. It is, however, geometrically similar to a section through the aft portion of the small aluminium survey boats operated by RAN.

On one occasion, one of these craft was presented for inclining with a list of just under three degrees. This would not normally be acceptable, in part because of the unreliability of tank dips at that angle, but it was decided to proceed with the experiment so that the results could be used in planning appropriate remedial measures. A heel of over two degrees each way was developed, but the vessel never came upright during the experiment. This inclining is of interest and will be referred to later at 5.5.

4.2 Model Construction

The model was of plywood construction and subsequent measurements showed it to be generally quite accurate.

Aluminium angle rails, forward and aft, were pre-drilled at 50 mm centres out to 550 mm either side of the centreline. This allowed quick and accurate movement of the inclining weight without having to measure the shift at each weight move and also provided a secure mounting for a pair of ballast weights.

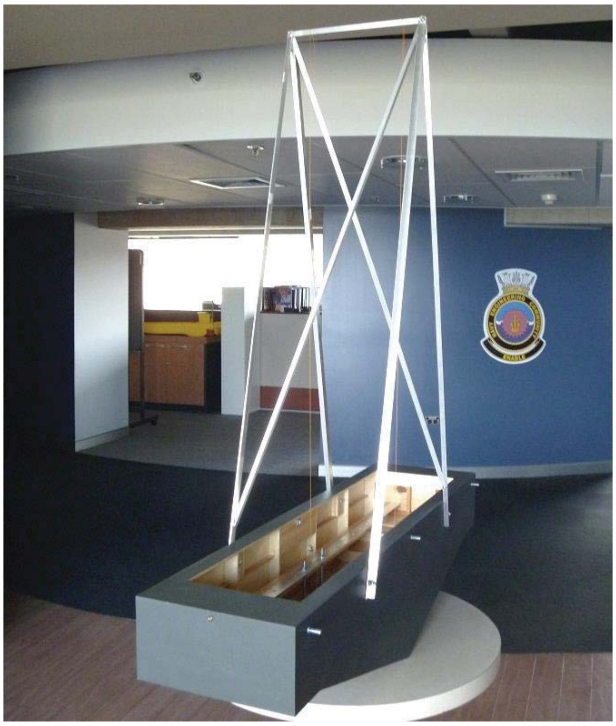

Figure 3 Model Arrangement

The ballast weights were suspended on threaded rods and could be wound up and down to vary *KG*. A disadvantage of this arrangement is that two pairs of the pre-drilled holes were occupied by the ballast and were therefore not available for inclining weight shifts. Ballast was chocked to prevent any movement.

To date, only one set of readings has been taken: at a single displacement and with ballast in its lowest position.

As ballasted, the model floated close to level trim (0.07°) and heel (0.05°) with the chine flat about 19 mm clear above the waterplane. The chine outer edge touched the waterplane at a heel of 1.9 degrees and the chine flat was completely immersed beyond 2.2 degrees. The significance of the small trim was not realised until the inclining results were analysed and is discussed later at 5.1.

4.3 Determining the Model Weight

An unexpected difficulty came with the determination of the model weight.

For the first measurement, a pair of scales, each with an upper limit of 5 kg, was used to weigh the bare hull and individual components. It was immediately apparent that the two scales gave different readings, but there was no way of determining which, if either, was correct.

A second measurement was taken by measuring the rise of water in the trough; from the trough dimensions and water density (0.9985 kg/l), the weight of the model could be calculated. Once the model was in the trough, the depth of water and height to gunwale were used to determine the freeboard at each corner. Both methods required measurement of water depths, but the height of the meniscus where the water surface met the measuring scale was difficult to determine. This was a concern as the effect of change of draught for the model is about 0.265 kg/mm.

A set of as-built offsets was lifted so that a new numerical model could be generated and the results re-worked.

