# APPLICATIONS OF 3D PARALLEL SPH FOR SLOSHING AND FLOODING

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**ABSTRACT**

Smoothed particles dynamics scheme is applied in sloshing and flooding problems in this paper. New solid boundary condition is used to simulate complex geometry. Parallelization of SPH scheme is carried out using MPI standard which makes 3D simulation acceptable. The numerical solutions obtained have been compared with both experimental results and other numerical solutions.

**Key words:** *SPH, sloshing, flooding*

## 1 INTRODUCTION

The smoothed particles hydrodynamics (SPH) is a meshless scheme, developed by Lucy [1] and Gingold and Monaghan [2] for astrophysical applications. It has been extended to hydrodynamics problems with free surface flow [3].

The advantages of SPH can be summarized as follow: SPH is conceptually both simple and easy for coding. The Lagrangian nature of SPH means that changes in density and flow morphology are automatically accounted for without the need for mesh refinement or other complicated procedures.

### 1.1. Basic algorithm

Governing equations

The governing equations for fluid flow are the mass and momentum conservation. In Lagrangian form, these governing equations can be written as

$$\begin{aligned} &\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot u = 0 \\ &\frac{Du}{Dt} = -\frac{1}{\rho}\nabla p + g + \nu\nabla^2 u \end{aligned} \quad (1)$$

where ρ is the fluid particle density; $t$ is time; $u$ is the particle velocity; p is pressure at the particles; $g$ is gravitational acceleration; and ν is the kinematic viscosity.

### 1.2. SPH formulation

The mass density at an arbitrary position is determined by a weighted average of the neighbouring particles:

$$\rho(x) = \sum_{b=1}^{N} m_b W_b(x) \quad (2)$$

And, the velocities are

$$v(x) = \sum_{b=1}^{N} V_b v_b W_b(x) \qquad V_b = \frac{m_b}{\rho_b} \quad (3)$$

Thus, we obtain the divergence of the velocity

$$\nabla v_a = \sum_{b=1}^{N} V_v (v_b - v_a) \cdot \nabla W_b(x_a) \quad (4)$$

Where,

$$\nabla W_b(x_a) = \frac{dW}{dr}\frac{1}{r_{ab}}(x_a - x_b)$$
$$r_{ab} = \|x_a - x_b\| \tag{5}$$

### 1.3. Kernel functions

The most commonly used kernel functions, $W_{ab}$, belong to a family of spline curves, and $\nabla W_{ab}$ is the gradient of interpolating kernel function. In our applications we refer to cubic B-spline, defined as follows:

$$W(r,h) = \frac{\alpha}{h}\begin{cases} 1 - \frac{3}{2}r^2 + \frac{3}{4}r^3 & if\, 0 \le r < 1 \\ \frac{1}{4}(2-r)^3 & if\, 1 \le r < 2 \\ 0 & if\, r \ge 2 \end{cases} \tag{6}$$

Here, $r = |x_a - x_b|/h$ and $h$ is called smoothed length. $\alpha$ is chosen to satisfy $\int W_{ab} dV = 1$

### 1.4. Equation of state

The constitutive model is determined by the equation of state. To close the system of equations, the most common used EOS is on derived from a relation proposed by Batchelor [4].

$$p = \kappa\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right], \tag{7}$$

with $\gamma = 7$. The density variation in fluid flow is proportional to the square of Mach number, M. An estimation of the upper bound of the velocity of the particles is

$$v = \sqrt{2gH}\ . \tag{8}$$

Monaghan 1992 shows 0.1 is a good estimation of Mach number for this problem and using Eq. (8) as an estimation for sound speed. The bulk modulus, $\kappa$, is re-evaluated as

$$\kappa = \frac{200gH\rho}{\gamma} \tag{9}$$

### 1.5. Viscosity

Basic SPH formulism suffers the absence of dissipation of energy. In order to increase the stability of SPH, many forms of artificial viscosity have been proposed, but the most commonly used artificial viscosity is obtained by writing the momentum equation as

$$\frac{dv_a}{dt} = -\sum_b m_b\left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi_{ab}\right)\nabla_a W_{ab} \tag{10}$$

Where $\Pi_{ab}$ is given by

$$\mathrm{P}_{ab} = \begin{cases} \dfrac{-6\bar{c}_{ab}M_{ab} + BM_{ab}^2}{\bar{c}_{ab}} & v_{ab}\cdot r_{ab} < 0; \\ 0 & v_{ab}\cdot r_{ab} > 0; \end{cases} \tag{11}$$

And

$$\mu_{ab} = \frac{hv_{ab}\cdot r_{ab}}{r_{ab}^2 + \eta^2} \tag{12}$$

The expression for $\Pi_{ab}$ contains a term that is linear in the velocity differences, which produces a shear and bulk viscosity.

