COMPUTATION OF HYDRODYNAMIC LOADS ON A SHIP MANOEUVRING IN REGULAR WAVES

S. Sutulo, Centre for Marine Technology and Engineering (CENTEC), Technical University of Lisbon, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001, Lisbon, Portugal, sutulo@mar.ist.utl.pt
C. Guedes Soares, Centre for Marine Technology and Engineering (CENTEC), Technical University of Lisbon, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001, Lisbon, Portugal, sutulo@mar.ist.utl.pt

ABSTRACT

The earlier developed algorithm for simulating manoeuvring motion in regular waves was augmented by computation of vertical and transverse shear forces and bending moments developing in the time domain. While the integrated forces and moments affecting motions of the ship as a rigid body were estimated relatively rigorously, using the auxiliary state variables method for representation of radiation forces in the time domain, the corresponding distributed loads are calculated in a simplified way exploiting the concept of a slowly varying encounter frequency. Simulations were carried out for straight runs, turning manoeuvres and zigzags for the S-175 container ship.

Keywords: *manoeuvring, simulation, regular waves, shear forces, bending moments*

1. INTRODUCTION

A ship's safety in a seaway is a multi-factor and multi-criterial issue. There can happen situations when the vessel is only partly incapacitated, that may happen because of high instantaneous inclinations and large local accelerations both impairing the crew's alertness and productivity and, possibly, functioning of the weapons systems. However, the ultimate danger to the ship's very existence is mainly associated with two events: (1) capsizing in roll, and (2) loss of global hull strength, most probable in longitudinal waves and due to longitudinal motions. Another undesirable phenomenon, green water shipping on deck does not present per se clear and present danger for any sea-going ship rigged appropriately for storm conditions but it can contribute to additional hull loads.

Information on the distributed dynamic loads acting upon the ship hull in waves is, as a rule, of the same importance as information on the ship roll and roll motion stability and it is sometimes even more important than the data on the heave and pitch amplitudes. That is why, most seakeeping codes provide estimates for the shear forces and bending moments computed simultaneously with the ship motions kinematics (Fonseca and Guedes Soares, 1998; Watanabe and Guedes Soares, 2000). Existing generalizations of seakeeping codes on the arbitrary base motion are mainly treated as codes for manoeuvring simulation in waves and do not provide information on distributed loads.

One of the first studies linking, in some sense, the ship bending moments to the manoeuvring motion was undertaken by

Guedes Soares (1990) who studied how voluntary heading changes could influence extreme expected magnitudes of bending moments on the rough sea. However. manoeuvring was considered there in the navigation sense i.e. a ship was supposed to always follow some straight path.

In some sense, such an approach is justified as the most heavy loads are expected to occur during a long straight run with the least favourable parameters as manoeuvring itself apparently does not increase longitudinal stresses. At the same time, importance of a capability to assess hydrodynamic loads in arbitrary curvilinear motion seems obvious from the viewpoint, for instance, of accumulating more realistic statistical data.

An attempt to combine computation of distributed loads on the ship hull with the true manoeuvring simulation is undertaken in the present study.

Mostly, these will be loads related to the wave excitation although the transversal loads include also the still-water manoeuvring contribution. Obviously, this contribution will not be significant for most vessels although once it happened to be critical to airships which suffered some cases of loss of global transverse strength during tight turns in the horizontal plane. Exactly that phenomenon stimulated development of a semi-empiric strip method for predicting transverse loads, shear forces and bending moments on airship hulls in curvilinear motion widely known nowadays as the Munk method (Munk, 1924).

The present approach is based on the unified mathematical model for ship manoeuvring in waves developed earlier by the authors. Although a brief outline of the method itself is presented below, its more detailed exposure can be found in (Sutulo and Guedes Soares, 2006a,b, 2008).

When the global ship strength is associated with the ship hull's integrity, are usually discussed vertical (approximately) loads acting in the centerplane and treated as the most important, torsion loads which can be important for cargo vessels with large deck openings, and transverse loads, less critical for the majority of ships.

A historically conditioned separation of static (or still water) and dynamic vertical loads is not very important for most mathematical models as the former is just a specific case of the latter. More meaningful is distinction between the "normal" hydrodynamic loads observed in absence of the ship slamming and green-water shipping and "additional loads" accompanying the mentioned phenomena. Also, hydrodynamic loads can be estimated with account for elastic effects which can be caused by springing vibrations of the ship hull excited by "normal" loads and also — by whipping from slamming shock loads. If elastic effects are thought to influence significantly the rigid-body motions, the primary formulation must be hydroelastic (Wu and Moan, 1996). Often, however, a simplified approach is sufficient. Namely, the loads are primarily determined on the rigid hull serving as input for vibrations analysis based on the modal shapes approach. The vibrations' eigen-frequencies and mode shapes are then pre-calculated using the finite-element method and the hull's approximation with a Vlasov or Timoshenko beam. The vibrations analysis results in estimates of equivalent quasi-steady loads which can exceed the corresponding "rigid hull" values by up to 20–30 percent.