Finally, a larger set of scales was used to weigh the entire model. Although the scales were not calibrated, this was thought likely to be the most accurate measurement and differs by less than 0.5 percent from the mean value of 17.560 kg which was taken to be the model weight for calculation purposes. Results are shown in Table 2.

Table 2 Results of the methods used to determine model weight

Method	Weight
Weigh parts	17.473 kg
Weigh whole	17.575 kg
Freeboards	17.928 kg
Draughts	17.223 kg
Displaced volume	17.698 kg
Mean weight	17.560 kg

4.4 Establishing *KG*

Six strong points were incorporated in the model to allow it to be freely suspended from a spreader bar, via solid wire strops, onto large washers; pendulums were hung from the same points – see Figures 4 and 5. On one face, all three pendulum lines intersected at a single point: on the other, they formed a very small triangle or ‘cocked hat’.

Values forward and aft of 162.0 mm and 161.5 mm respectively were found and the mean value of 161.75 mm was taken to be the model *KG*.

4.5 Inclining the Model

By sighting across the two pendulums, both during the measurement of *KG* and during the inclining at each weight move, parallax errors in the readings were minimised. All readings were photographed for later analysis. A short video of each would have been useful as it

Figure 4 Establishing *KG* by suspension

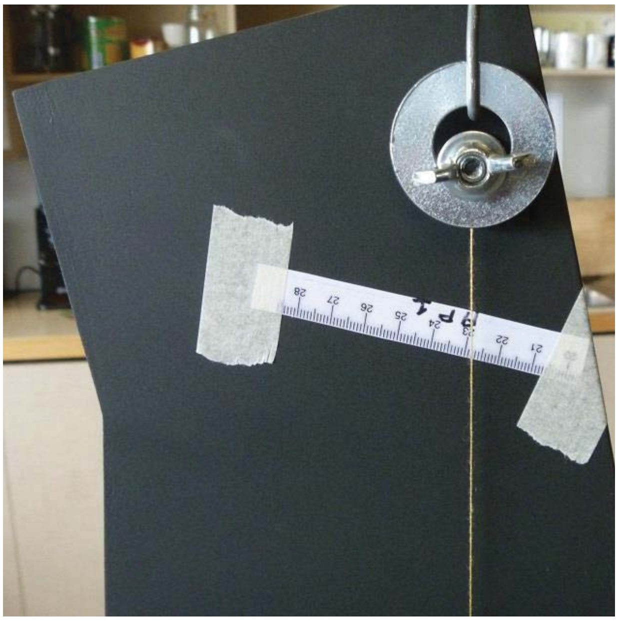

Figure 5 Suspension point detail

would have shown the extent of any movement at pendulum readings.

The inclining weight was then moved progressively along the rails with pendulum readings taken at each move. In addition to the pre-determined positions, the weight was also placed as close as practicable to either side of each ballast weight (moves 11, 12, 14 & 15), and the positions measured, to give additional readings close to upright.

In analysing the results, deflections were normalised to the respective mean zero readings. The full set of inclining readings is shown in Table A–1.

For the classic workup, $w\ d$ and $\tan\varphi$ are calculated and the derived data over all weight moves is included in Table A–1.

Table A–2 shows the derived data for the new method over all weight moves. There was a small initial list of 0.054 degrees measured by freeboards. This was added to each heel determined by pendulum deflection to give actual heels. *KN*s were then calculated from the numerical hull mesh model.

5. INTERPRETING THE MODEL TEST RESULTS

5.1 Balancing Heeling and Righting Arms

The concept of the new method is based on heeling and righting arms being equal after each weight shift. Knowing KG_I, upright TCG_I can be calculated and a set of righting arms developed from the *KN* values. These righting arms are, in effect, *GZ* values shifted to take account of TCG_I. The comparison between experimental heeling arms and calculated righting arms is included in Table A–2.

It can be seen that the greatest differences occur at weight moves 9 and 17. These are symmetrical port and starboard and it is possible that they resulted from the small initial trim. This would have caused the low corners of the chine flats to touch the water surface early, generating small additional righting moments at those weight moves.