## 2 BOUNDARY CONDITIONS

In SPH method, in order to apply the correct boundary conditions for the equations in the SPH formulation, the detection of boundary particles is needed to impose correct boundary conditions on these particles. Concerning bounded domain problems, there are several strategies and mathematical artifacts that allow for modeling the presence of boundaries with different degrees of accuracy.

In the first approach, the boundaries are replaced by interacting particles which exert a repellent force on fluid particles. In general, Lennard-Jones force is applied [3]. The repellent forces on the boundaries produce oscillations and this force only depends on the distance between particles and boundaies.

Quasi-ghost particles (fig.1) are defined to satisfy the wall boundary no penetration condition [5]. Several layers of ghost particles are lying within some specified distance (smoothing length) from the boundary. They carry the attributes with time history like fluid particles but the position is fixed and velocity is zero at every time step. The effects of the fictitious ‘ghost’ particles are explicitly included in the summation for the fields and for their gradients.

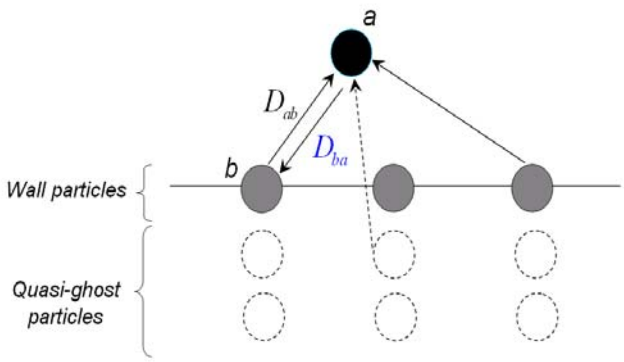


Figure 1. quasi-ghost particles.

$D_{ab}$ means the internal force on particles due to ghost particles which depends on pressure gradient.

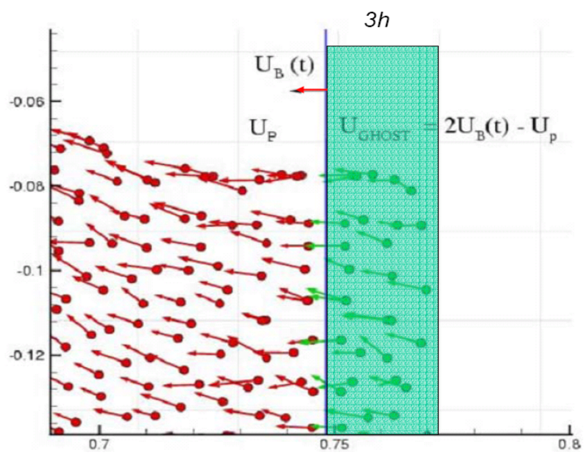


Figure 2. dynamic ghost particles.

Quasi-ghost particles can give some accurate results in our tests but they disturb the fluid domain too much. Alternatively dynamic ghost particles boundary conditions (fig.2) are available for the tests which give some more accurate results particular in the pressure evaluation on the boundary. Different from quasi-ghost particles, dynamic ghost particles are mirroring particles from fluid domain near the wall boundary and they carry the same attributes as the fluid particles but the opposite velocity in normal direction of wall boundary.

Dynamic ghost particles boundary conditions suffer penetration problem because of tensile instability. To avoid this kind unphysical phenomenon we restrain the particle motion and velocity when they are too close to solid boundary.

## 3 TIME EVOLUTION AND PARALLEL SPH

### 3.1. Time stepping and evolution

The numerical integration of the ordinary differential equations for the physical variables at each particle can be carried out by standard methods with a time-step control that involves the courant condition, the force terms, and the viscous diffusion term. The time step should be defined following Courant Friedrichs Levy (CFL) condition based on the local smoothing length and local sound speed. Then the resulting ordinary differential equation system can be integrated in time by schemes such as Runge-Kutta, Leap-Frog or any Predictor-Corrector, to ensure at least second order convergence in time.

### 3.2. Parallel SPH

In general, SPH calculation is very computationally demanding, both in memory and in CPU time.

Firstly, SPH method usually involves a large number of particles to be geometrically enough to model the deformation of fluid body.

In three dimensional cases, the SPH model may involve several millions of particles.

Secondly, the evolution of the particle information should be very time consuming. Besides the governing equations themselves, neighbour search, boundary treatment, and interactions between particles manifold the complexity of the problem.

