GENERAL PRINCIPLES FOR SHIP DYNAMIC PROBLEMS SOLUTION IN HIGH WIND AND WAVES CONDITIONS

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ABSTRACT

Approach to development of general mathematical tool for ship dynamic problems solution in intensive wind and wave conditions (complex SD-system) is discussed. Main attention is focused on formulation of conceptual basis and principles of data processing in complex dynamic environments (mainly for on-board system). Formal description of logical space for computational models integration is shown. Model of solution algorithm choosing is formulated. Interpretation of computational SD-system as parallel architecture is presented

Keywords: *ship stability problems, hieratical decomposition of parallel applications, problem solving environment*

1. INTRODUCTION

Synthesis of data processing models in complex SD-system defines methods of complex ship-environment interaction problems solution. It is especially true for conditions when object dynamics and environment continuously change that typical for on-board intelligent systems operation. In this case problems of evolutionary self-organizing knowledge bases and adaptive computer systems development are very important. In aggregate these problems define general problem of theoretical foundations development for complex SD-system's software-hardware design [Degtyarev et al., 2007, 2007a, France et al., 2001]. Software suite of intelligent decision support in complex SD-systems is shown on fig.1.

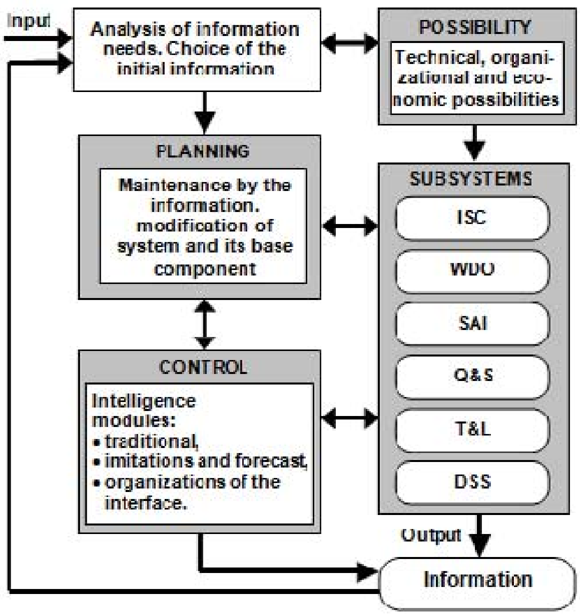

Figure 1. Information flow realizing decision support system concept in SD-system: ISC – integrated subsystem of computation control; WDO – data management subsystem; SAI – search and access to information; Q&S – control of decisions quality; T&L – testing and learning; DSS – decision-making.

SD-system combines robust formal methods of data analysis and interpretation for ship dynamics problem solution with heuristic methods and models based on hydrodynamics, expert's knowledge, and cumulative experience. The system includes some cooperative modules carrying out determinate functions in accordance with general strategy. Besides traditional intelligent support modules the system contains modules for simulation, analysis and forecast of problem situation, different interfaces.

Problem's analysis permits to emphasize some characteristics:

- algorithms complexity and huge amount of initial data with essentially different structure;
- rigid requirements to computer performance, necessity of calculations in real time mode (especially for on-board systems);
- operations with great number of different kinds objects (characteristics of ship, wind and waves).

Adaptive synthesis and computer program parallelization are carried out by the way of systematic programs transformation (which are presented by system rules). However it is much more convenient to assign many parallelizing transformation in procedural view. It is possible to store and use rules of knowledge transformation (system rules) about code parallelization methods with the help of data organization subsystem of complex SD-system.

The main problems of high performance realization of ship dynamics codes are the following: high linkedness of CFD problems which are foundation of SD-systems, multiscale physical processes that take place, various data sources, etc. Lack of universal ways for such applications mapping on parallel architecture of supercomputers or clusters makes such approach very important.

On-board SD-systems obtain initial data about current state by the way of information processing from sensors of measurement system (fig.2). Processing of this information is implemented in real-time mode. It permits "turn" SD-system for solution of current situation interpretation problem. It carries out in accordance with algorithms of waves and ship parameters identification (source is patents of Prof. Yu.Nechaev), construction of real values of membership functions determining logic of dynamical knowledge base functioning (estimation of situation danger and forecast of its development).