The use of readings with large differences between heeling and righting arms should be avoided in further calculations if practicable.

5.2 Simulating Actual Inclining Experiments

The full data set was analysed, together with four combinations of weight moves to represent inclining experiment scenarios. Not all sets use the same number of weight moves.

5.3 Case 1: Full Data Set

Using the full set of results the new method gives an accurate *KG* of 0.162 m. Despite a high coefficient of determination of 0.998, the classic method is significantly in error with *KG* of 0.010 m. See Figure 6 and Table 3.

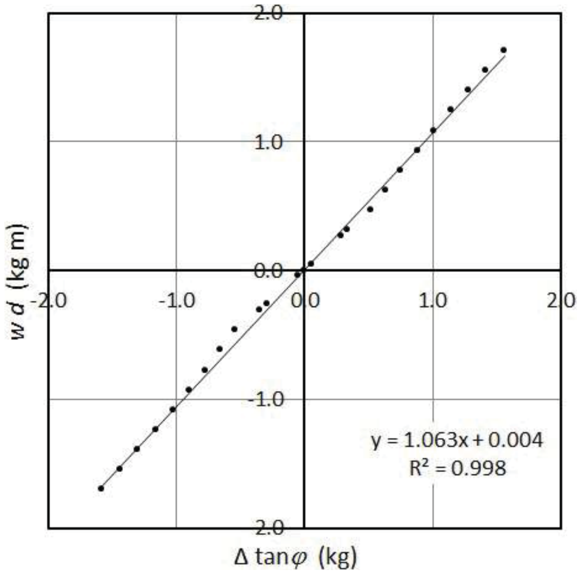

Figure 6 Plot of full data set $w\,d$ vs. $\Delta \tan\varphi$

Table 3 Results using the full data set

Classic – Full Set		
GM_0	(m)	1.063
KG_l	(m)	0.010

New – Full Set		
TCG_l	(m)	0.001
KG_l	(m)	0.162

5.4 Case 2: Typical

A typical set of inclining results can be selected with a heel range a little over two degrees each way. Results are shown in Table A–3. *KG* by the new method is 2 mm low, but significantly better than by the classic method.

5.5 Case 3: Large Initial List to Port

A set can be selected which is similar to the inclining experiment on the survey boat described previously at 4.1. *TCG* is known to be −0.053 m from the inclining weight shift at the initial state (move 20) and values for the classic method have been adjusted to reflect this.

The resulting *KG* is 0.163 m by the new method (1 mm error) with *TCG* slightly in error at −0.050 m. The small error in *TCG* may been caused by extrapolation of the trendline through *HZ* vs. Heel to obtain the intersection at upright. *KG* is −0.063 m by the classic method – placing the centre of gravity below the baseline. Results are shown in Table A–4.

The fact that the classic method can give a negative *KG*, well below the underside of keel, is alarming – though hopefully would not go unnoticed.

5.6 Case 4: Restricted Heel

It has been pointed out that, since the new method relies on division by sinφ, the results can be erratic when measurement of heels close to upright is not completely accurate. When calculating individual results, division by zero would occur at upright.

This is not an issue with a reasonable range of heels to either side of upright. By using the slope of a trendline, values close to the mean have little effect as they tend to shift, rather than skew, the line of best fit. However a set of inclining values over a small range near upright may cause a problem. In fact, the model test shows a good result by the new method, even with less than one degree heel to either side of upright.

Though still a little low, the result for the classic method is almost correct – as would be expected with such a small range of heel. Results are shown in Table A–5.

5.7 Case 5: One Extreme Heel Close to Upright

Division by sinφ may also result in error if one of the extreme heels is close to upright.

The current set of model results cannot be used to reliably illustrate the problem. Upright was the starting point for model readings and the mean of three readings (0, 13 & 26) gives better accuracy than would be expected in practice. Results in Table A–6 show that the new method has an error of 2 mm but, as in Case 3, the classic method gives a nonsensical result with *KG* below the baseline!

