NUMERICAL AND EXPERIMENTAL INVESTIGATION ON THE PARAMETRIC ROLLING OF A TRIMARAN SHIP IN LONGITUDINAL REGULAR WAVES

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ABSTRACT

This paper presents a combined numerical experimental investigation on the parametrically excited rolling motion in regular longitudinal waves for a trimaran hull for wave lengths shorter than the ship length. The tested hull form shows significant rolling amplitudes even in case of short and/or steep waves. The obtained experimental results are compared with the predictions given by a nonlinear 1-DOF mathematical model with a generally good agreement. The linear stability is investigated numerically by a direct application of the Floquet theory, i.e. by a direct construction of the monodromy. The used method can be applied also to cases with more DOFs. The time domain analysis of the nonlinear model shows a series of interesting features connected to the strong nonlinear behaviour of the righting arm, in particular the presence of multiple coexisting solutions of different types. Parametric roll is analysed also in resonance regions different from the first one. A risk index is introduced in order to rationally deal with the presence of multiple coexisting solutions by combining a severity measure and a distribution of initial conditions. Some examples of application are reported.

Keywords: *Parametric roll ; Floquet theory ; Multiple solutions ; Sub-harmonic resonance ; Ship safety ; Large amplitude motions*

1. INTRODUCTION

Multihull ships are often selected for their superior performances in terms of ratio between deck area and displacement. Moreover they allow for a favourable reduction in resistance with respect to competing similar monohull designs. From the stability point of view lateral outriggers have the positive effect of increasing the vertical position of the transversal metacentre. However, in order to reduce the overall resistance of the ship, lateral outriggers are sometimes designed with a shallow draught: the indirect effect of this choice is an early-emergence of the outriggers during the heeling of the ship, with a consequent strong sudden variation of the slope of the restoring lever at relatively small heeling angles. The consequences of such a strongly nonlinear restoring have already been studied in the beam sea case (Bulian & Francescutto, 2007, 2009) showing the possibility of a significant bending of the roll response curve with associated multiple steady states.

As a natural prosecution of the previous research, the interest of this paper concentrates on the case of longitudinal regular waves with the consequent possible inception of parametrically excited rolling motion also in view of the less attention up to now given to the case of multi-hulls. Some examples of investigations concerning the magnitude and/or effect of variations of restoring in waves for multihulls ships can be found in (Bulian et al., 2004; Smith, 1982). Gee et al. (2009)

investigated the possible inception of parametric roll in the case of a pentamaran design, but the phenomenon did not occur. Starting from the lack of experience on parametric roll for multihulls, this research has concentrated on the case of a trimaran.

In literature the attention has been given, more often, to wave lengths close to the ship length, or longer. However parametric roll can occur also in case of shorter waves if the appropriate conditions are met (see, e.g., (Brunswig et al., 2006)). As it is well known, the length of waves at sea is physically limited (at least waves with not negligible steepness) while ship dimensions are growing, with a tendency towards longer and longer ships. For very long ships, therefore, the possibility of parametric roll due to waves having length close to the ship length could become quite small. On the other hand, the possibility of parametric roll due to wave lengths shorter than the ship length could become a serious threat.

In view of the above, the model of a trimaran ship has been tested at the towing tank of the University of Trieste in longitudinal regular waves of different lengths and amplitudes (Sinibaldi, 2008) and the experimentally observed behaviour has been the subject for a subsequent theoretical/numerical investigation (Fucile, 2008). In addition to the 1:1 ratio between ship length L and wave length λ_w in this research also the case of shorter waves, covering the range $\lambda_w / L \in [0.5,1]$ has been investigated: the parametrically excited rolling motion was observed to be significant.

A numerical investigation has also been performed using a 1-DOF model. The use of a 1-DOF modelling for the simplified description of the parametrically excited rolling motion in longitudinal waves can be considered nowadays a well established simplified procedure. A simplified modelling based on an uncoupled roll equation has pros (simplicity and relatively good accuracy (Bulian, 2006a; Francescutto, 2001; Hashimoto and Umeda, 2004; Spyrou, 2000)) and cons (lack of proper description of the dynamic coupling of roll with heave and pitch (Neves & Rodriguez., 2007; Neves et al., 2008; Oh et al., 2000)). The numerical investigation reported in this paper has been performed in three stages:

1) The instability regions have been analysed by a direct numerical application of the Floquet theory (Hayashi, 1964; José & Saletan, 1998; Nayfeh & Mook, 1979; Simakhina, 2003; Simakhina & Tier, 2005; Turhan & Bulut, 2005) to the linearised mathematical model, retaining hence all the features of the Hill equation obtained by the linearization of the nonlinear 1-DOF model.
2) Time domain numerical simulations have been carried out using the fully nonlinear 1-DOF mathematical model. The obtained roll response curves have been compared with experimental results with a generally good agreement, and a series of interesting features have been highlighted.
3) Finally, a risk-measure is applied in this paper as a rational tool for dealing with the case of multiple coexisting steady states (or in general to deal with a significant dependence of ship motions on initial condition) in a deterministic environment.