In the present study, however, which is considered as one of the first steps to generalization of the loads estimation methods for the case of an arbitrary curvilinear base motion, only "normal" loads on the rigid hull will be dealt with.

2. PROBLEM STATEMENT AND MATHEMATICAL MODEL

2.1 Frames of Reference and Main Definitions

It is assumed that a common monohull surface displacement ship is performing and arbitrary curvilinear motion on the fluid surface in presence of regular monochromatic two-dimensional surface waves of small steepness.

The primary Cartesian frame of reference $O\xi\eta\zeta$ is Earth-fixed with its origin lying on the undisturbed free surface, with the ζ-axis pointing vertically downwards, the ξ-axis lying in the ship's centerplane at the initial time moment $t=0$, and with the η-axis directed initially to the starboard. The ship is treated as a rigid body with its shape described in the body-fixed axes $Cxyz$, where the origin C is the intersection of the centerplane, midship plane and of the initial equilibrium waterplane. At $t=0$, the body axes are supposed to coincide completely with the Earth-fixed axes while deviating from them in the further development.

As the rigid-body formulation is, obviously, 6 degrees of freedom, the instantaneous position of the ship with respect to the Earth-fixed frame is described by usual six generalized co-ordinates: advance/surge ξ_C, transfer/sway η_C, sinkage/heave ζ_C, roll angle φ, pitch angle θ and heading/yaw angle ψ. The generalized co-ordinates form a 6-dimensional arithmetic vector $\mathbf{O}$ represented usually as a column matrix. projections of the ship's instantaneous velocity vector $\mathbf{V}$ in the body axes are the linear velocities of surge u, sway v, and heave w, and the angular velocities of roll p, pitch q, and yaw r. All these velocities form a 6-vector $\mathbf{U}$.

The body axes are used to write down the basic dynamic equations of motion and for final representation of hull loads. But most of hydrodynamic forces and loads will be determined primarily in the auxiliary semi-fixed frame of reference $C_1\xi_1\eta_1\zeta_1$ which only differs from the body-fixed frame $Cxyz$ by not being involved into motions of heave, pitch and roll.

2.2 Brief Description of Mathematical Models for Ship Dynamics

The equations of motion of the ship can be written in the following general form:

$$\begin{aligned} &\mathbf{M}\dot{\mathbf{U}} + \mathbf{Ц}(\mathbf{M}_0, \mathbf{U}) = \mathbf{Ш}(\mathbf{O}, \mathbf{U}, a_w, \omega, \delta_R, n, t), \\ &\dot{\mathbf{O}} = \mathbf{T}(\mathbf{O})\mathbf{U}, \end{aligned} \qquad (1)$$

where $\mathbf{M}$ is the inertial matrix including the ship mass m, ship moments of inertia $I_{xx}, I_{yy}, I_{zz}, I_{xz}$ and added mass coefficients corresponding to the infinite frequency, $\mathbf{Ц}$ is the vector function describing centripetal effects, $\mathbf{M}_0$ is the proper inertial matrix with the added-mass terms excluded, $\mathbf{Ш}$ is the vector of total hydrodynamic forces and moments, a_w is the wave amplitude, ω—the wave frequency, δ_R the current rudder angle, n is the propeller rotation frequency; $\mathbf{T}$ is the matrix comprising sine and cosine functions of the Euler angles and linking time derivatives of the generalized co-ordinates with the velocities in the body axes. Explicit forms of the introduced matrices and vector functions are in fact established in general mechanics but can also be restored from the co-ordinate form of the same equations given in (Sutulo and Guedes Soares, 2006a,b).

The thus outlined ship mathematical model is somewhat simplistic as in reality it includes also models for the steering gear and the main engine but this is less relevant to the topic of the present study.

The model devised by Sutulo and Guedes Soares (2006a,b, 2008) presumes that each of the force/moment components is described in the time domain and consists of the following contributions:

manoeuvring forces estimated according to any method recognized in the still-water manoeuvring: the classic semi-empiric method developed by Inoue et al. (1981);

hydrostatic and Froude–Krylov forces; as the algorithm is nonlinear in this respect, these two parts cannot be separated;

diffraction excitation forces;

radiation or inertial-and-damping hydrodynamic forces.

Computation of the last two categories of forces presumes the linear free-surface condition and provisions for their estimation at an actual instantaneous position of the ship were made. However, at present, due to some technical difficulties they are computed at the equilibrium position of the hull i.e. the code is fully linear in this respect. At the same time, the zero-frequency contribution related to the Munk forces is also estimated and subtracted from the radiation forces as it corresponds to the still-water manoeuvring forces estimated independently, see (Sutulo and Guedes Soares, 2008) for details.