5.8 Summary of the Simulation Cases

Figure 7 shows the *KG*s for the five cases by each method. The horizontal line is at the *KG* found by suspending the model. Clearly the new method is the more reliable for this model.

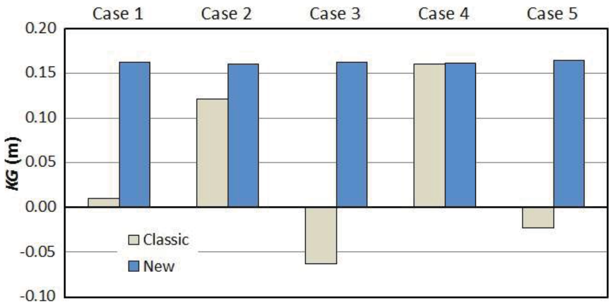

Figure 7 Comparison of inclining simulations

6. GENERAL OBSERVATIONS

6.1 Discarding Readings Near Upright

The new workup method benefits from values near upright for calculating *TCG* but, in the subsequent calculation of *KG*, there may be justification for discarding any readings with the vessel close to upright unless good accuracy in the measurements of those heel angles can be assured.

6.2 Measuring Pendulum Deflections

The problem of measuring small pendulum deflections can be overcome to some extent by the use of longer pendulums. However, these can often fail to settle at a measurable point and are influenced by ship movement transmitted through the pivot point. The practice of hanging pendulums high in the ship is not helpful and, wherever practicable, they should be low down with the pivot point close to the waterline for best results.

Damping of the pendulum bob in a trough of viscous fluid will assist. In the past, DNPS has recommended the use of spent oil as a damping medium, but now advocates the use of a thin wallpaper paste – about 5 g/l. This has several advantages: it is readily available, light (it can be taken in powder form to the inclining) and, being basically starch, is more environmentally friendly. There are no special precautions or equipment required for disposal.

7. EXPERIENCES WITH THE NEW METHOD

7.1 Applying the New Method

RAN generally employs contractors to conduct inclining experiments and to produce the associated reports. Since the use of the new workup method was introduced in 2013, a number of inclining experiments have been conducted on RAN ships and the new method used with varying degrees of success.

Some contractors have chosen to use the resources of DNPS to provide the as-inclined characteristics, but several have used the new method for themselves. DNPS offers an Excel spreadsheet which will perform the necessary calculations.

Apart from *KN*s, the data input required is the same as for the classic method.

7.2 Generating *KN* Values

The calculation of *KN* values is clearly an issue. Few contractors hold good numerical models of our ships and even those who do so are not necessarily able to produce *KN*s at the specific heel angles, and to sufficient accuracy, to be of use.

One solution has been to provide *KN* data tabulated at fine intervals of heel, trim and volume which can be interrogated by linear interpolation. Volume is used rather than displacement as it is independent of the water density at the time of the inclining experiment. Considerable effort has gone into determining how fine the intervals need to be and experience has shown that the requirements are specific to each hull form. To date, no general rules have been found which will enable the intervals to be determined by simple inspection of the hull characteristics.

8. CONCLUSIONS

Although some issues, particularly the generation of accurate *KN* values, need to be resolved before the new method can be readily and universally applied, the practical model inclining has shown how robust and versatile it is. By comparison, on only one occasion did the classic method come closer than 0.04 m (25%) to the correct *KG* in the scenarios which were simulated for this particular model.

It has been clearly demonstrated that this is a superior method for application to non- wall-sided hull forms, even when they are inclined to extreme angles, and its adoption is strongly recommended.

9. ACKNOWLEDMENTS

The author is grateful for the assistance given by members of the Stability and Hydrodynamics Sections of DNPS in the conduct of the model testing described above.