2. NUMERICAL MODELLING

2.1 Description of the numerical model

The employed numerical model is a 1-DOF model for regular longitudinal waves, where roll restoring in waves is calculated using a quasi-static approach for heave and pitch. Smith effect is not considered and a purely hydrostatic pressure under the non-flat sea surface is assumed for the computation of the roll restoring moment in waves. This approximation is usually acceptable in case of long waves, where the ship's draught is small in comparison with the wave length. In case of shorter waves this approximation could be criticised and hence needs further attention. The differential equation describing the roll motion in longitudinal waves is taken as:

$$\ddot{\phi}+d\left(\dot{\phi},V\right)+ \\ +\frac{\Delta}{J'_{xx}}\cdot\overline{GZ}\left(\phi,V,x_c\left(t\right),\lambda_w,a_w\right)=0 \tag{1}$$

where: ϕ [rad] is the roll angle, V [m/s] is the ship speed, $d\left(\dot{\phi},V\right)$ [rad/s^2] is the damping function (explicitly considering a dependence on forward speed), Δ [N] is the ship displacement (assumed constant according to the quasi static assumption for heave), J'_{xx} [kg*m^2] is the roll moment of inertia comprising the effect of added inertia, $\overline{GZ}\left(\phi,V,x_c\left(t\right),\lambda_w,a_w\right)$ [m] is the roll righting lever (depending on: instantaneous inclination ϕ, ship speed V, instantaneous wave crest position $x_c\left(t\right)$ [m] of the wave along the ship, wave length λ_w [m], wave amplitude a_w [m]).

In (1) a speed dependence is considered in the damping function and this is a quite standard procedure that takes into account the fact that roll damping at forward speed is different from zero speed roll damping. Although forward speed in principle also affects the hydrodynamic roll added inertia, in (1) this dependence has been neglected (note that the quasi static assumption allows to us a constant displacement). A forward speed correction has, on the other hand, been considered in the restoring term. The evidence of possible significant forward speed effects on the restoring of a trimaran hull can be found in the recent numerical investigation of Walree and Jong (2008). In the past Bulian (2006a) considered forward speed effects on restoring as a variation of the roll natural frequency. However this latter approach is partially inappropriate, since the variation of frequency is an indirect effect of the lift generated by the loss of port-starboard symmetry when the ship heels. The situation is similar to the case of a not symmetric airfoil at zero angle of attack. Such phenomenon shall not be confused with lift effects associated to the roll angular velocity.

According to the above, forward speed effects in the restoring term in (1) have been incorporated in an approximate way as follows. Firstly we have separated the righting lever in a zero speed term and a forward speed contribution $\delta\overline{GZ}\left(\phi,V\right)$ depending on the heeling angle and on the ship forward speed, namely:

$$\overline{GZ}\left(\phi,V,x_c\left(t\right),\lambda_w,a_w\right)= \\ =\overline{GZ}_{zs}\left(\phi,x_c\left(t\right),\lambda_w,a_w\right)+\delta\overline{GZ}\left(\phi,V\right) \tag{2}$$

then the speed dependent part of the righting lever was represented by a metacentric approximation:

$$\delta\overline{GZ}\left(\phi,V\right)\approx\delta\overline{GM}\left(V\right)\cdot\sin\left(\phi\right) \tag{3}$$

where the variation of metacentric height $\delta\overline{GM}\left(V\right)$ is the speed dependent part of $\delta\overline{GZ}\left(\phi,V\right)$. The effect of the wave on $\delta\overline{GZ}\left(\phi,V\right)$ has been neglected at this stage. In order to determine $\delta\overline{GM}\left(V\right)$ we started from the measured roll natural frequency at forward speed $\omega_0\left(V\right)$ and we have considered the approximation that the variation of the roll natural frequency is solely due to a variation in the righting moment, i.e. the variation $\delta\overline{GM}\left(V\right)$:

$$\delta\overline{GM}\left(V\right)=\overline{GM}_{zs}\cdot\left(\frac{\omega_0^2\left(V\right)}{\omega_0^2\left(V=0\right)}-1\right) \tag{4}$$

where $\overline{GM}_{zs}$ is the metacentric height at zero speed in calm water.

Concerning the model for roll damping, we consider a quite general linear+quadratic+cubic model. Accordingly, the final model takes the following form:

$$\ddot{\phi}+2\cdot\mu(V)\cdot\dot{\phi}+\beta(V)\cdot\dot{\phi}\left|\dot{\phi}\right|+\delta(V)\cdot\dot{\phi}^3+ \\ +\frac{\omega_{0,zs}^2}{\overline{GM}_{zs}}\cdot\begin{bmatrix}\overline{GZ}_{zs}\left(\phi,x_c(t),\lambda_w,a_w\right)+\\+\delta\overline{GM}(V)\cdot\sin(\phi)\end{bmatrix}=0 \tag{5}$$

where $\omega_{0,zs}$ is intended to be the roll natural frequency at zero speed in calm water. For the linear stability analysis of (5) a linearization close to $\phi=0$ is necessary:

$$\ddot{\phi}+2\cdot\mu(V)\cdot\dot{\phi}+ \\ +\frac{\omega_{0,zs}^2}{\overline{GM}_{zs}}\cdot\begin{bmatrix}\overline{GM}_{zs}\left(x_c(t),\lambda_w,a_w\right)+\\+\delta\overline{GM}(V)\end{bmatrix}\cdot\phi=0$$

where (6)

$$\overline{GM}_{zs}\left(x_c(t),\lambda_w,a_w\right)= \\ =\left.\frac{\partial\overline{GZ}_{zs}\left(\phi,x_c(t),\lambda_w,a_w\right)}{\partial\phi}\right|_{\left(\phi=0,x_c(t),\lambda_w,a_w\right)}$$

Note that since the average of $\overline{GM}_{zs}\left(x_c(t),\lambda_w,a_w\right)$ in a single wave passage is different, in general, from $\overline{GM}_{zs}$, the actual roll natural frequency in waves depends not only on speed, but also on the wave length and the wave amplitude. Summarising, forward speed effects are accounted for as:

- Variation of damping
- Variation of natural roll frequency
- Variation of restoring

2.2 Linear stability analysis

The most distinctive feature of roll motion in longitudinal waves is the loss of stability of the upright position in certain ranges of parameters. According to the analysis of the basic Mathieu equation this could happen when the encounter frequency and the natural roll frequency are in a ratio of $2/n$ with $n=\pm1,\pm2,\pm3,\ldots$. The instability regions in the space of parameters can be determined by means of analytical approximate techniques (see, e.g., (Hayashi, 1964; Nayfeh & Mook, 1979; Tondl et al., 2000)). In the particular case of parametrically excited ship roll motion such approach has been used extensively (see, e.g. (Bulian, 2006a; ITTC, 2006; Neves & Rodriguez, 2007; Shin et al., 2004)). In the majority of cases the attention has been given to the first parametric resonance region or, in addition, to the second parametric resonance region. As an alternative the instability regions have also been determined in some cases by direct numerical simulation (e.g. (McCue et al., 2007; Neves & Rodriguez, 2007)) with "practical" definitions of unstable condition that, unfortunately, have some drawbacks when compared with the formal definition of stability for the upright position. An alternative numerical approach, based on continuation analysis, has been used by Spyrou et al. (2008): such approach (based on an analytical model closely resembling that in (Bulian, 2005)) allows to trace, in a consistent way, loci in the parameters' space associated to different types of bifurcation, comprising the loss of stability of the upright position.

In this work, we have approached the determination of the linear stability of the upright position by a significantly different method, namely the direct numerical application of the Floquet theory.

Floquet theory provides a criterion for deciding whether the solutions of a linear system of first order differential equations with periodic coefficients could be unstable or not. For more mathematical details see (Hayashi, 1964; José & Saletan, 1998; Nayfeh & Mook, 1979; Simakhina, 2003; Simakhina & Tier, 2005). Here the basic ideas behind the used numerical application of the Floquet theory are given.

Consider a linear first order dynamic model with periodic coefficients with period T:

$$\begin{aligned}&\dot{\underline{u}} = \underline{\underline{P}}(t)\cdot\underline{u}\\&\underline{\underline{P}}(t+T) = \underline{\underline{P}}(t)\\&\underline{u}\in\mathbb{R}^{N\times 1}\;;\;\underline{\underline{P}}\in\mathbb{R}^{N\times N}\end{aligned} \tag{7}$$

In not degenerate cases it is possible to construct a base consisting of N independent fundamental solutions. The $N\times N$ monodromy matrix $\underline{\underline{\Sigma}}(t)$, having the set of fundamental solutions as columns, can be obtained as solution of the following initial value problem in matrix form:

$$\begin{cases}\dot{\underline{\underline{\Sigma}}}(t) = \underline{\underline{P}}(t)\cdot\underline{\underline{\Sigma}}(t)\\ \underline{\underline{\Sigma}}(t=0) = \underline{\underline{I}}_N\end{cases} \tag{8}$$

where $\underline{\underline{I}}_N$ is the $N\times N$ identity matrix. What is really important for the stability of the system is the monodromy matrix evaluated at $t=T$, i.e. $\underline{\underline{\Sigma}}(T)$. It can indeed be proved that, given a set of initial conditions $\underline{u}_0$ the solution $\underline{u}(t=nT)$ can be expressed as

$$\underline{u}(nT) = \underline{\underline{\Sigma}}^n(T)\cdot\underline{u}_0 \tag{9}$$

Relation (9) is basically a linear (Poincaré) mapping $\mathbb{R}^N\to\mathbb{R}^N$ with parameter n. The analysis of the stability of the system (7) can finally be obtained by looking at the behaviour of $\underline{u}(nT)$ for $n\to\infty$. It is indeed sufficient to look at the eigenvalues λ_j of the monodromy matrix $\underline{\underline{\Sigma}}(T)$. Such eigenvalues are called the Floquet multipliers. The system is unstable when at least one of them has modulus larger than one, i.e.

$$\exists j:\left|\lambda_j\right|>1\Rightarrow \text{the system is unstable} \tag{10}$$

It is therefore just sufficient to check the moduli of the eigenvalues of the monodromy matrix.

In this work we have calculated the monodromy matrix $\underline{\underline{\Sigma}}(T)$ by numerically integrating the linearised equation of motion (6) recast in the form (7) using the transformation $\underline{u}=\left(\phi,\dot{\phi}\right)^T$. The period of the coefficients' matrix $\underline{\underline{P}}(t)$ is equal to the ship-wave encounter period T_e (in our modelling the ship speed is constant). The equation is numerically integrated in $\left[0,T_e\right]$ using two different initial conditions, namely $\underline{u}_0^1=(1,0)^T$ and $\underline{u}_0^2=(0,1)^T$ (see (8)). From the knowledge of $\underline{u}^j(T_e)=\left(\phi^j(T_e),\dot{\phi}^j(T_e)\right)^T$ $j=1,2$ the monodromy matrix $\underline{\underline{\Sigma}}(T_e)$ can be created and the calculation of eigenvalues allows to apply the criterion (10). The method is easy to apply, can be easily introduced in more complex models, and it is relatively efficient. The gain in the application of this methodology is based on the fact that only N time domain numerical integrations shall be performed for one encounter period each and the obtained outcome from the stability check is formally correct (although influenced by the numerical accuracy of the integration scheme) and does not show any influence of arbitrary choices made for the initial conditions.