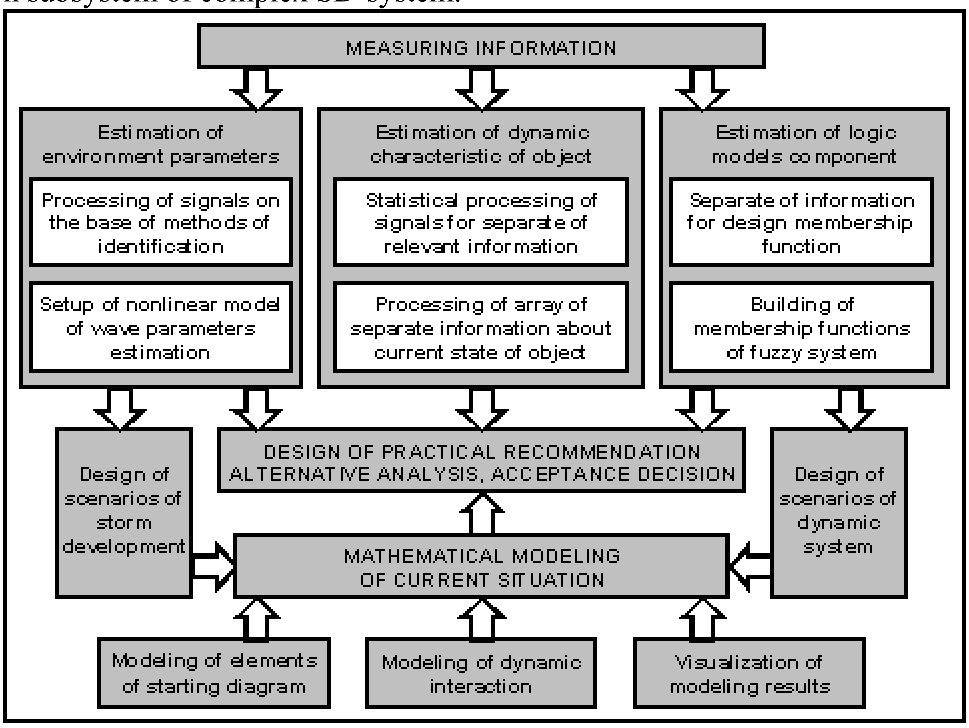

Figure 2. General principles of measurement data processing algorithms using for environmental dynamics assessment, object parameters estimation, modeling and visualization of current situation.

2. CONCEPT OF COMPLEX SD-SYSTEM

We can consider complex SD-system as a new generation of computer environment – problem solving environment (PSE) [Houstis et al., 2000, Walker et al., 2000]. Main character of such PSE is complication of information processing algorithms in SD-systems results in necessity of high performance methods application for search of new effective computer procedures and there parallel implementation [Degtyarev et al., 2007, On-board, 2006].

SD-system concept is formulated as generalization and development of information processing common methods utilizing high performance computer tools [Nechaev, 2003]. Such model foresees complex SD-system using both for complex ship dynamics modeling and for software-hardware complexes development (on-board intelligent system, virtual testbed, etc.). We can note the following principles of the concept: adaptability, distribution, service orientation, virtualization, fault resistance. Complex SD-system has possibility of evolution extension in conditions of permanent changing of object dynamics and environment.

Improvement of assessment and situation forecast reliability in complex ship dynamics problems is achieved with the help of new approach to data processing based on “soft computing” concept development [Zadeh, 1994]. This approach foresees theoretical principles using. It has to permit data processing computer technology organization and formalization of data flow in parallel/distributed computer environment. So complex SD-system is complicated multiprocessor computer complex which we can consider as self organized dynamic informational space of unified data and knowledge representation about ship dynamics and sea.

Methods of alternative algorithms forming are developed at formal description of computational space in complex SD-system. Principles of data processing in multiprocessor environment are realized with the help of these methods. In this case different approaches to ship dynamics problems solution are used: classical mathematical methods (standard algorithms), fuzzy and neural networks models [Nechaev, 2003]. Such methods form set of equivalent (functionally close) algorithms. Improvement of algorithms efficiency with the same functionality is achieved by the way of there adaptation to initial data. Choice of the favorite computer technology is carried out by the way of alternatives analysis [Degtyarev et al., 2007].

3. CHOICE OF SHIP DYNAMICS MODEL ON THE BASIS OF COMPLEX SD-SYSTEM

Different approaches of ship dynamics problems solution, choice of initial data, mathematical models and algorithms of there realization are considered in process of complex SD-system operation. Comparison of obtained results and choice of optimum decision is achieved subject to practicability parameters of complex problem in distributed computing environments. Developed approach to adaptation of algorithms in complex SD-system permits to realize methods of alternative algorithms formation on the basis of superiority degree.

Hierarchical model uniting functional modules of applied programs (information processing, ship dynamics problem solving, etc.) permits to describe complex SD-system at various levels of abstraction. Decomposition model of complex SD-system is represented by the set of models of sublevels:

$$M = \langle M^{sl}(S_i), R^T \rangle \qquad (1)$$

where $M^{sl}(S_i)$ is sublevel model generated by subsystem S_i; $R^T \in M^{sl}$ is tree relation.