The mentioned radiation forces in the time domain are computed with the help of an extra set of ordinary differential equations for auxiliary state variables (Sutulo and Guedes Soares, 2007). This sets resulted from application of the inverse Fourier transform to the integrated forces in the frequency domain which, in their turn, are obtained through the integration of the sectional loads over the hull's length. As result, the time-domain representations of the radiation hull forces used in this mathematical model do no longer contain any information about distributed loads. In general, this is not *conditio sine qua non* for application of the auxiliary state variables method but any alternative algorithm keeping this information would have require a much greater number of required auxiliary state variables and corresponding differential equations. That is why, a less rigorous but more efficient approximate method was chosen here for estimating distributed radiation loads in the time domain.

2.3 Brief Description of Mathematical Models for Ship Dynamics

The transverse loads must be finally considered in the local axes parallel to the body axes y and z. Using traditional notations: $v_2(x)$ for the transverse load and $v_3(x)$ for the "vertical" load it is possible to assemble them into the arithmetic vector $\left|v\right\rangle = (v_2, v_3)$. Then, the d'Alembert principle can be applied to each section resulting in

$$\left|v\right\rangle = \left|v\right\rangle_{\text{iner}} + \left|v\right\rangle_{\text{act}}, \tag{2}$$

where $\left|v\right\rangle_{\text{iner}}$ is the inertial load and $\left|v\right\rangle_{\text{act}}$ is the active load.

The inertial load can be represented as

$$\left|v\right\rangle_{\text{iner}} = \left|-m(x)\dot{v}(x), -m(x)\dot{w}(x)\right\rangle, \tag{3}$$

where $m(x)$ is the ship's mass distribution and the local accelerations are defined as (Lourie, 2002):

$$\begin{aligned} \dot{v}(x) &= \dot{v} + ur - wp + \dot{r}x + pqx, \\ \dot{w}(x) &= \dot{w} - uq + wp - \dot{q}x + prx. \end{aligned} \tag{4}$$

However, the local accelerations can be computed in this way if only the integrated active forces on the ship hull are formed strictly from calculated sectional loads as otherwise the forces will not be balanced and the shear forces and bending moments will not be estimated correctly. This is exactly what happens in the present formulation and while the vertical loads could have been treated in a more consistent way, albeit at the expense of increased complexity of the code, this would be much more problematic with the horizontal-plane loads as the most appropriate empiric models do not contain information about longitudinal loads distribution. A similar situation happens in purely seakeeping codes when the loads are estimated with account for slamming and/or green water while the ship motions are supposed to be not affected with these factors. In such cases, the parameters $\dot{u}(x)$ and $\dot{v}(x)$ in eq. (3) must be substituted with the effective local accelerations a_y and a_z restored using the d'Alembert principle applied to the whole hull. Namely,

$$\begin{aligned} a_y(x) &= a_y + (x - x_G)\, a_r, \\ a_z(x) &= a_z - (x - x_G)\, a_q, \end{aligned} \tag{5}$$

where a_y, a_z are the effective accelerations of the centre of mass of the ship and a_r, a_q are the effective angular accelerations, all defined as

$$\begin{aligned} a_{y,z} &= \frac{1}{m}\int_L v_{2,3\text{act}}(x)\,\mathrm{d}\,x, \\ a_r &= \frac{1}{I_{zz}}\int_L v_{2\text{act}}(x)(x - x_G)\,\mathrm{d}\,x, \quad \ldots ..(6) \\ a_q &= -\frac{1}{I_{yy}}\int_L v_{3\text{act}}(x)(x - x_G)\,\mathrm{d}\,x. \end{aligned}$$

The active loads are primarily estimated in the semi-fixed axes where the projections will be $\gamma_2 \equiv \gamma_{\eta_1}(\xi_1)$ and $\gamma_3 \equiv \gamma_{\zeta_1}(\xi_1)$ where it will be assumed that $\xi_1 \equiv x$ which is accurate enough for any realistic pitch angles observed on surface ships. The active loads in the body axes will be:

$$\begin{aligned} v_{2\text{act}} &= \gamma_2 \cos\varphi + \gamma_3 \cos\theta \sin\varphi, \\ v_{3\text{act}} &= -\gamma_2 \sin\varphi + \gamma_3 \cos\theta \cos\varphi. \end{aligned} \tag{7}$$

Also, the active loads can be decomposed like this:

$$\left|v\right\rangle_{\text{act}} = \left|v\right\rangle_{\text{RAD}} + \left|v\right\rangle_{\text{FK+HS}} + \left|v\right\rangle_{\text{DIF}} + \left|v\right\rangle_{g}, \tag{8}$$

where the subscripts stand, left to right, for: radiation, Froude–Krylov–hydrostatic, diffraction, and gravitational components.