3. SHIP DESCRIPTION AND EXPERIMENTAL SETUP

The ship used in this work is a trimaran hull characterised by the shallow draught of the outriggers (see Figure 1). The main particulars of the tested ship are reported in Table 1 at full scale, and the model scale is 1:50. The same hull form, although with slightly different loading conditions, has been tested in beam sea by Bulian & Francescutto (2009). The ship is characterized by a strongly nonlinear $\overline{GZ}$ curve both in calm water and in waves (see Figure 2). The most important feature of the righting arm is the strong change of slope at moderate heeling angles due to the emergence of one outrigger as the ship heels.

Table 1. Main particulars of "Trim-S1". Data at full scale. Scale of the model 1:50.

L_{BP}	$[m]$	105.6		B_{WL}	$[m]$	22.5
T	$[m]$	4.416		∇	$[m^3]$	2204
A_{WL}	$[m^2]$	765		$\overline{KB}$	$[m]$	2.72
$\overline{BM}$	$[m]$	5.64		$\overline{KG}$	$[m]$	5.90
$\overline{GM}$	$[m]$	2.46		$\omega_{0,zs}$	$[rad/s]$	0.8

Experiments have been carried out at the towing tank of the University of Trieste. The model was connected to the towing carriage with a couple of elastic connections at bow and stern (Bulian, 2006b; Francescutto, 2002; Thomas et al., 2008). Before tests in waves, a series of decays with different forward speeds have been carried out in calm water (Dall'Aglio, 2008) to determine the damping model parameters (see Figure 3): the linear damping coefficient $\mu(V)$ has been actually taken as speed dependent, while nonlinear damping coefficients β and δ have been taken as speed independent. Concerning forward speed effects on roll restoring, Figure 3 also shows the experimentally determined roll oscillation frequency from roll decays, and the corresponding approximated variation in the metacentric height. It can be seen that $\delta\overline{GM}(V)$ reaches up to 15% of the zero speed $\overline{GM}$ (see Table 1).

Afterwards, regular waves were generated and the model was towed at different speeds in head and following waves. The total test matrix is reported in Table 2: from the table it can be noticed that most of attention was given to the case of relatively short waves in comparison with the ship length.

Table 2. Test matrix.

λ_w / L_{BP}	0.5	0.6	0.824	1.0
$s_w = H_W / \lambda_w$	1/20 1/30 1/50 1/100	1/28.2 1/50 1/100	1/50 1/100	1/50

4. COMPARISON BETWEEN EXPERIMENTS AND PREDICTIONS

4.1 Stability of upright position

The nonlinear roll model (5) has been linearized as in (6). The numerical determination of the linear instability regions has been performed according to the described numerical application of the Floquet theory. In (Fucile, 2008) experiments and predictions are reported for all the tested conditions. Here we will give only a few examples. First of all a global view of the instability regions for the upright position is shown in Figure 4 (circles indicates conditions considered as unstable from numerical simulations). From the figure a large first parametric resonance is visible, but also the second parametric resonance region extends over a not negligible range of parameters. Higher parametric resonance regions are also visible.

Appropriate sections of the three dimensional plot in Figure 4 can be performed to obtain the stable / unstable conditions in the plane of forward speed and wave amplitude, for a given wave length. In Figure 5 some comparisons between calculated and experimentally determined stability maps are reported. The experimental stability check for the upright position has been carried out by assessing the model response to a manual perturbation of the upright position. Of course this is an experimental practice, and hence it is impossible to provide an "infinitesimal perturbation" to accurately check the stability of the upright position. Nevertheless, the manual perturbation was sufficiently small to be considered "infinitesimal" for practical purposes. In case of stable conditions different perturbations were used in order to be sure about the actual stability of the upright position. In a large number of cases the upright position was actually stable: the ship's tendency was to come back to $\phi = 0$ for sufficiently small initial perturbation. Large enough perturbations, however, were able to trigger the inception of a large amplitude oscillation. In such case the condition was considered as "stable", because the linear stability analysis deals only with the behaviour of the linearised system. It means that the stability maps reported in this section only deal with the stability / instability of the upright position and shall not be confused with maps reporting the possible presence of a large amplitude motions as in, e.g., (Neves & Rodriguez, 2007). In some cases a clear decision about the stability / instability of the upright condition was not reached, hence such cases are reported as doubtful. According to Figure 5 it can be seen that the agreement between experiments and simulations is quite good, with some problems in the region of higher speeds in head sea. The first parametric resonance region is usually the largest, however also the second parametric resonance region has a not negligible extension (as well as other additional regions of instability). In some cases the minimum calculated wave steepness for the inception of parametrically excited rolling motion in the second parametric resonance region can be lower than the corresponding quantity for the first parametric resonance region: in this case this is also due to the forward speed effects. A bending of the instability regions is evident and it could be related to differences between the average roll restoring in waves and the roll restoring in calm water. It is interesting to see that, as the wave length approaches the ship length, the major growing in the instability regions is predicted for the second and higher parametric resonance zones.

4.2 Results in the nonlinear range

The stability maps reported in previous section are inherently the result of a linear approach to the problem of parametric roll in longitudinal regular waves. In this section we report some key examples of the type of responses observed and/or numerically simulated using the nonlinear model (5). For a thorough description see (Fucile, 2008).