Formation of models hierarchy of integrated system is carried out with the help of standard bases of decomposition. At any level of hierarchy subsystems and interrelations between them are allocated. Formally subsystem model could be described as the follows:

$$M^S(S_i)=<C(S_i),\{V_j(S_i)\}>, \qquad (2)$$

where $C(S_i)$ is class description of subsystem S_i; $\{V_j(S_i)\}$ is the set of samples (variants) of subsystem S_i.

Description of class $C(S_i)$ contains set of attributes. Identifier (name), type and range of values (domain), relations set (dependences) between attributes and methods set (attached procedures) are set for each of classes:

$$C(S_i)=<\{x_k,t_k,D_k\},G,F>, \qquad (3)$$

where x_k is attribute identifier; t_k is attribute type; D_k is attribute domain; $G=\{g_k\}$ is the set of relations between attributes; F is methods set.

As attributes of object-subsystem every possible properties and parameters describing subsystem function and its elements can be. Simple types of operators are used for properties description: Text, Number, Time, Date. Complex types of attributes are used for modeling of subsystems elements: Structure, Collection. Attributes of structure type are described by set of attributes, i.e. they are objects.

Rational form of data presentation is developed for analysis and interpretation of the information. Mathematical approximation of data is represented in the form of matrix model:

$$X=\begin{bmatrix} x_1^{(1)}\dots x_1^{(i)}\dots x_1^{(M)} \\ \dots \\ x_j^{(1)}\dots x_j^{(i)}\dots x_j^{(M)} \\ \dots \\ x_N^{(1)}\dots x_N^{(i)}\dots x_N^{(M)} \end{bmatrix} \qquad (4)$$

Rows of a matrix represent set of used models, columns represent features of model (information attributes) and opportunity of its application in a considered problem. Various approaches such as weighing of terms, transition to new system of attributes (factor analysis, component analysis), statistical approach, theoretical-information approach are used for informative attributes revelation.

Development of ship dynamics PSE is carried out in form of situational model of game with dynamically varying class of strategy and operated script. For this problem solution script described by finite graph is preliminary formulated.

$$G = (S_S, W_S), \qquad (5)$$

where S_C are strategies, P_S are transitions between them. It permits to develop algorithm of problem solution in the view of the following procedures:

- to present S_C in the form of

$$S_C=\bigcup_{t_j} S_C^{t_j} \qquad (6)$$

i.e. as association of strategy (alternatives) S_C^{tj} and control moments t_j, (j=1,…,N);

- to present P_S in the form $P_S \subseteq S_C \times S_C$ describing transients between strategies with the help of set of efficiency strategies I_1 in the form of two sets: the first is arc set $\delta_A t_j : A^{l(tj)} \rightarrow P_S$, the second is the set of utility of such strategies $\delta_U t_j : U^{l(tj)} \rightarrow P_S$.

The problem of decision choice at data interpretation in complex SD-systems consists in realization of hierarchical structure analysis. Elementary subtasks allocated in this structure preliminary are analyzed by method of hierarchies [Saaty, 1993].

Such approach to ship dynamics PSE design results in necessity of hierarchical decomposition of parallel applications. Such decomposition could be realized in two specifically different ways – in homogeneous and heterogeneous computational environment [Korkhov, 2009].

By the first way workload is decomposed among *C* Computing Elements (CE's), and each

CE is a parallel computing system with p nodes that have the same computing capacity Δ. Thus a two-level hierarchical decomposition is introduced: first, the workload is decomposed between C CE's, and then for each CE the portion of workload is again decomposed between the processors of this CE (Fig.3). In this picture, illustrating the hierarchical decomposition, three levels can be identified. The first level is the full non-decomposed workload W – the level of sequential computing. The next *level 1* introduces the division of the workload between the CE's. This decomposition induces a *level 1* overhead $Q_1(W,C)$, that reflects communication between CE's or load imbalance among CE's. Next, within each CE the workload W/C is again decomposed between the p processors of the CE. On this level another overhead $Q_2(W/C;p)$ is encountered. In homogeneous grid environment each CE receives an equal share of the total workload. Moreover, the same overhead function $Q_2(W, p)$ applies within each CE.

By the second way such reasoning is not true, because workload and overhead have not equal share and depend on computing capacities of nodes p_i – Δ_i. So we have not exact algorithms of hierarchical decomposition of parallel applications in heterogeneous environment. It is separate very important problem going beyond this paper.

User interface in PSE of complex SD-system foresees at the minimum the following solution variants: situation analysis and forecast; visualization of interaction dynamics; operative control. In this case graphic window of the interface «Situation analysis and forecast» **Analysis - Forecast** (fig.4) contains three areas: area of initial data – **Input Data**, area of tree of model – **Domain** and viewport – **Results**. Initial data area contains processed data of estimations of situation danger and forecast of its development. Model tree area includes the following elements: **Physics** – set of problem conditions, **Model** – model choice (classical, fuzzy, neural network); **Simulation** – work with model. Viewport **Results** contains results of analysis (choice of preferable computing technology) with indication of corresponding considered stability characteristics curve and practical recommendations on vessel speed choice - **Conclusion**.