10. FUTURE WORK

10.1 Extending the Experimental Data Set

To date, only a single set of readings has been taken on this model at one displacement and *KG*. The work should be extended to cover a range of hull forms, displacements and *KG*s. This should include a true wall-sided model.

10.2 Investigating Sensitivity

The sensitivity of the new method to both hull type, and to inaccuracies in measurements taken at the inclining experiment, needs to be investigated.

10.3 Deviations in Hull Form

Deviations of the hull from the original design and errors in numerical modelling may be significant. Work is needed to establish the extent of this problem in practice and the influence it has on inclining experiment results.

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Appendix A – Tables of Data and Results

Table A–1 Pendulum deflections and classic workup

Move No.	Weight Shift mm	Forward Pendulum			Aft Pendulum			Mean Heel (φ) deg.	***w d*** kg m	**Δ tanφ** kg
		Reading mm	**Deflection** mm	**Heel (φ)** deg.	**Reading** mm	**Deflection** mm	**Heel (φ)** deg.			
0	0	151.5	0.00	0.000	153.2	0.00	0.000	0.000	0.0000	0.0000
1	550	54.4	96.90	5.097	249.9	96.73	5.094	5.095	1.7039	1.5658
2	500	63.7	87.60	4.610	241.0	87.83	4.627	4.619	1.5490	1.4186
3	450	72.0	79.30	4.175	232.2	79.03	4.165	4.170	1.3941	1.2803
4	400	80.5	70.80	3.729	223.8	70.63	3.724	3.726	1.2392	1.1437
5	350	88.6	62.70	3.303	215.5	62.33	3.287	3.295	1.0843	1.0110
6	300	96.6	54.70	2.883	207.8	54.63	2.882	2.882	0.9294	0.8841
7	250	104.5	46.80	2.467	200.0	46.83	2.471	2.469	0.7745	0.7571
8	200	111.8	39.50	2.082	192.5	39.33	2.076	2.079	0.6196	0.6375
9	150	119.0	32.30	1.703	185.2	32.03	1.691	1.697	0.4647	0.5202
10	100	130.2	21.10	1.113	174.1	20.93	1.105	1.109	0.3098	0.3399
11	84.50	133.5	17.80	0.939	171.0	17.83	0.941	0.940	0.2618	0.2881
12	16.00	147.8	3.50	0.185	156.5	3.33	0.176	0.180	0.0496	0.0553
13	0	151.2	0.00	0.000	153.2	0.00	0.000	0.000	0.0000	0.0000
14	-14.00	154.2	-2.90	-0.153	150.3	-2.87	-0.151	-0.152	-0.0434	-0.0466
15	-84.25	168.9	-17.60	-0.928	135.5	-17.67	-0.933	-0.930	-0.2610	-0.2852
16	-100	172.3	-21.00	-1.107	132.0	-21.17	-1.117	-1.112	-0.3098	-0.3410
17	-150	184.5	-33.20	-1.750	120.3	-32.87	-1.735	-1.743	-0.4647	-0.5342
18	-200	191.5	-40.20	-2.119	113.2	-39.97	-2.109	-2.114	-0.6196	-0.6482
19	-250	198.8	-47.50	-2.504	105.8	-47.37	-2.499	-2.501	-0.7745	-0.7671
20	-300	206.5	-55.20	-2.909	98.2	-54.97	-2.899	-2.904	-0.9294	-0.8908
21	-350	214.5	-63.20	-3.330	90.4	-62.77	-3.310	-3.320	-1.0843	-1.0186
22	-400	222.4	-71.10	-3.745	82.0	-71.17	-3.752	-3.748	-1.2392	-1.1504
23	-450	231.2	-79.90	-4.206	73.1	-80.07	-4.219	-4.213	-1.3941	-1.2935
24	-500	239.8	-88.50	-4.657	64.8	-88.37	-4.655	-4.656	-1.5490	-1.4302
25	-550	248.9	-97.60	-5.134	55.8	-97.37	-5.127	-5.130	-1.7039	-1.5765
26	0	151.2	0.00	0.000	153.1	0.00	0.000	0.000	0.0000	0.0000
Mean Zero		151.30			153.17					