We start, in Figure 6, from the case $\lambda_w / L = 0.5$ with wave steepness $s_w = H_w / \lambda_w = 1/50$. The figure reports the average roll amplitude from experiments and simulations. The predicted roll response curve very well agrees with experimental results. A region of instability for the upright position is visible at very low speed in following sea and at moderate speeds in head sea. Due to the softening behaviour of the righting arm, the peak of the response curve is found in following waves in a region where the large amplitude oscillation coexists with a stable upright position. Despite the fact that the wave length is small in comparison with the ship length, a significant roll is experimentally observed (around 13deg) while in numerical simulations the observed peak reached 20deg. This result confirms that also short waves could significantly endanger the ship even with waves having not extreme steepnesses as in the reported case.

The next example is the case $\lambda_w / L = 0.6$ with $s_w = 1/100$ (Figure 6). This case is very peculiar because, for this condition, the upright position is stable for any speed (see also Figure 5). Nevertheless a resonant branch, completely disconnected from the horizontal axis, can be observed. Numerical results are very well confirmed by numerical simulations. The reason for the presence of a resonance branch with an always stable upright position is to be sought in the peculiar restoring of this trimaran ship. Roughly speaking what is happening is that, for small perturbations of the upright position, the $\overline{GZ}$ is basically linear, and a metacentric approximation is suitable, leading to the conclusion that this wave is below threshold for any speed. However, for larger initial perturbations, the strong change of slope of the $\overline{GZ}$ curve leads to a significant departure from the linear behaviour, as if she had a reduced "equivalent" righting arm, with a consequent reduction in the large amplitude natural oscillation frequency. This reduction shifts the instability regions towards lower encounter frequencies, hence smaller speeds in head sea and partially higher speeds in following waves. When the instability region is shifted towards lower speeds in head sea the roll damping is lower, and this helps in triggering the inception of roll. To some extent we obtain a sort of "locking", in the sense that if the ship rolls at small rolling amplitudes her natural oscillation frequency is high enough to have a case below threshold. On the other hand, when the ship is started from large heeling angles, she has a lower oscillation frequency (at least for the first few cycles of the decay) and the corresponding condition can be considered, to some extent, above a sort of "instantaneous stability threshold". In such case the sub-harmonic response is triggered and the ship remains "locked", without the possibility of ending the initial decay.

The final example is the case $\lambda_w / L = 0.824$, $s_w = 1/50$ (Figure 7). This case summarises a series of very interesting features. First of all a very large amplitude roll (25deg) is observed in the first parametric resonance zone, with a good agreement between predictions and experiments. Numerical simulations show the coexistence of two stable resonant solutions in a very limited range of low speeds in following sea. During experiments, however, a check for the presence of multiple steady states in this zone was not performed. The numerical roll response curve shows resonances also for the second, third, etc. resonance zones. The numerically predicted peak in the second parametric resonance region reaches about 8deg at about 4m/s in following waves. Roll in such range of speeds has also been observed experimentally, though with a quite disturbed time history. An additional very interesting characteristic of the roll behaviour has been observed, for this condition, in numerical simulations close to 3m/s in following waves, namely the coexistence of a 1:1 resonance typical of the second parametric resonance region with a fourth order sub-harmonic response (i.e. a roll response having period that is four times the encounter period). The numerically obtained roll time histories are also shown in Figure 7. In case of the 4:1 sub-harmonic response, a Fourier analysis showed the first harmonic of the roll response as well as all the other odd harmonics, this being a characteristic associated to the symmetry of the response. On the other hand, for the 1:1 resonance, both even and odd harmonics appear in the Fourier analysis (with a not zero mean), starting from the first harmonic of the response that has a frequency equal to the encounter frequency.

5. DEALING WITH MULTIPLE STEADY STATES: A POSSIBLE GLOBAL RISK INDEX

When a regular sea is considered in the framework of a safety assessment, it is necessary to take into account the fact that multiple solutions can be present that deterministically depends on the selected initial conditions. This point has been discussed by Bulian and Francescutto (2008). Here the intention is to briefly report the bases of a possible extension of the proposed approach in (Bulian and Francescutto, 2008) with the aim of providing an overall, though semi-empirical, measure of risk based on a regular sea environment, taking into account the possible presence of multiple solutions and the possible occurrence of capsize. The idea shares some similarities with the approach used by Umeda & Yamakoshi (1993) for the estimation of the capsize probability, though here we are aware that the developed risk index is by no means an approximation of the actual capsize probability in an irregular seaway.

The idea is to mix a "severity measure" $S\left(\underline{x}_0\right) \in \left[0,1\right]$ (0 for low severity and 1 for high severity) to be associated to the ship motions, and hence to the initial conditions $\underline{x}_0$, with a semi-empirical probability density function $pdf\left(\underline{x}_0\right)$ of the initial conditions. To some extent this idea is not very dissimilar also from what is reported in (Blocki, 1980, 1994) and actually could be considered to represent a sort of simplification.

Let $\underline{x}(t)$ be the N-dimensional state vector of the system (with $N = 2$ for a 1-DOF model), and let $\underline{x}_0 = \underline{x}(t = 0)$ be the vector of the initial conditions (in case of a 1-DOF model for roll $\underline{x}_0 = (\phi(t = 0), \dot{\phi}(t = 0))^T$). It follows that the severity measure S is actually a function of $\underline{x}_0$, i.e. $S = S(\underline{x}_0)$. Let now $pdf(\underline{x}_0)$ be an assumed probability density function for the initial conditions. If we limit our attention to the expected value of S, and if we consider it as a risk index $RI \in [0,1]$, it is possible to write:

$$RI = E\{S\} = \\ = \int_{x_{0,1}} \dots \int_{x_{0,N}} S(\underline{x}_0) \cdot pdf(\underline{x}_0) \, dx_{0,1} \dots dx_{0,N} \qquad (11)$$

$$\underline{x}_0 = (x_{0,1}, x_{0,2}, \dots, x_{0,N})^T$$

The closer the risk index to 1, the "more dangerous" is the condition.