The gravitational component is the easiest to define:

$$\gamma_{g2} = 0; \quad \gamma_{g3} = m(x)g. \tag{9}$$

If the oncoming waves potential φ_w and the corresponding free surface elevation ζ_w are written as

$$\begin{aligned} \phi_w &= \mathrm{Re}\left[\frac{iga_w}{\omega} e^{-k\zeta - i(k_1\xi_1 + k_2\eta_1)} e^{i\omega t}\right], \\ \zeta_w &= \mathrm{Re}\left[a_w e^{-k\zeta - i(k_1\xi_1 + k_2\eta_1)} e^{i\omega t}\right], \end{aligned} \tag{10}$$

where k_1, k_2 are projections of the wave number vector and $k = \sqrt{k_1^2 + k_2^2}$, The Froude–Krylov and hydrostatic loads will be expressed as

$$\begin{aligned} \gamma_{\text{FK+HS}\,j} =& -\rho g \int_{C(\xi)} \zeta n_j \,\mathrm{d}\, C(\xi) - \rho g a_w e^{i[\omega t + \Phi(t)]} \\ & \times \int_{C(\xi)} e^{-k\zeta - ik[\xi\cos(\chi_w - \psi) + \eta\sin(\chi_w - \psi)]} n_j \mathrm{d}C(\xi), \end{aligned} \tag{11}$$

where $j = 2,3$; C is the contour of the hull section, $\Phi = -k(\xi_C \cos\chi_w + \eta_C \sin\chi_w)$ is the total wave phase and χ_w is the waves propagation angle counted from the axis $O\xi$.

The diffraction loads are :

$$\gamma_{DIF\,2,3}(x) = \mathrm{Re}\left[f_{d\,2,3}(x) e^{i\omega_e t}\right], \tag{12}$$

where $\omega_e = \omega - uk_1 - vk_2$ is the encounter frequency, and

$$f_{d\,j} = -\rho\omega a_w e^{i\Phi} e^{-ik\xi\cos(\chi_w-\psi)} \int_{C(x)} [n_2 \sin(\chi_w - \psi) - in_3] e^{-k\zeta - ik\eta\sin(\chi_w-\psi)} \tilde{\varphi}_j \,\mathrm{d}C(x), \tag{13}$$

where $\tilde{\varphi}_j(\eta_1, \zeta_1)$ are the two-dimensional radiation functions calculated at the absolute wave frequency.

The complex amplitudes of the radiation loads in the frequency domain can be represented as (Sutulo and Guedes Soares, 2006a):

$$\begin{aligned} \tilde{f}_j(x) &= -i\omega \sum_{\ell=2}^{4} \tilde{m}_{j\ell}(\omega, x)\tilde{v}_\ell(x) \\ &+ u \sum_{\ell=2}^{4} \lambda_{j\ell} \tilde{m}'_{j\ell}(\omega, x)\tilde{v}_\ell(x) \\ &+ u \sum_{\ell=2}^{4} \tilde{m}_{j\ell}(\omega, x)\tilde{v}'_\ell(x), \end{aligned} \tag{14}$$

where $\tilde{m}_{j\ell}$ are the frequency-dependent sectional added masses; $\lambda_{j\ell}$ are the load-reduction functions implicitly accounting for viscosity; the prime means derivative with respect to x, and $\tilde{v}_\ell$ are complex amplitudes of the sectional horizontal and vertical velocities.

Assuming that the ship is always oscillating with the encounter frequency and applying usual formal transformations the following approximate formulae for the time-domain radiation loads can be obtained:

$$\begin{aligned} \gamma_j(x,t) = &-\sum_{\ell=2}^{4} \Big[\mu_{j\ell}(\omega_e, x)\dot{u}_\ell(x,t) \\ &+ \nu_{j\ell}(\omega_e, x) u_\ell(x,t) \Big] \\ &+ u \sum_{\ell=2}^{4} \Big[\lambda_{j\ell}\mu'_{j\ell}(\omega_e, x) u_\ell(x,t) \\ &- \frac{\lambda_{j\ell}}{\omega_e^2} \nu'_{j\ell}(\omega_e, x)\dot{u}_\ell(x,t) \Big] \\ &+ u \sum_{\ell=2}^{4} \Big[\mu_{j\ell}(\omega_e, x) u'_\ell(t) \\ &- \frac{1}{\omega_e^2} \nu_{j\ell}(\omega_e, x)\dot{u}'_\ell(t) \Big], \end{aligned} \tag{15}$$

where $\mu_{j\ell}$ and $\nu_{j\ell}$ are respectively the traditional frequency-dependent added mass and damping coefficient and the local velocities and accelerations are:

$$\begin{aligned} &u_2 = v^1 + xr^1; \quad u_3 = w^1 - xq^1; \quad u_4 = p^1; \\ &u'_2 = r^1; \qquad u'_3 = -q^1; \qquad u'_4 = 0; \\ &\dot{u}_2 = \dot{v}^1 + x\dot{r}^1; \quad \dot{u}_3 = \dot{w}^1 - x\dot{q}^1; \quad u_4 = \dot{p}^1; \\ &u'_2 = \dot{r}^1; \qquad u'_3 = -\dot{q}^1; \qquad u'_4 = 0; \end{aligned} \tag{16}$$

where the velocities with the superscript "1" are similar to corresponding parameters without the subscript but transformed to the semi-fixed axes.