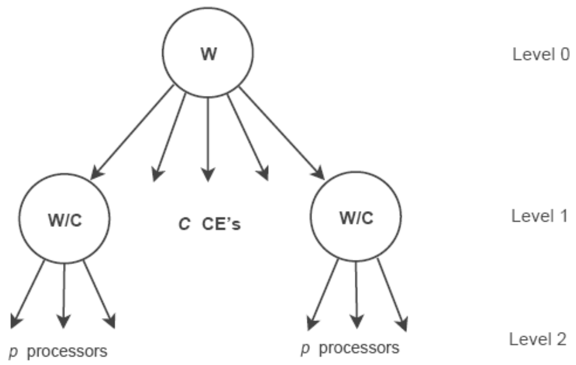

Figure 3. Hierarchical decomposition of a parallel application on a homogeneous computational Grid: the total workload W is distributed between C computing elements with p processors each.

In order to make the general system user-friendly we need to hide the complexity of the underlying components. For that we have to develop an advanced graphical user interface that seamlessly integrates the disparate distributed modules into one transparent user environment, which is presented to the user via a Web-interface and a Grid portal. The PSE architecture supports interaction on various levels: interaction with the workflow through the user interface, interactive visualization, control over the simulation processes and interactive job control on the middleware level.

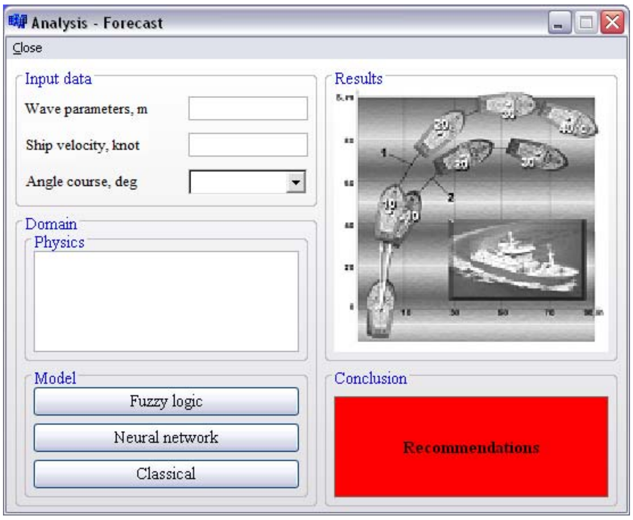

Figure 4. Graphic window «Situation analysis and forecast»: in the window «Results» dynamic picture of ship turning on waves in broaching regime is shown.

4. GENERAL CHARACTERISTICS SHIP DYNAMICS PROBLEMS ON WAVES

Among ship dynamics models on waves the greatest interest is problems of ship stability assessment. Complexity of such problems solution results in necessity of taking into account the following characteristics at effective algorithms realization [Degtyarev et al., 2007, Nechaev, 1989]:

(1) uncertainty in external forces description (wind and wave excitations) determining difficulty of problem initial conditions formation;

(2) necessity to take into account complicated spatial nonlinear function describing restoring moment in waves;

(3) complexity of mathematical description of some stability problems, which formalization can be achieved on the basis of hypotheses and generalizing assumptions only.

The specified characteristics have determined new paradigm determining various approaches to ship dynamics problems solution:

- deterministic approach when system dynamics is completely determined by initial conditions formulation of discussed mathematical model;
- stochastic approach when problem initial conditions are not determine, poor formalizable and cannot be correctly formulated owing to stochasticity of investigated process.

At first direction problems solution formulation of interaction model is based on enough simple calculation schemes of wind and waves excitations. In particular, wind influence is represented in the form of carried out calculation schemes which have found application in stability regulation practice, and waves are approximated by harmonious functions, or in trochoid form [Nechaev, 1989]. At stochastic systems investigation the most effective way consists in initial conditions formulation «on the other end» of problem and system of differential equations integration in return time using action functional method [On-board, 2006]. Really, with reference to ship stability assessment problem it is much easier to develop algorithm of initial conditions search by limiting rolling estimation. Usually two-three iterations are enough as search area is known in advance depending on GM-curve kind. For the majority of single-line vessels this area covers rather small range of dynamic roll. E.g. this range is 45 – 55^{o} for small fishing vessels.

Below general formulation of a ship stability problem in waves is given. By way of illustration we consider problems of hydrodynamic analysis of ship stability and on-board intelligent systems. Close inspection of this formulation shows that these problems could not be solved in principle in frameworks of old approaches for obtaining of numerical results due to hierarchical structure of considered models. Only PSE principles could be considered as the way for solution of general complex problems. Alternative computing technologies (fuzzy and neural network models) are described in [Nechaev, 2003].