Of course both the severity measure S and the probability density function $pdf(\underline{x}_0)$ for the initial conditions are, to a large extent, arbitrary. However, it is likely possible to obtain for them models that are reasonable for practical applications. In this paper we have used a severity measure S and a probability density function $pdf(\underline{x}_0)$ defined as follows:

For the severity measure, we have considered "totally safe" a steady state rolling amplitude up to 2deg, and totally unsafe rolling amplitudes above 28deg (or capsize), with a linear interpolation between 2deg and 28deg (a proper tuning of the severity measure S could allow to embed the so called "total" and "partial" stability failure according to the definitions in (IMO, 2008)). For the probability density function of initial conditions we have used a (truncated) joint Gaussian probability density function for the roll angle and roll velocity, with uncorrelation between them (diagonal covariance matrix) and with both variables having zero mean with a renormalization to obtain unitary probability in the domain of tested initial conditions. The standard deviation of the roll motion σ_ϕ [rad] and of the roll velocity $\sigma_{d\phi/dt}$ [rad/s] are considered to be related as follows:

$$\sigma_{d\phi/dt} \approx \omega_0 \cdot \sigma_\phi \qquad (12)$$

It shall be mentioned that, however, in the definition of the risk index, there is also a "hidden" parameter, namely the position of the wave crest at $t = 0$. Indeed the actual outcome concerning RI computed according to (11) depends on the initial wave crest position. In general an appropriate additional averaging shall be carried out assuming, for instance, a uniform distribution of the wave crest position in the interval $[0, \lambda_w]$. In this paper we have not considered explicitly this aspect and the initial wave crest position is always at the aft perpendicular.

As an example of application we have calculated the "risk index" in the case $\lambda_w / L = 0.5$ with $H_w / \lambda_w = 1/50$. Initial conditions have been considered only in the box $\phi_0 \in [-40, 40]\deg$ and $\dot{\phi}_0 \in [-20, 20]\deg/s$. Results of calculations are shown in Figure 8. Different values for the assumed standard deviation of initial roll angle are considered, namely 2.5deg, 5deg and 7.5deg. For sake of reference Figure 8 also depicts the domains of attraction of different solutions for the case of a ship speed of 2m/s in following sea. The numerical response curve for this case has already been shown in Figure 6. From Figure 8 it can be seen that:

- For $\sigma_{\phi_0} = 5\deg$ and $\sigma_{\phi_0} = 10\deg$ the peak of the risk index is out from the region of instability of $\phi = 0$. This is due to the fact that the resonant solution, though coexisting with a stable upright position, has a significantly large attraction basin (namely, a high attraction index according to Bulian and Francescutto (2008)). The majority of initial conditions lead to a large amplitude motion at steady state, hence a high risk index (see the domains of attraction).

- When the standard deviation of the initial heeling angle is reduced to 2.5deg the probability density function concentrates in the region of the origin of the plane of initial conditions. In this region the attractor of the stable upright position (if existing) is dominant, hence the risk index tends to reduce out from the instability zone for the upright position with respect to calculations based on larger σ_{ϕ_0}. In case of the smallest value for σ_{ϕ_0} the "most dangerous" condition is found close to zero speed of advance, i.e. close to the point where the pitchfork bifurcation occur leading to the disappearance of the attractor of $\phi = 0$.

6. FINAL REMARKS

This paper has presented a combined numerical experimental investigation on the parametrically excited rolling motion in regular longitudinal waves for a trimaran hull form characterised by the shallow draught of its outriggers. The interest concentrated in the range of wave lengths shorter than the ship length. The tested hull form in the considered loading condition showed significant rolling amplitudes even in case of short and/or steep waves.

The obtained experimental results have been compared with the predictions given by a nonlinear 1-DOF model, finding a general good agreement. In the proposed model forward speed effects are taken into account in an approximate way both in damping and in restoring.

The linear stability has been investigated numerically by a direct application of the Floquet theory. The numerical construction of the monodromy matrix allowed to determine all the regions of instability for the upright position in the considered range of speeds, without requiring any simplification of the time dependence of the restoring term (thus considering the complete Hill's equation coming from the linearization of the nonlinear model). The experience gained from this work concerning the direct application of the Floquet theory supports the idea that it is actually a powerful tool that is worth to be further considered and from which other analytical approaches could gain a significant benefit without requiring significant extra effort in terms of coding. In this paper we have applied it to a 1-DOF model, but it is applicable to models with more DOFs without major problems after a proper analytical/numerical linearization procedure is applied. In particular the direct application of the Floquet theory could provide significant insight into the stability limits in case of strongly not sinusoidal variations of $\overline{GM}$, in case of 3(4)-DOF models (with or without tanks) and in case of models with surge dynamically accounted for.

The time domain analysis of the nonlinear model showed a series of interesting features connected to the strong nonlinear behaviour of the righting arm. In particular multiple solutions have been found in a large range of speeds. For small wave steepness it was also found that a stable resonant solution can exist also when the wave is such that the system is completely below threshold.