Presence of the squared encounter velocity in the denominator in eq. (15) is a common result of artificial manipulation with frequency-domain expressions. It does not bring any harm as the damping coefficients also vanish accordingly as the frequency goes to zero. However, in purely seakeeping formulations this denominator could be avoided if quasi-coordinates π_ℓ such as $\dot{\pi}_\ell = u_\ell$ were introduced. Though, this is not applicable here as the velocities can be aperiodic and the corresponding quasi-coordinates can go to infinity resulting in unrealistic estimates of the loads.

When the distributed total loads are computed, the shear forces V_j, $j = 2,3$ and the bending moments M_j are calculated as usual:

$$V_j(x,t) = \int_x^{\text{bow}} v_j(\xi,t)\,\mathrm{d}\xi;$$
$$M_j(x,t) = \int_x^{\text{bow}} V_j(\xi,t)\,\mathrm{d}\xi. \tag{17}$$

Here, the first integration must be performed in a special way for the diffraction loads as the load defined by eq. (13) resulted from the Tuck transformation (Salvesen et al., 1970), and is not quite the real load density.

3. NUMERICAL EXAMPLES

The algorithm described in the previous section was used for augmenting the combined manoeuvring-and-seakeeping code developed earlier by Sutulo and Guedes Soares (2006a,b). Some computations have been performed for the well-known benchmark vessel S-175 described in detail in many sources including (Watanabe and Guedes Soares, 2000). The ship has the length 175m between perpendiculars and its initial speed was always 16.7knots which corresponds to the Froude number 0.2. Some simulations were performed in still water but mostly in regular waves with the length 175m and steepness 1/40. These were always heading waves, at least in the approach phase of the manoeuvre which always lasted 37.5s.

Time histories for the shear forces and bending moments in straight run are shown on Figures 1 through 4. Always are shown the midship values but on stretched graphs also are presented minimum and maximum values along the ship’s hull. Detailed verification of the numerical results still was not the main objective of the present study but the amplitudes of the loads fairly well correspond to those obtained previously for the same vessel in similar conditions (Watanabe and Guedes Soares, 2000). The midship vertical bending moment is positive in hogging.

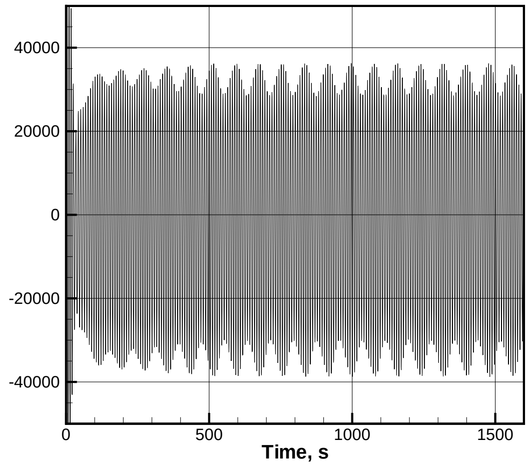

Figure 1. Straight run in heading waves: time history for the midship shear force.

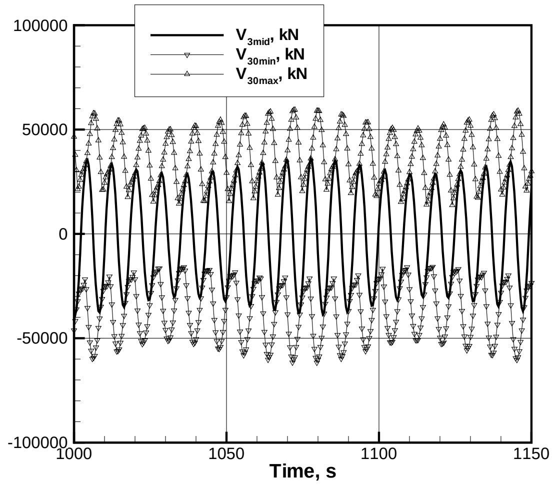

Figure 2. Straight run in heading waves, zoomed: time histories for the midship, minimum and maximum, shear forces.