4.1 Formulation of the problem of ship stability in waves

The hydrodynamic problem of ship dynamics in waves at program realization with the help of complex SD-system is represented in standard form used in [Boroday, 1982, Handbook, 1985]. Generally hydrodynamic action is reduced to principle force F applied in the center of moving coordinates and principal moment M. Values F and M are determined by formulas:

$$\begin{aligned}
&\vec{F} = -\int_S p\vec{n}\, dS = \frac{\partial \vec{n}}{dt} - \vec{\omega} \times \vec{N} \\
&\vec{M} = -\int_S p\left(\vec{r} \times \vec{n}\right) dS = -\frac{\partial \vec{Q}}{\partial t} - \vec{\omega} \times \vec{Q} - \vec{V} \times \vec{N} \\
&\vec{N} = -\int_S \Phi_K \vec{n}\, dS,\;\; \vec{Q} = -\rho \int_S \Phi_K \left(\vec{r} \times \vec{n}\right) dS \\
&\Phi_K = \sum_{m=1}^{6} u_m \Phi_m,\; m = 1,...,6
\end{aligned} \tag{7}$$

where $\vec{r}$ is radius-vector of the point of wetted surface S; u_m are projections of vectors $\vec{V}$ and $\vec{\omega}$ defined forward and rotary movement; $\vec{N}$ and $\vec{Q}$ are principle vector and principle moment of the system of instantaneous impulses of hydrodynamic pressure; Φ_K are harmonious functions satisfied the conditions of disturbed motion damping on infinity and flow on surface S.

On the basis of expressions (7) by the traditional transformations accepted in hydromechanics, the system of differential equations describing ship motion is determined. Compact view of this system looks like [Nechaev, 1989]:

$$\begin{aligned}
&F_i(\ddot{x}_i, \dot{x}_i, x_i, t, X_{i1}, \ldots, X_{im}, \\
&\quad Y_{i1}, \ldots, Y_{in}) = 0,
\end{aligned} \tag{8}$$

where $F_i(\bullet)$ are functions; x_i are linear and angular displacements; $X_{i1},\ldots,X_{im}$ are parameters characterized ship as dynamic system (inertial, damping and restoring components); $Y_{i1},\ldots,Y_{in}$ are disturbing forces and moments $i = 1,2,\ldots,6$.

Structure of mathematical models describing ship motion in waves is based on the classical approach of division of acting forces and includes inertial, damping, restoring and disturbing components. Forces of the hydromechanic and aerodynamic nature are considered.

It permits to allocate specific characteristics in mathematical models and to investigate character of considered phenomena.

At research of stability on random waves stochastic equations of ship dynamics are considered. Research of such problems is carried out either by Monte-Carlo method using description of random waves in the form of correlation function or spectral density, or on the basis of representation of wave models in the form of wave packages of various form and intensity.

In practical stability problems it is accepted to consider some special cases of ship state (beam sea, following sea, etc.) or special situations/modes (broaching, breaking waves, bulk cargo, etc.).

4.2 3-D problem of hydrodynamic theory of stability

Determination of minimal capsizing moment usually is carried out on the basis of static calculation schemes in accordance with diagrams of static and dynamic stability. Thus it is considered that the restoring moment at ship inclinations is determined by hydrostatic pressure upon an underwater part of ship hull. Meanwhile, heelling ship disturbs surrounding water changing pressure field and deforms free surface. Therefore this problem is hydrodynamical problem. It is very important, because such diagram (GZ-curve) is a reference curve for calculation of restoring moment in waves and accuracy of its calculation determines the general error of methods of ship stability estimation.

3-D problem of hydrodynamical theory of stability is considered in [Sisov, 1998]. It is based on fundamental conservation laws: law of conservation of mass, law of conservation of momentum, law of conservation of energy, etc. Fluid and floating body are considered as uniform hydrodynamical system. The fluid is considered as ideal, with unlimited depth. Exact boundary conditions are considered at free surface and on wetted surface. All system is supposed being in quiescent state up to the moment of application of external pair forces. The author [Sisov, 1998] obtains theoretical solution of the problem. Value of capsizing moment is the numerical solution as the least bifurcational value of the moment for ship inclination equation.