It was also found that the roll response could be large even in case of relatively short waves that are often neglected in the analysis of parametric roll. Moreover the roll response predicted by the numerical model in regions of parametric resonance higher than the first one was found to be not negligible, especially in case of the second parametric resonance. In some cases the minimum wave steepness for the inception of parametric roll was found to be smaller for the second parametric resonance than for the first one. It is not clear whether this is due solely to forward speed effects or also to the effect of nonlinearities of restoring leading to a recombination of harmonics.

In the region of the second parametric resonance harmonic 1:1 responses have been numerically found to coexist with 4:1 sub-harmonic responses in certain ranges of speeds. In particular, the fourth order sub-harmonic response was found to have a larger amplitude than the harmonic response.

A risk measure has been introduced in order to rationally deal with the presence of multiple coexisting solutions. Such risk index is based on the definition of a severity measure and a distribution of initial conditions. Although both the severity measure and the probability density function of the initial conditions are arbitrary quantities, it is thought that the proposed risk index could serve in using more rationally the regular sea environment for the assessment of the risk of parametric roll inception. Some examples have been reported concerning the calculation of the risk index, and the outcomes have been discussed in connection with the topology of the domains of attraction for the different coexisting solutions.

7. ACKNOWLEDGMENTS

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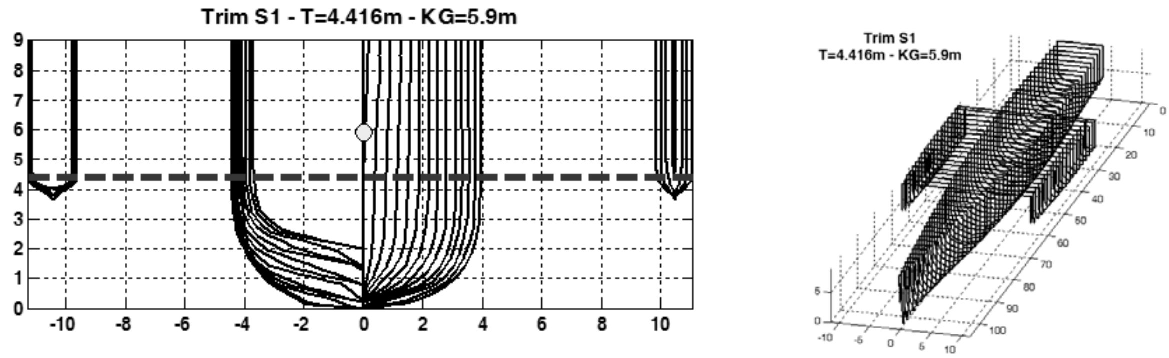

Figure 1. Trimaran "Trim S1" - Body plan. The thick dashed line indicates the draught and the dot indicates the position of the centre of gravity. A 3D view of the used sections is also reported.

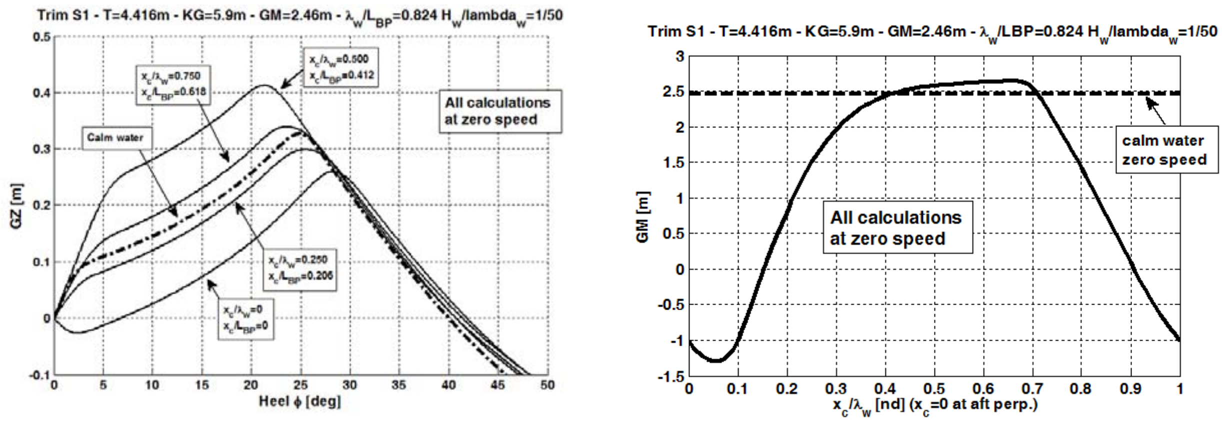

Figure 2. $\overline{GZ}$ and $\overline{GM}$ hydrostatic calculations in waves for Trim S1. Regular wave with $\lambda_w / L = 0.824$ and $H_w / \lambda_w = 1/50$.

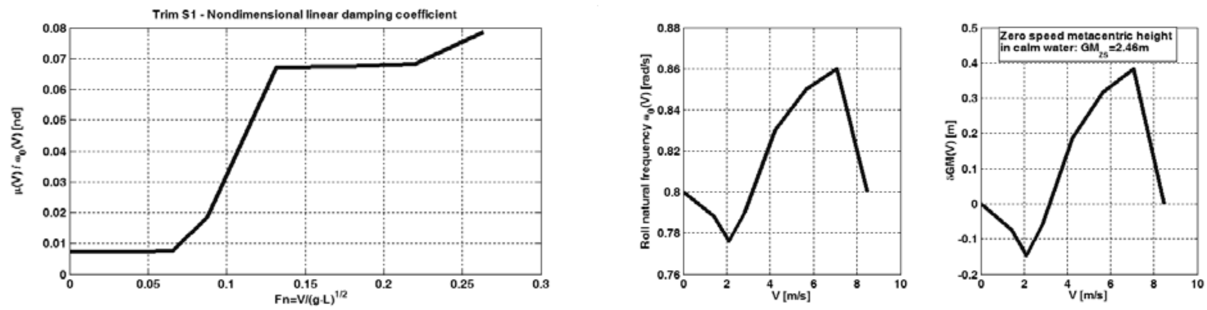

Figure 3. Speed dependent linear damping coefficient and forward speed effects on restoring. Nonlinear damping coefficients taken as speed independent: $\beta = 0.3 rad^{-1}$, $\delta \cdot \omega_0 = 2.4 rad^{-1}$.