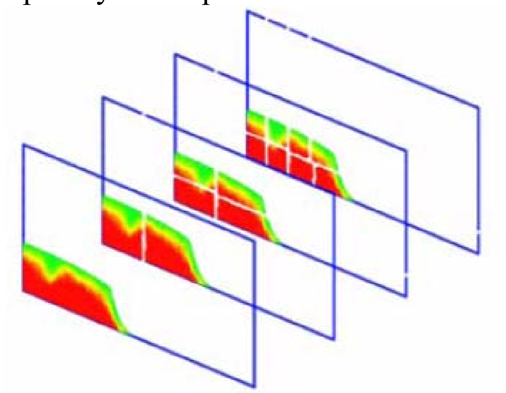

Figure 3. Parallel SPH.

The barge has the following features:

| | |
|---|---|
| Length | 3m |
| Width | 1m |
| Height | 0.267m |
| Mass without tank | 127 kg |
| Draft | 0.108m |
| Moulded volume | 0.285m3 |
| Center of gravity above keel | 0.237m |
| Giration radius (in roll) | 0.414m |

The model is installed in the basin in the middle of the testing section; with its longitudinal axis parallel to the wave-maker line (only beam waves were considered).

The mooring system is ensured by 4 cables equipped by springs. The mooring lines are attached at the 4 corners of the barge at 0.262m height according to the keel. The natural sway period of the mooring system (direction of the waves) is 20s.

The reference point for the measurements of the translation of the barge motion is at the deck level (0.267 m above the keel level).

For these reasons, standard MPI technique is adopted for parallel SPH code. The computational domain is decomposed into several parts for each process working.

# 4 BENCHMARK RESULTS

## 4.1. Case 1 - Validation case for sloshing

Tests have been carried out in 2001 and 2002 by the GIS-Hydro in the wave basin of La Seyne/mer – France (BGO First facilities). The experimental set up consists of a rectangular barge model supporting two rectangular tanks partly filled with water. Regular and irregular wave tests results are available for beam seas. Barge motions and internal wave elevations have been measured. Results are free to use which can be provided by Principia.

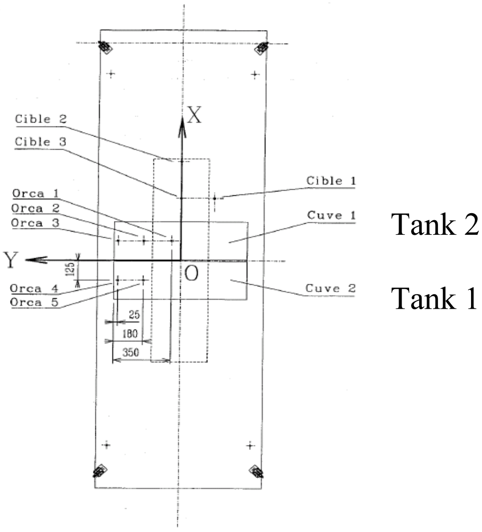


Figure4. Sketch of the model.

The two tanks are set on the deck of the barge, at mid-ship, with their length in the transverse direction. The elevation of the inner bottom of the tanks, with respect to the keel, is about 0.3m.

The characteristic of the two rectangular tanks are the following:

| | |
|---|---|
| Length | 0.8m |

| | |
|---|---|
| Width | 0.25m |
| Height | 0.6m |
| Mass | 37kg |

The water motion inside the tank is measured by 5 probes:

- 3 in the tank 2 (higher filling level) at 24mm, 180mm and 350mm from the wall closest to the wave-maker.
- 2 in the tank 1 at 25mm and 180mm from the corresponding wall.

The sloshing motion in the tanks is recorded with a video camera.

Waves elevation is measured with 5 probes. All are set on the longitudinal axis of the basin at different spacing.

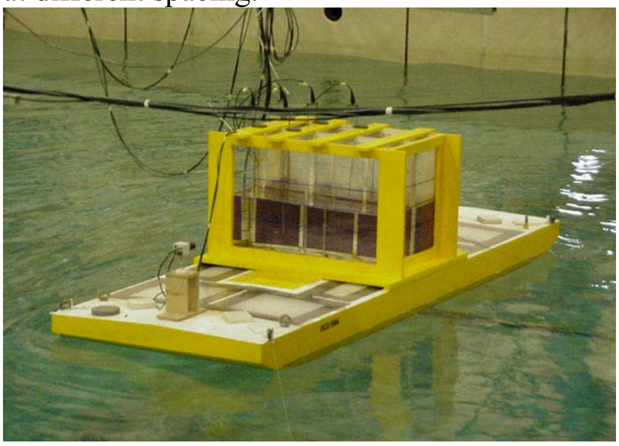

Figure 5. View of the model test set-up in BGO First.