The method of problem solution consists in the following. Differential system is based on representation of vector function W^* and pressure p in the form of series on time with coefficients depending on initial conditions (a, b, c):

$$\overline{W}(a,b,c,t)=\sum_{k=0}^{\infty}\overline{W}_k(a,b,c)t^k \tag{9}$$

$$p(a,b,c,t)=\sum_{k=0}^{\infty}p_k(a,b,c)t^k \tag{10}$$

In expansion $\overline{W}$ vector $\overline{W}_0$ gives initial position of particulates, $\overline{W}_1$ – initial velocities. In accordance with initial conditions $\overline{W}_1$=0, $\overline{W}_2$ determines accelerations, next terms give high order accelerations. Substitution of (9) in continuity equation

$$\overline{W}_a\cdot\overline{W}_b\times\overline{W}_c=1 \tag{11}$$

and equating of coefficients at equal powers t gives:

$$\nabla\overline{W}_k=\Phi_k;\ \text{for}\ k\le 3\ \ \Phi_k=0. \tag{12}$$

Substitution of the same series in fluid equations

$$\begin{aligned}\overline{W}_a\,\overline{W}_{bt}-\overline{W}_b\,\overline{W}_{at}&=0\\ \overline{W}_b\,\overline{W}_{ct}-\overline{W}_c\,\overline{W}_{bt}&=0\\ \overline{W}_c\,\overline{W}_{at}-\overline{W}_a\,\overline{W}_{ct}&=0\end{aligned} \tag{13}$$

takes possibility to write

$$\nabla\times\overline{W}_k=\overline{\Phi}_k,\ \text{for}\ k\le 4\ \ \overline{\Phi}_k=0. \tag{14}$$

These equations result in equations for coefficients $\overline{W}_k$:

$$\Delta\overline{W}_k=\Phi_k^{(1)} \tag{15}$$

If k ≤ 3 we obtain Laplace equations for $\overline{W}_k$, if k ≥ 4 we have Poisson equations where $\Phi_k^{(1)}$ depend on solutions of low orders. Substitution of series (9) and (10) in equations for pressure

$$-(1/\rho)\Delta p=\left(\overline{W}_{tt}\cdot\overline{W}_a\right)_a+\left(\overline{W}_{tt}\cdot\overline{W}_b\right)_b+\left(\overline{W}_{tt}\cdot\overline{W}_c\right)_c+g\ \Delta Z \tag{16}$$

(where Δ is Laplace operator) gives:

$$\Delta p_k=\Phi_k^{(2)} \tag{17}$$

If k=0 or k=1 $W_k^{(2)}$=0 and $p_{0,1}$ satisfy Laplas equation, if k ≥ 3 – Poisson equation. Expansion of initial conditions p(a,b,0,t) = 0 on free surface results in

$$\sum_{k=0}^{\infty}p_k(a,b,c)t^k=0 \tag{18}$$

whence it follows that $p_k(a,b,0) = 0$ for all k.
Let us consider condition

$$\overline{\rho}(e,t)=\overline{W}(e,t)-\overline{r}_A(t) \tag{19}$$

determined relation between fluid particles motion on wetted surface of the body and its motion in fluid. Similarly previous let us present function setting progressive motion of the floating body in the form of the series

$$\overline{r}_A(t)=\sum_{k=0}^{\infty}\overline{r}_{Ak}\,t^k \tag{20}$$

and radius-vector determining position of particles of the wetted surface:

$$\rho(a,b,c,t)=\sum_{k=0}^{\infty}\rho_k(a,b,c)t^k \qquad (21)$$

Than condition (15) could be written in form

$$\sum_{k=0}^{\infty}\left[\overline{W}_k(a,b,c)-\overline{\rho}_k(a,b,c)-\overline{r}_{Ak}\right]t^k \qquad (22)$$

It follows that:

$$\overline{W}_k(a,b,c)-\overline{\rho}_k(a,b,c)-\overline{r}_{Ak}=0 \qquad (23)$$

If k = 0: $\overline{W}_0(a,b,c)=\overline{\rho}_0(a,b,c)$; $\overline{r}_{A0}=0$ and condition (19) is satisfied automatically.

If k = 1: $\overline{W}_1=\overline{\rho}_1=\overline{r}_{A1}=0$ and condition (19) is satisfied owing to initial conditions.

Projections of vector equation (23) are complex. They are expressed through variables that are trigonometric functions of angles between axes of coordinate system. Euler's angles defining rotary movement of a body are represented by series:

$$\theta=\sum_{k=0}^{\infty}\theta_k\,t^k\ ,\ \varphi=\sum_{k=0}^{\infty}\varphi_k\,t^k\ ,\ \psi=\sum_{k=0}^{\infty}\psi_k\,t^k\ , \qquad (24)$$

where θ, φ, ψ are constants. Projections of instantaneous angular velocities are expressed through these constants. Substitution of series (20) and (24) in equations of progressive and rotary motion result in system of ordinary equations system for series coefficients determination.

For principle vector and principle moment expansion consecutive solutions of boundary problems (17) for term of series determining pressure are used. Besides equality (23) and expansion of function σ given by expression:

$$\sigma=\frac{1}{F_\eta^2}\left(\begin{vmatrix}B_1 & C_1\\ B_2 & C_2\end{vmatrix}^2+\begin{vmatrix}C_1 & A_1\\ C_2 & A_2\end{vmatrix}^2+\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}^2\right)^{1/2} \qquad (25)$$

are used.