Table A–2 New workup and heeling / righting arm comparison

Move No.	Actual Heel (φ) deg.	*KN* m	sin φ	cos φ	*HZ* m	*KG* sin φ m	*HZ* mm	*GZ'* mm	Delta mm
0	0.054	0.0013	0.0009	1.0000	0.0000	0.0002	0.00	0.00	0.00
1	5.149	0.1123	0.0898	0.9960	0.0966	0.0146	96.64	96.65	0.01
2	4.673	0.1022	0.0815	0.9967	0.0879	0.0131	87.92	87.85	-0.07
3	4.224	0.0924	0.0737	0.9973	0.0792	0.0121	79.18	79.30	0.12
4	3.780	0.0824	0.0659	0.9978	0.0704	0.0108	70.42	70.55	0.14
5	3.349	0.0724	0.0584	0.9983	0.0616	0.0096	61.64	61.78	0.14
6	2.936	0.0625	0.0512	0.9987	0.0529	0.0085	52.86	53.11	0.25
7	2.523	0.0524	0.0440	0.9990	0.0441	0.0072	44.06	44.14	0.08
8	2.133	0.0426	0.0372	0.9993	0.0353	0.0062	35.26	35.41	0.15
9	1.751	0.0334	0.0306	0.9995	0.0265	0.0058	26.45	27.27	0.82
10	1.163	0.0221	0.0203	0.9998	0.0176	0.0033	17.64	17.69	0.05
11	0.994	0.0189	0.0173	0.9998	0.0149	0.0029	14.91	14.99	0.08
12	0.234	0.0047	0.0041	1.0000	0.0028	0.0007	2.82	2.87	0.04
13	0.054	0.0013	0.0009	1.0000	0.0000	0.0002	0.00	0.00	0.00
14	-0.098	-0.0016	-0.0017	1.0000	-0.0025	-0.0002	-2.47	-2.42	0.05
15	-0.876	-0.0162	-0.0153	0.9999	-0.0149	-0.0024	-14.51	-14.81	-0.30
16	-1.058	-0.0196	-0.0185	0.9998	-0.0176	-0.0031	-17.64	-17.72	-0.08
17	-1.689	-0.0315	-0.0295	0.9996	-0.0265	-0.0062	-26.45	-27.84	-1.39
18	-2.060	-0.0396	-0.0359	0.9994	-0.0353	-0.0055	-35.26	-34.89	0.37
19	-2.447	-0.0494	-0.0427	0.9991	-0.0441	-0.0064	-44.07	-43.59	0.48
20	-2.850	-0.0593	-0.0497	0.9988	-0.0529	-0.0076	-52.86	-52.41	0.46
21	-3.266	-0.0693	-0.0570	0.9984	-0.0616	-0.0088	-61.65	-61.22	0.42
22	-3.694	-0.0793	-0.0644	0.9979	-0.0704	-0.0100	-70.42	-70.02	0.40
23	-4.159	-0.0899	-0.0725	0.9974	-0.0792	-0.0118	-79.18	-79.26	-0.08
24	-4.602	-0.0996	-0.0802	0.9968	-0.0879	-0.0129	-87.93	-87.78	0.15
25	-5.076	-0.1098	-0.0885	0.9961	-0.0967	-0.0143	-96.65	-96.60	0.05
26	0.054	0.0013	0.0009	1.0000	0.0000	0.0002	0.00	0.00	0.00