Distributions of the vertical load and shear force and of the bending moment, as functions of the longitudinal body co-ordinate non-dimensionalized by the ship’s half-length and corresponding to the arbitrarily chosen time moment $t = 100\text{s}$, are shown on Fig. 5 and 6.

All further plots correspond to the manoeuvring motion. Trajectory of the ship in still water turn corresponding to the 35deg rudder starboard is shown on Figure 7. The same manoeuvre executed in waves resulted in the trajectory presented on Figure 8. Figures 9 through 12 show time histories of the horizontal shear forces and bending moments during the turning motion in waves. As expected, the absolute values in the horizontal plane are definitely smaller than in the vertical one.

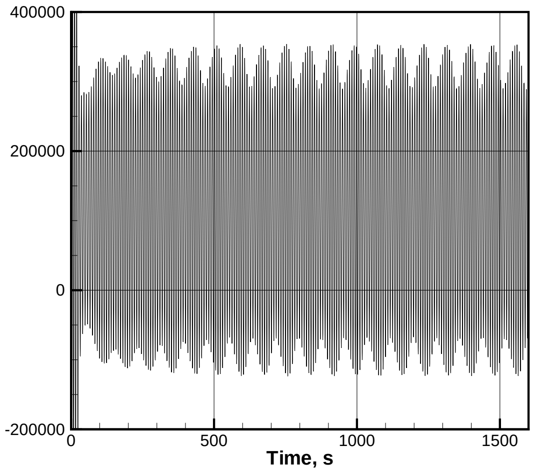

Figure 3. Straight run in heading waves: time history for the midship bending moment.

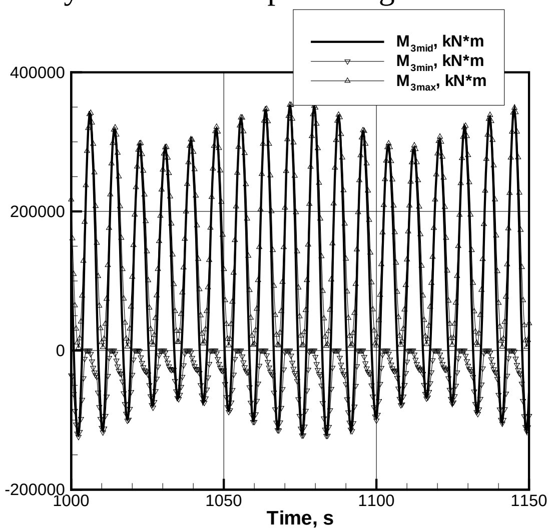

Figure 4. Straight run in heading waves, zoomed: time histories for the midship, minimum and maximum bending moments.

A time history for the vertical bending moment is shown on Figure 13. As expected, the pattern of the time history looks more complicated mainly due to the continuous alteration of the relative wave direction but the extreme values do not exceed those obtained in the straight path run although, in general, this is less evident for another wave parameters and, probably, more comparative simulations are required.

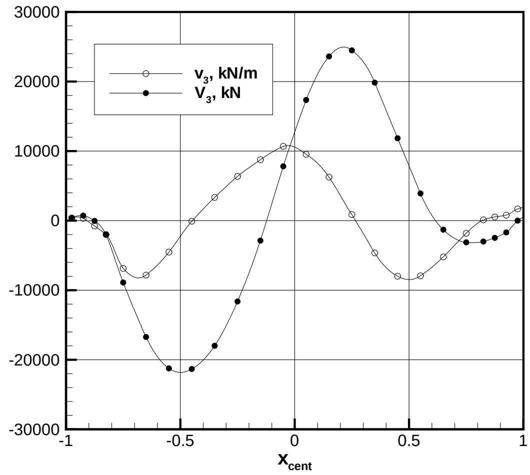

Figure 5. Snapshot of the vertical load density and shear force distribution.

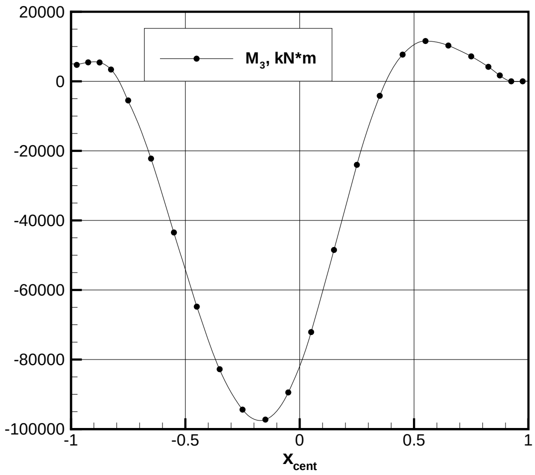

Figure 6. Snapshot of the vertical bending moment distribution.

Finally, the Figures 14 and 15 present time histories of the horizontal and vertical bending moments in the standard 20°–20° zigzag manoeuvre executed with the starboard initial helm and with the initial course heading the waves.