**Definition of the test cases**

Sloshing in tank without considering the barge: the discretized domain is a rectangular tank animated with a motion which is deduced from the motion of the barge. The input are the height of the free surface, the 6 DOF (degree of freedom) displacement of the tank (motions of the barge) and the position of the tank according to the barge centre of gravity.

| Test cases irregular waves | Motions of the barge | Filling level |
|---|---|---|
| Irreg 1 | 6 DOF | 39cm |
| Irreg 2 | 6 DOF | 19cm |

The following figures show an example of the 6 DOF motions of the barge (and of the tank) in irregular waves

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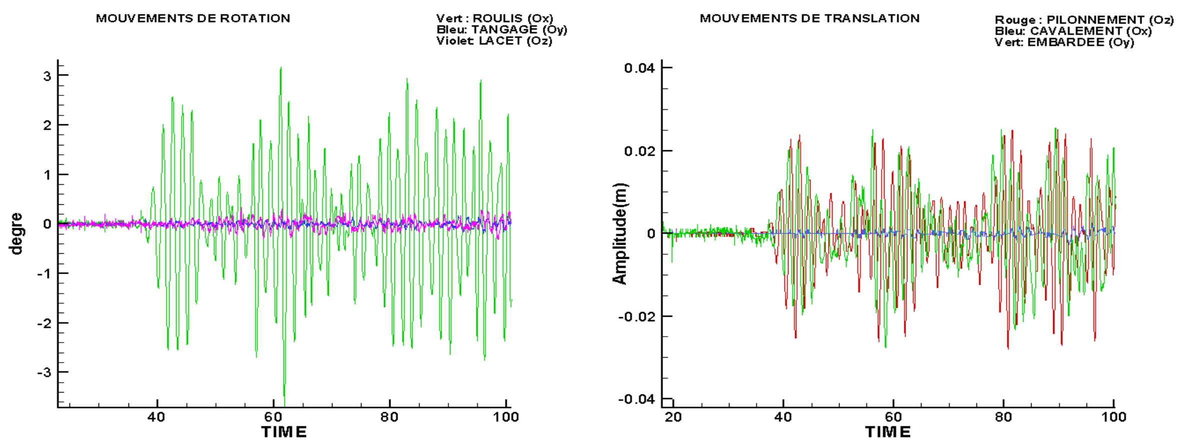



Figure 6. Irregular wave motion in time series.