Table A–3 Case 2: Data set representing a typical inclining experiment

Move No.	Shift m	*w d* kg m	Δ tanφ kg	Heel (φ) deg.	*KN* m	sinφ	cosφ	*HZ* m	*KG* sinφ m
13	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0004
10	0.100	0.3098	0.3399	1.1629	0.0221	0.0203	0.9998	0.0176	0.0036
8	0.200	0.6196	0.6375	2.1330	0.0426	0.0372	0.9993	0.0353	0.0065
13	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0004
16	-0.100	-0.3098	-0.3410	-1.0584	-0.0196	-0.0185	0.9998	-0.0176	-0.0028
18	-0.200	-0.6196	-0.6482	-2.0602	-0.0396	-0.0359	0.9994	-0.0353	-0.0052
13	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0004
		GM_0	0.952		TCG_I	0.001			
		KG_I	0.121		KG_I	0.160			

Table A–4 Case 3: Data set representing an inclining experiment with large initial list

Move No.	**Shift** m	***w d*** kg m	**Δ tan*φ*** kg	**Heel (*φ*)** deg.	***KN*** m	**sin*φ***	**cos*φ***	***HZ*** m	***KG* sin*φ*** m
20	0.000	0.0000	0.0000	-2.8502	-0.0593	-0.0497	0.9988	0.0000	-0.0094
22	-0.100	-0.3098	-0.2587	-3.6942	-0.0793	-0.0644	0.9979	-0.0176	-0.0118
25	-0.250	-0.7745	-0.6826	-5.0763	-0.1098	-0.0885	0.9961	-0.0439	-0.0161
20	0.000	0.0000	0.0000	-2.8502	-0.0593	-0.0497	0.9988	0.0000	-0.0094
18	0.100	0.3098	0.2421	-2.0602	-0.0396	-0.0359	0.9994	0.0176	-0.0072
15	0.216	0.6676	0.6052	-0.8764	-0.0162	-0.0153	0.9999	0.0380	-0.0042
20	0.000	0.0000	0.0000	-2.8502	-0.0593	-0.0497	0.9988	0.0000	-0.0094
		GM_0	1.136		TCG_l	-0.050			
		KG_l	-0.063		KG_l	0.163			

Table A–5 Case 4: Data set representing an inclining experiment with restricted heel

Move No.	**Shift** m	***w d*** kg m	**Δ tan*φ*** kg	**Heel (*φ*)** deg.	***KN*** m	**sin*φ***	**cos*φ***	***HZ*** m	***KG* sin*φ*** m
13	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0002
11	0.085	0.2618	0.2881	0.9941	0.0189	0.0173	0.9998	0.0149	0.0029
13	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0002
15	-0.084	-0.2610	-0.2852	-0.8764	-0.0162	-0.0153	0.9999	-0.0149	-0.0024
11	0.000	0.0000	0.0000	0.0540	0.0013	0.0009	1.0000	0.0000	0.0002
		GM_0	0.912		TCG_l	0.001			
		KG_l	0.161		KG_l	0.163			

Table A–6 Case 5: Data set representing an inclining experiment with an extreme heel close to upright

Move No.	**Shift** m	***w d*** kg m	**Δ tan*φ*** kg	**Heel (*φ*)** deg.	***KN*** m	**sin*φ***	**cos*φ***	***HZ*** m	***KG* sin*φ*** m
8	0.000	0.0000	0.0000	2.1330	0.0426	0.0372	0.9993	0.0000	0.0142
6	0.100	0.3098	0.2462	2.9362	0.0625	0.0512	0.9987	0.0176	0.0166
3	0.250	0.7745	0.6412	4.2241	0.0924	0.0737	0.9973	0.0440	0.0201
8	0.000	0.0000	0.0000	2.1330	0.0426	0.0372	0.9993	0.0000	0.0142
11	-0.116	-0.3578	-0.3491	0.9941	0.0189	0.0173	0.9998	-0.0204	0.0110
13	-0.200	-0.6196	-0.6375	0.0540	0.0013	0.0009	1.0000	-0.0353	0.0082
8	0.000	0.0000	0.0000	2.1330	0.0426	0.0372	0.9993	0.0000	0.0142
		GM_0	1.096		TCG_l	0.028			
		KG_l	-0.023		KG_l	0.164			