Term of zero order of this function equal:

$$\sigma_0[\rho_0,0]=\left(1/F_\eta\right)\left(F_\xi^2+F_\eta^2+F_\zeta^2\right)^{1/2} \qquad (26)$$

Calculation of initial terms of series gives basic data for determination of the subsequent terms. However difficulties of calculations thus quickly increase.

4.3 Problem based on action functional method

Algorithm of action functional [On-board, 2006] permits to use actual information about external excitations obtained during dynamic measurement. Advantages of the method are opportunities to take into account essential nonlinearity of considered problems (ship dynamics) and changing of parameters of the system. Action functional method is solution of inverse problem. Under the condition of capsizing we determine input process realization resulting system in this condition with maximum probability.

Ship dynamics in nonlinear rolling problem is described by the nonlinear stochastic differential equation:

$$\left(\dot{X}_t^\varepsilon\right)=b(X_t^\varepsilon)+\sigma(X_t^\varepsilon)\,\xi_t,\ X_0^\varepsilon= \\ x\in D\subset R^n, \qquad (27)$$

where ξ_t is stationary Gauss process. It is input of linear generating filter:

$$\left(\dot{\xi}_t\right)=F\xi_t+\varepsilon Gu_t,=R^r,\ \ \xi_0=\xi=R^r, \qquad (28)$$

Here input is “white noise” u_t

$$u_t=\left(\dot{w}_t\right) \qquad (29)$$

i.e. formal derivation of Wiener process w_t, ε>0 is perturbation.

Reduction of initial mathematical model is achieved due to use of actual information about incoming wave parameters and features of oscillation motion in these conditions.

Let us assume correctness of Lipschitz condition for functions b and σ in (27)

$$\|b(x) - b(y)\| + \| \sigma(x) - \sigma(y) \le k\|x - y\|,\quad x,y \in R^n. \tag{30}$$

Non-perturbed equation corresponding to (27)

$$\left(\dot{X}_t\right) = b(X_t),\ X_0 = x \in D, \tag{31}$$

is stable in D domain, i.e if $x \in D$ than trajectories (31) are attracted to single equilibrium $x^* \in D$. The question on stability of equation (27) consists in determination of probability of border achieving by its solutions with the same initial conditions for finite time. Exact solution of this problem is the solution of boundary problem for Fokker - Planck - Kolmogorov equation.

Approximate probability estimations can be obtained with the help of optimum control problem. This is Lagrange variational problem in Pontryagin form which is formulated as follows: for any T>0 it is necessary to choose $0<t_f \le T$ and control u_t from class $C_{0T}(R^m)$ (continuous functions on [0, T]) by such a way that conditions $X_0^\varepsilon= x$, $\xi_0=\xi$, $X_{tf}^\varepsilon \in \partial D$ are satisfied, and functional

$$S_{0tf} = \int_0^{tf} u_t^T u_t \, dt \tag{32}$$

is minimized on motions (let us mark them $\varphi_t=(\varphi_t)_1$ and $(\varphi_t)_2$) of system (9) – (11).

The main result consists in the following. Let us assume that boundary domain coincides with boundary of its closing. Then functional $\varepsilon^{-2}S_{0T}(\varphi)$ is action functional for X_t^ε processes family defined by equations (27)–(29) and

$$\lim \varepsilon^2 \ln P_x\{X_t^\varepsilon \in \partial D\} = -\min S_{0T}(\varphi);\quad \varphi \in C_{0T}(R^{r+n}),\ \varphi_0 = (x, \xi),\ (\varphi_t)_1 \in \partial D. \tag{33}$$

The last takes possibility to estimate probability P_x. It is probability of critical set ∂D reaching by process (33) outgoing from point x with the help of deterministic problem of optimal control for criterion (32). Solution of this problem in accordance with (33) gives dominant term of logarithmic asymptotic of pointed probability when $\varepsilon \to 0$.

In a considered problem of ship capsizing equation (27) describes one-dimensional motion (n=2) with coordinates $X_1=(X_t^\varepsilon)_1 = \theta$, $X_2=(X_t^\varepsilon)_2 = \dot{\theta}$ which correspond to angular and angular velocity of roll. This system is almost conservative (small damping). It permits to determine critical set D interesting us. From conditions of local maximum of potential energy $dU/d\theta=0$, $d^2U/d\theta^2<0$ we obtain value of critical angular θ^* and, hence, set ∂D which in this case represents hyperplane in space R^{r+n}

$$(X_t^\varepsilon)_1 - \theta^* = 0. \tag{34}$$

Condition of our assumption is satisfied, and (33) could be used for capsizing probability estimation.