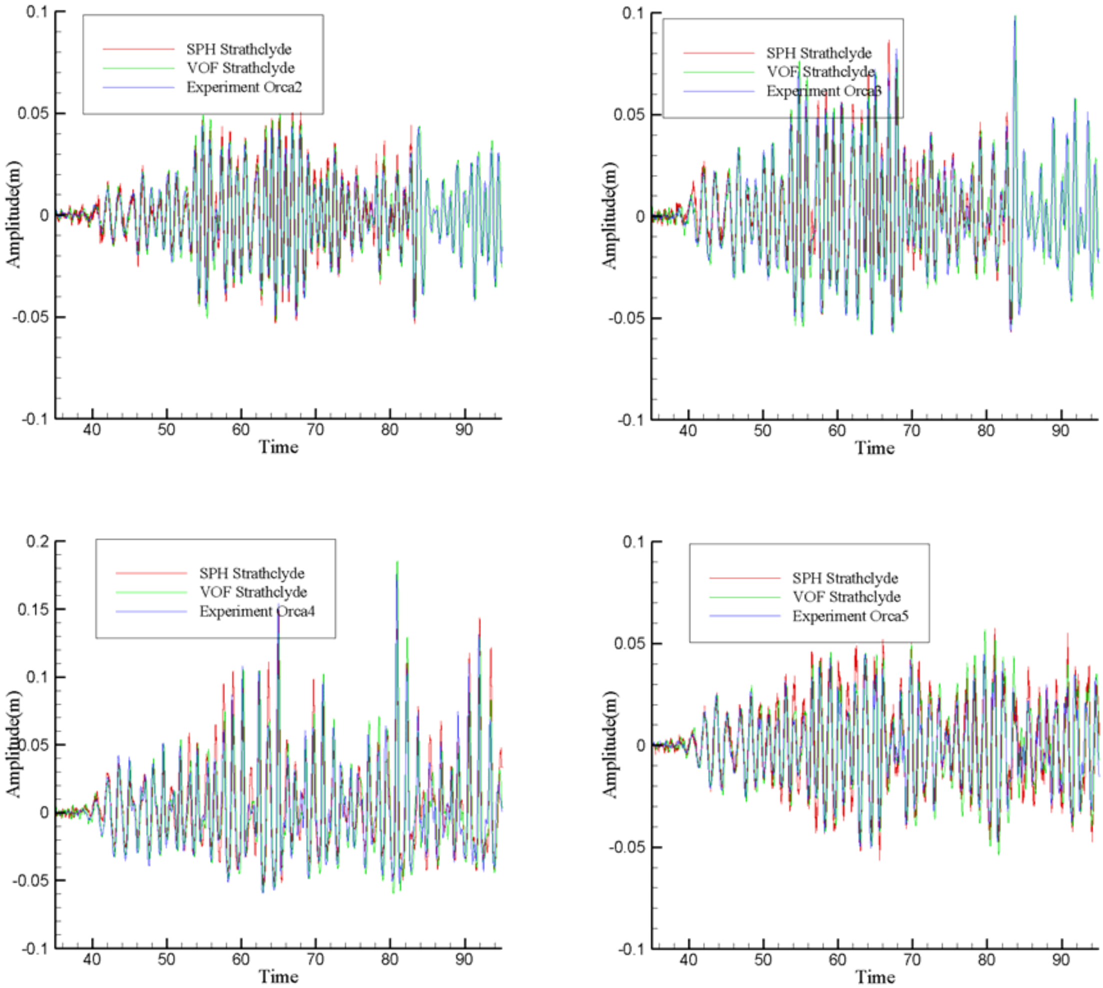


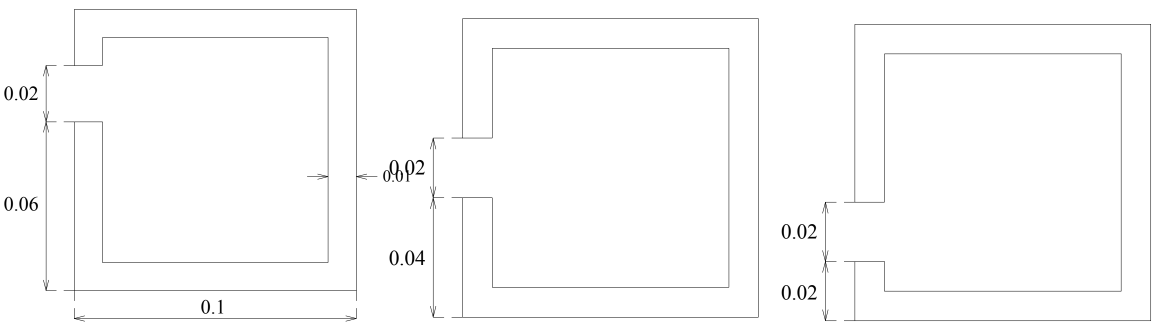

Figure 7. Results comparison of wave evaluation.

The numerical results in Fig 7 were carried out by SPH code and Fluent which both give good results of wave height on probes.

## 4.2. Case 2 – 2D rectangular box

To validate flooding case with SPH concerning coupling free motion tank we proposed numerical models and simulate the cases with both SPH code and Fluent.

Validation case for 2d transient flooding under free motion



Case A Case B Case C

| | |
|---|---|
| Weight | 7.5kg |
| Moment of inertia | 0.075kg.m2 |
| Length | 0.1m |
| Height | 0.1m |
| Gravity center above bottom | 0.025m |
| Horizontal gravity center | 0.05m |
| Thickness | 0.01m |
| Drought | 0.075m |

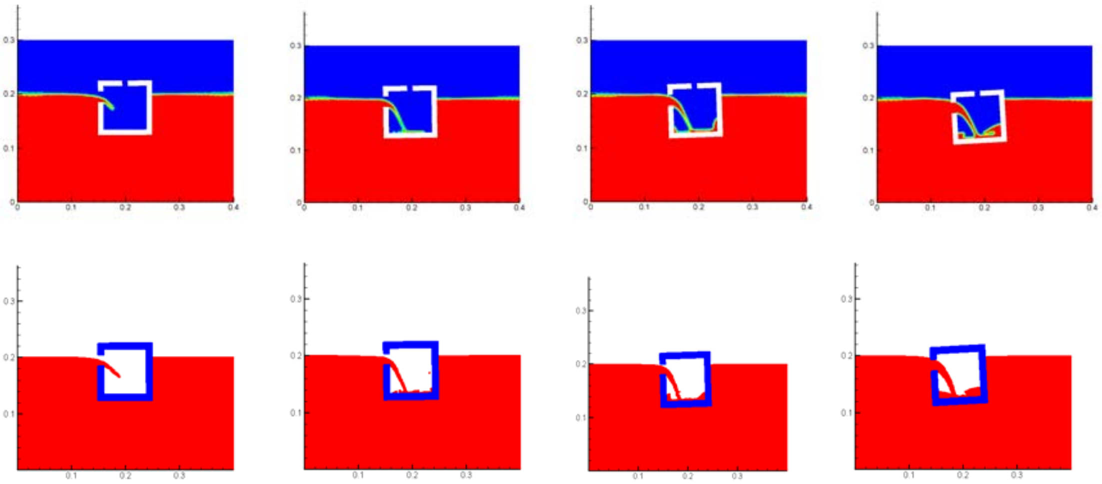


Figure 8. Case A From left to right: 0.1s, 0.2s, 0.3s, 0.4s.From top to bottom: VOF, SPH.