4. CONCLUSION

The earlier developed manoeuvring code possessing an option of the 6DOF motion in

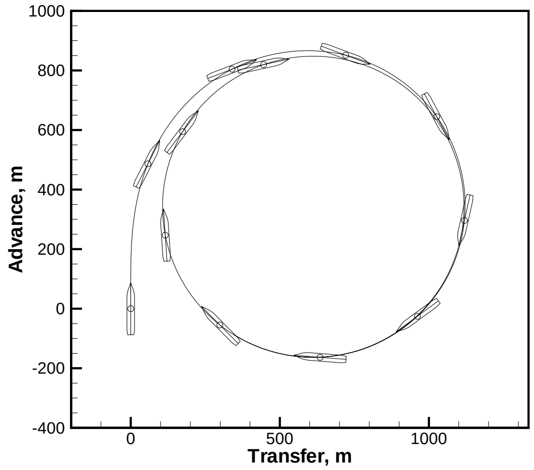

Figure 7. Trajectory in the 35deg helm turning manoeuvre in still water.

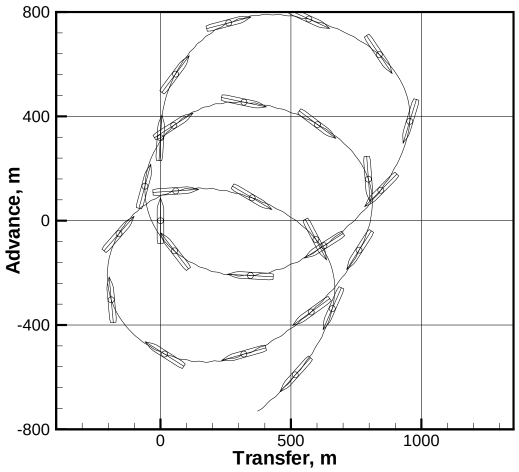

Figure 8. Trajectory in the 35deg helm turning manoeuvre in waves.

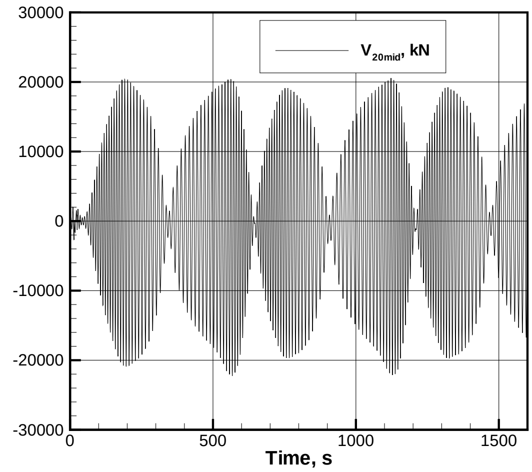

Figure 9. Turning in waves: time history for the midship horizontal shear force.

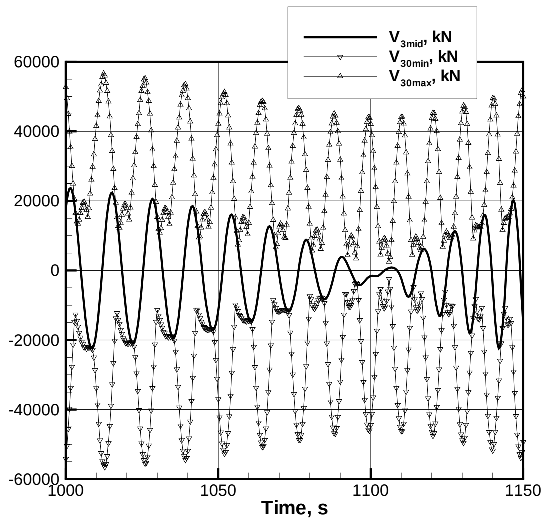

Figure 10. Turning in waves: time histories for the horizontal midship, minimum and maximum, shear forces (zoomed).

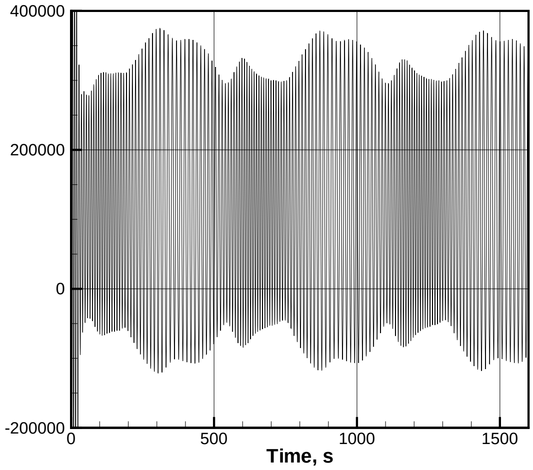

Figure 11. Turning in waves: time history for the horizontal midship bending moment.