In (27) we have:

$$B\left(X_t^\varepsilon\right) = \begin{bmatrix} b_1\left(X_t^\varepsilon\right) \\ b_2\left(X_t^\varepsilon\right) \end{bmatrix},\quad \left(X_t^\varepsilon\right) = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}; \tag{35}$$

$\sigma_1 = 0$ and $b_1 = \dot{\theta} = X_2$;

$$b_2 = -bD\left(K_1\theta + K_2\theta^3 + K_3\theta^5\right) + \dot{\theta}\nu_\theta\dot{\theta} + a_C;$$

$$\sigma_2 = -bD/\ C_1\theta + C_2\theta^3 + C_3\theta^5$$

Here b_2 is the sum of three terms: average component of restoring moment, damping moment and constant moment from gust.

Multiplication of $b^{-1}\sigma_2$ on noise $\xi_{1t} \equiv X_3$ in equation for $X_2 \equiv \dot{\theta}$ in system (35) is stochastic component of restoring moments in waves.

The model with the following parameters of second order (r=2) is assumed for waves

$$F = \begin{bmatrix} 0 & 1 \\ K_4 & K_5 \end{bmatrix}\quad G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}, \varepsilon = 1;$$
$$g_1 = g,\ g_2 = q((\omega^2 + E^2)^{1/2} - 2E),$$
$$K_4 = -2E,\ K_5 = -\omega^2 - E^2, \tag{36}$$

They are determined by values of dominate wave frequency and spectrum width E.

Deterministic Lagrange problem in first part of (35) has Hamiltonian

$$H = \psi_0 u_t^2 + \psi_1 X_2 + \psi_2(b_2(X_1) + \sigma_2(X_1)X_3 + \nu_\theta X_2 + a_C) + \psi_3(X_4 + g_1 u_t) + \psi_4(K_4 X_4 + K_3 X_3 + g_2 u_t) \quad (37)$$

It could be numerically integrated with the help of inverse substitution of time. The point is that on the right side of the problem there are less uncertain conditions. It is the only one for conjugate variable $X_5 \equiv \psi_1(t_f) = \psi_{1f}$, corresponding basic variable. From condition for Hamiltonian (zero condition) we have:

$$\dot{\theta}(t_f) \equiv X_2(t_f) = 0. \quad (38)$$

Value $\psi_{1f} = X_5$ is easily obtained. Boundary conditions for optimal conjugate variables are equal zero in accordance with transversality conditions, i.e.

$$X_6(t_f) \equiv \psi_{2f} = X_7(t_f) \equiv \psi_{3f} = X_8(t_f) \equiv \psi_{4f} = 0. \quad (39)$$

Let us obtain equations for conjugate variables by differentiation of (37) on variables (S_{OT}, θ, $\dot{\theta}$). Taking into account that conjugate variables are determined accurate within common factor, let us choose $\psi_0=1$. Further

$$\begin{aligned}
\dot{X}_5 &\equiv \dot{\psi}_1 = -\partial H / \partial X_1 = -b_{2\theta}(X_1)\psi_2 - \sigma_{2\theta}(X_1)X_3\psi_2; \\
\dot{X}_6 &\equiv \dot{\psi}_2 = -\partial H / \partial X_2 = \psi_1(t) - \nu_\theta \psi_2; \\
\dot{X}_7 &\equiv \dot{\psi}_3 = -\partial H / \partial X_3 = -\sigma_2(X_1)\psi_2 - K_5\psi_4; \\
\dot{X}_8 &\equiv \dot{\psi}_4 = -\partial H / \partial X_4 = -\psi_3(t) - K_4\psi_4(t);
\end{aligned} \quad (40)$$

where expression $u_4 = u_4^*$ for optimal control is took into account:

$$u_t^*(t) = -g_1\psi_3 - g_2\psi_4, \quad (41)$$

After t_f and u_t^* determination capsize probability is calculated by the following

$$P(t) = \exp\{-\varepsilon_1^{-2} S_{uf}\} = \exp\{\varepsilon_1^{-2} X_9(t)\}, \quad (42)$$

where

$$X_9(t) = (u_t^*)^2, \quad X_9(t_f) = 0. \quad (43)$$

Problem (27)-(29), (34), (38)-(40), (43) is integrated from right to left as Cauchy problem for 9 ordinary differential equations.

5. CONCLUSION

Analysis permitted to formulate the concept of development of complex SD-system as hierarchical system in multiprocessor computing environment. It is shown that hierarchy of considered system and corresponding models results in hierarchy of computer implementation. Modern IT way for this problem solution is problem solving environment.

Within the limits of considered concept system principles of information processing models development are proposed. Approach to choice of multilevel models of ship dynamics and practicability of problems solution on the basis of complex SD-systems is determined.

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