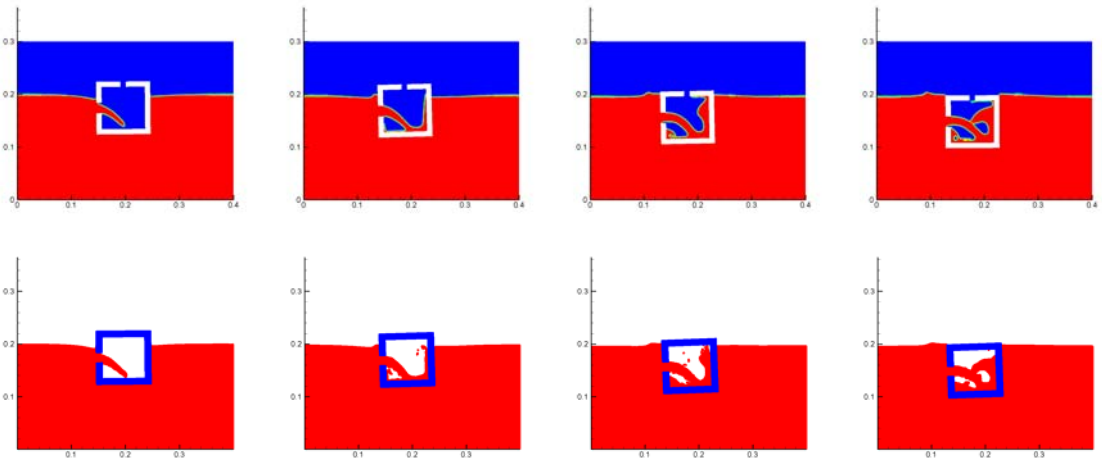


Figure 9. Case B From left to right: 0.1s, 0.2s, 0.3s, 0.35s.From top to bottom: VOF, SPH.

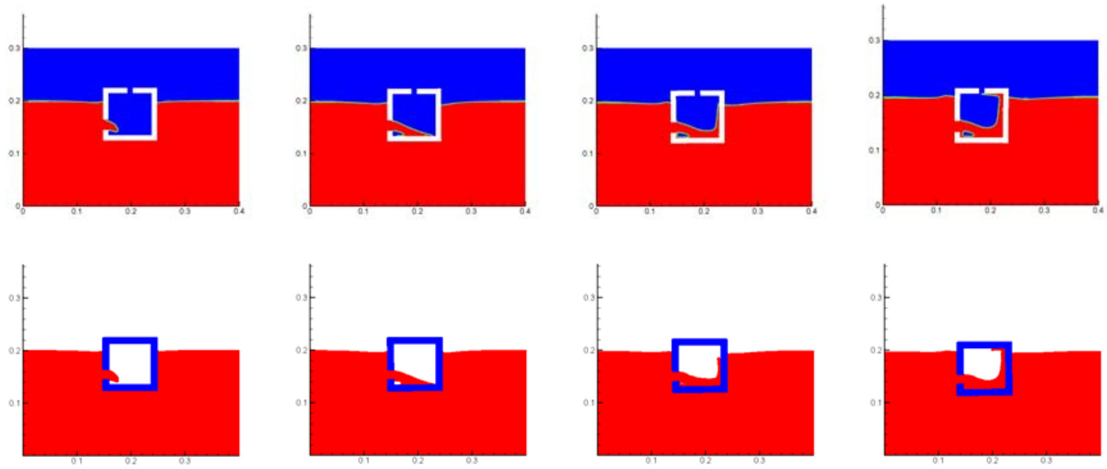


Figure 10. Case C From left to right: 0.1s, 0.2s, 0.3s, 0.4s.From top to bottom: VOF, SPH.