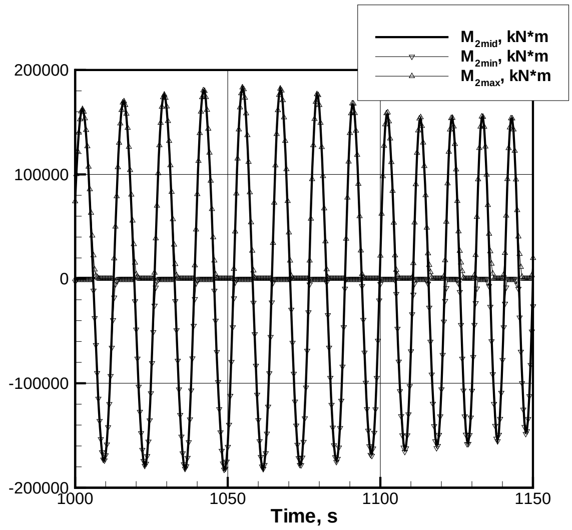

Figure 12. Turning in waves: time histories for the horizontal midship, minimum and maximum, bending moment (zoomed).

regular waves has been enriched with the option for approximate computation of vertical and horizontal loads on the ship's hull including the shear forces and bending momemts. The numerical results obtained so far in simulations carried out with the benchmark container ship looked reasonable. At the same time, it was noticed that the problem of accurate estimation of horizontal loads in general manoeuvring motion contains a major difficulty stemming from extreme scarcity of data on the manoeuvring forces' longitudinal distributions corresponding to typical empiric mathematical models preferably used in ship manoeuvring. Further development of the code in this direction should thus depend on the evaluation of importance of the output of this nature.

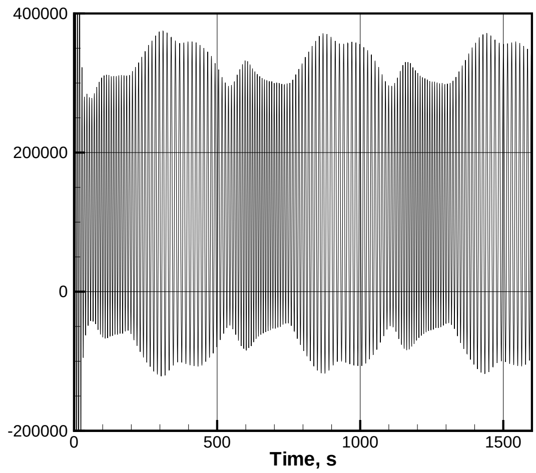

Figure 13. Turning in waves: time history for the vertical midship bending moment.

5. ACKNOWLEDGMENTS

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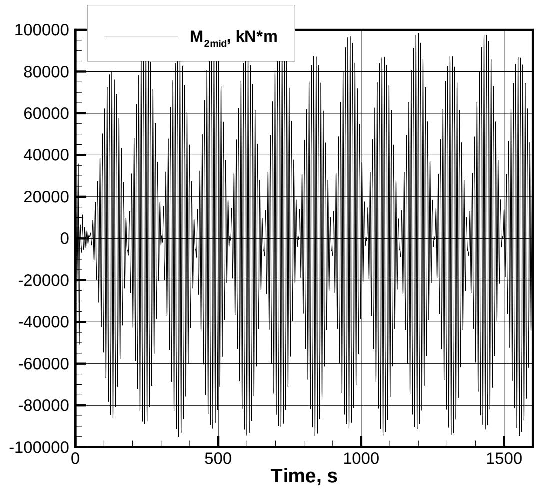

Figure 14. Zigzag manoeuvre in waves: time history for the horizontal midship bending moment.

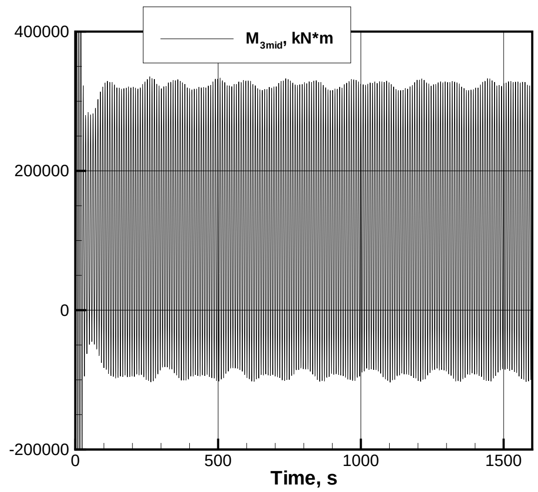

Figure 15. Zigzag manoeuvre in waves: time history for the vertical midship bending moment.

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