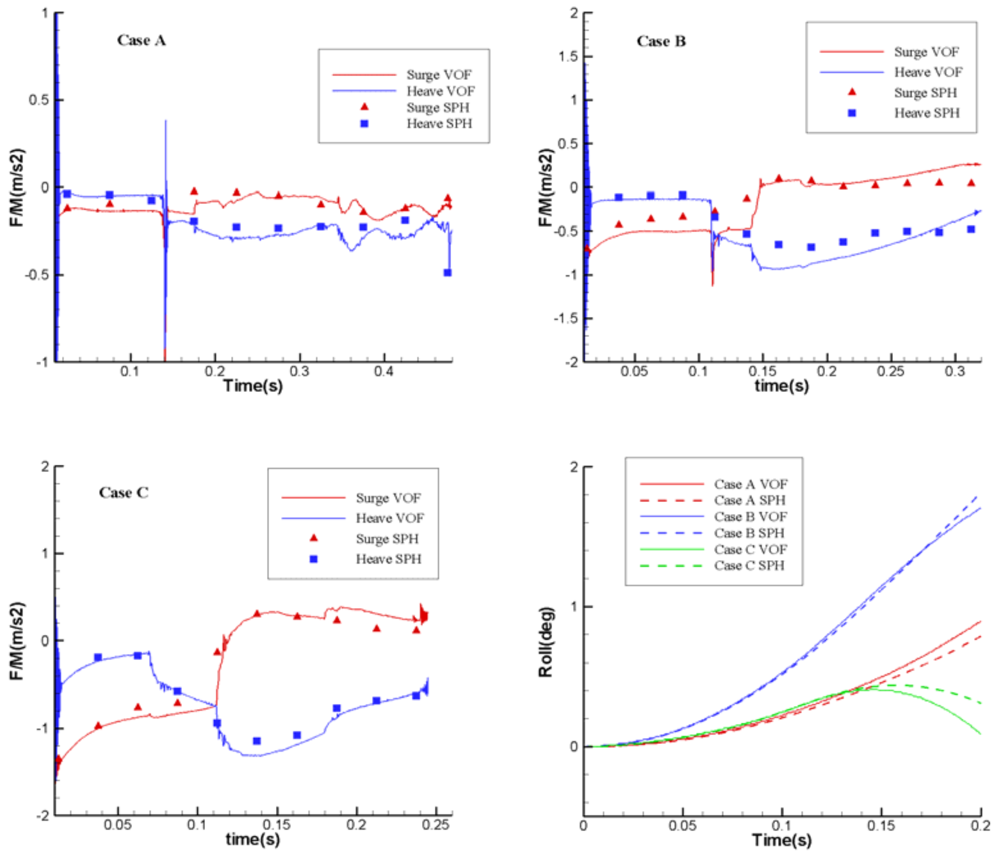
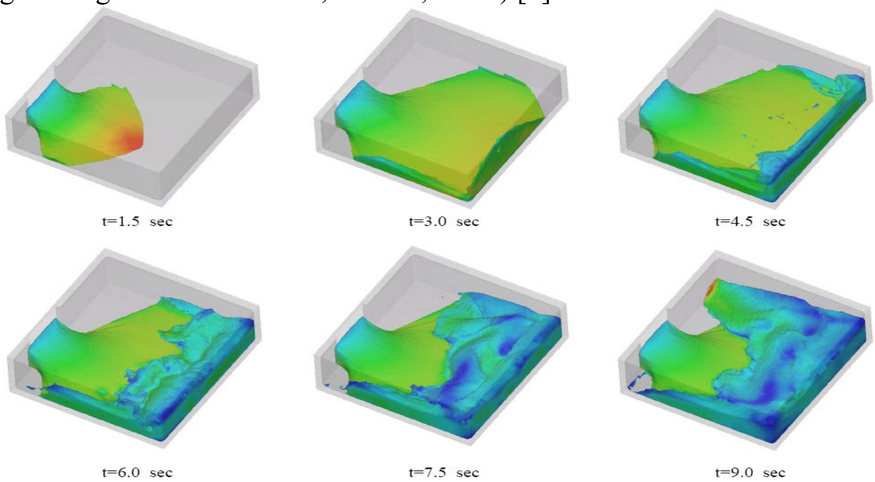


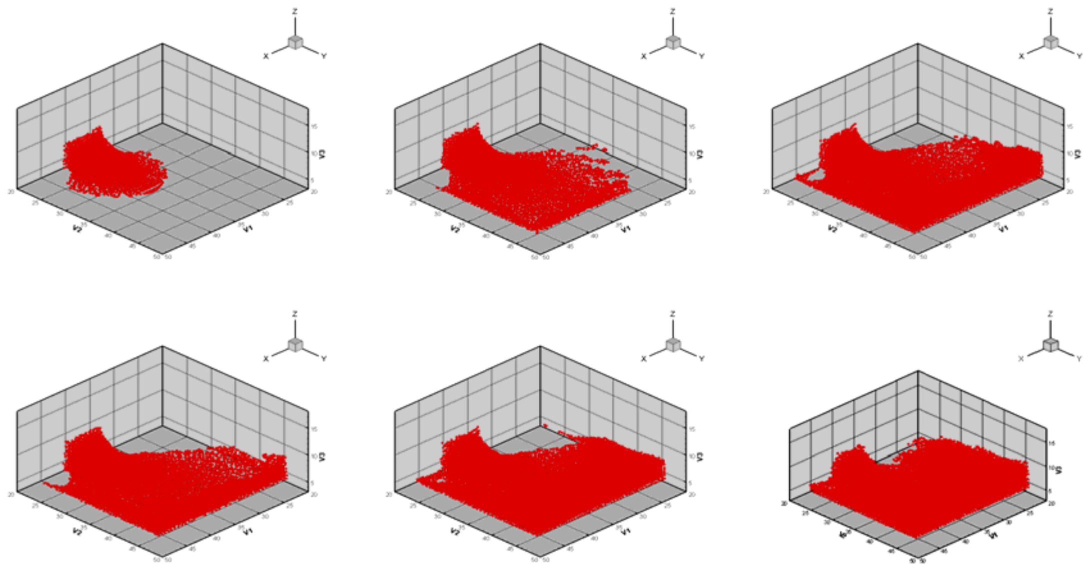

Figure 11. Surge, heave and roll motion of damaged box.

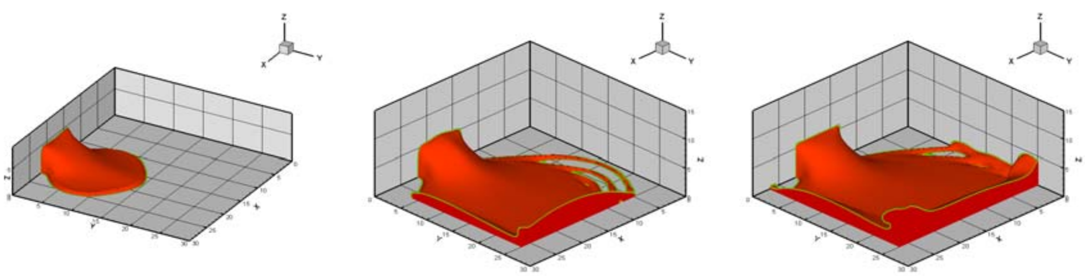
Fig. 8-11 gives the results of water surface and motion in time series which prove SPH's ability of handling transient flooding coupling the response of tank.

### 4.3. Case 3 – 3D rectangular box

Flooding test of Roro ship is proposed in 24th ITTC to investigation the stability under damaged flooding situation. Simplified model was tested by Cho (2005,Maritime and Ocean Engineering Research Institue,KORDI,Korea) [7].





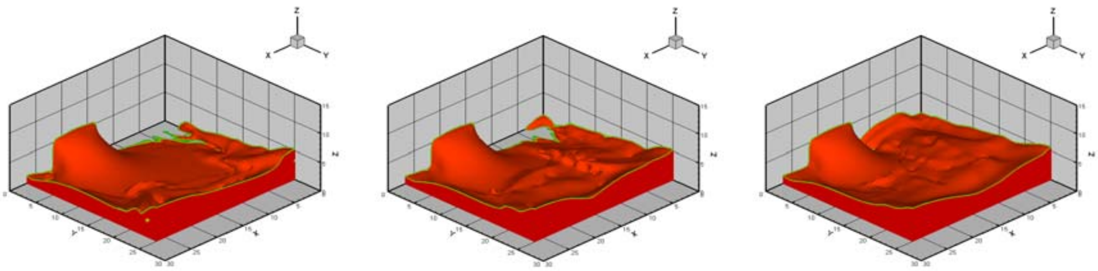


Figure 12. From top to bottom: Flow 3D,SPH,Fluent.

In fig 12 flooded water surfaces in the damaged tank was given at 1.5s, 3s, 4.5s, 6s, 7.5s, 9s. More than one million of particles are used in SPH. Because of efficiency parallel scheme, SPH could simulate 3D flooding correctly and quickly.

## 5 SUMMARY AND CONCLUSIONS

In this paper we presented the applications of SPH on the hydrodynamics problem: slamming, sloshing and flooding. SPH scheme shows the ability to predict correct answer to the violent free surface flow.

After parallelization of SPH scheme and variable particles smooth length, large 3D simulation became acceptable for SPH.

Further application of this method will concern real ship flooding and coupling SPH with other CFD method.

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