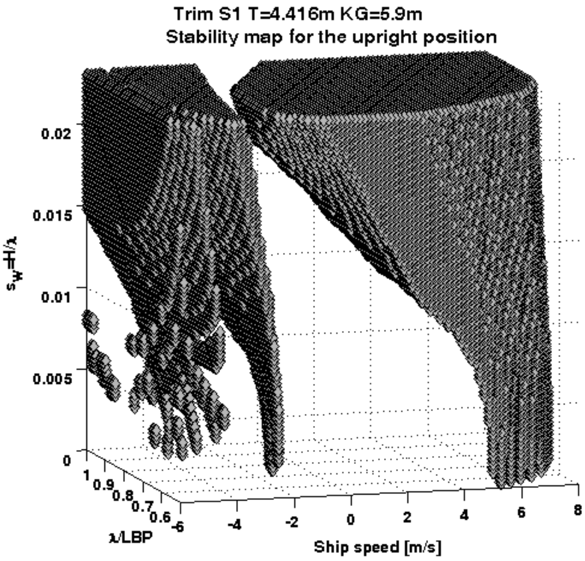

Figure 4. Stability map in a three parameters space: forward speed, wave length and wave steepness.

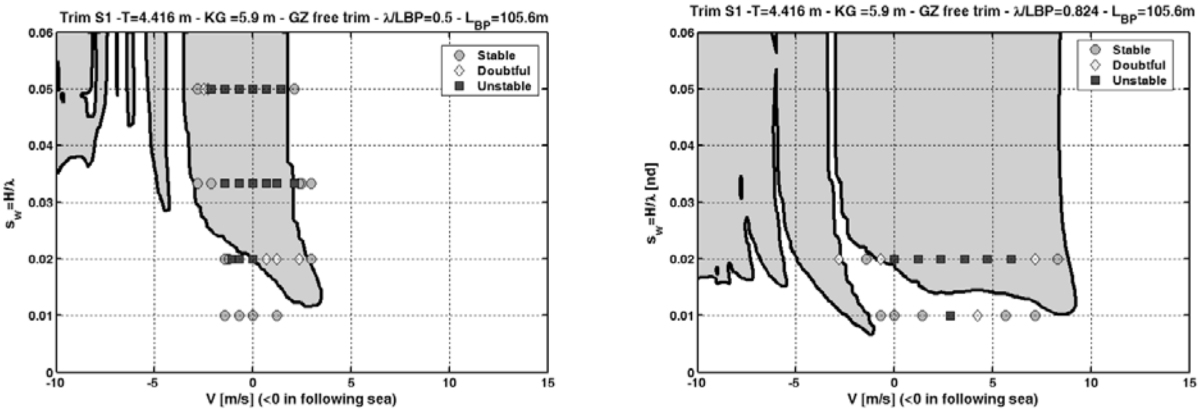

Figure 5. Examples of comparison between calculated and experimentally determined stability / instability regions for the upright position. $\lambda_w / L = 0.5$ and $\lambda_w / L = 0.824$.

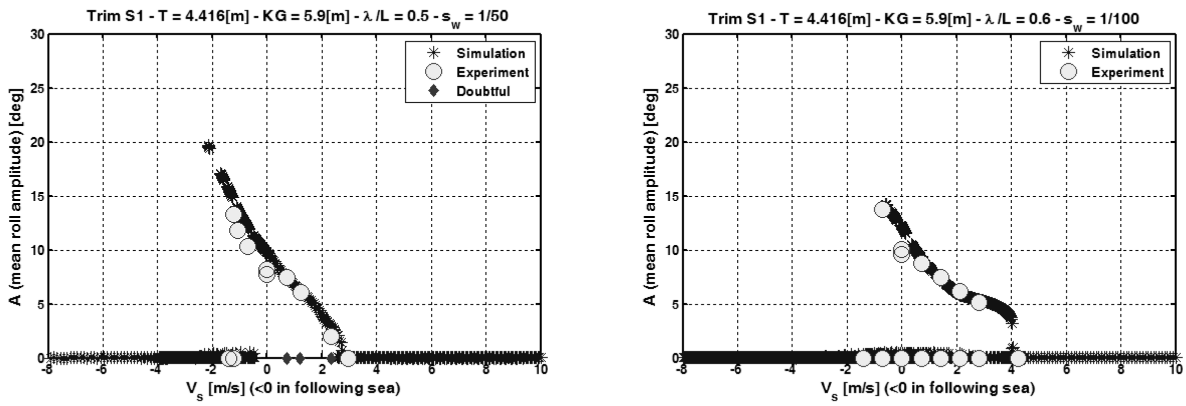

Figure 6. Experimental roll response curve and numerical simulations. $\lambda_w / L = 0.5$, $H_w / \lambda_w = 1/50$ and $\lambda_w / L = 0.6$, $H_w / \lambda_w = 1/100$.

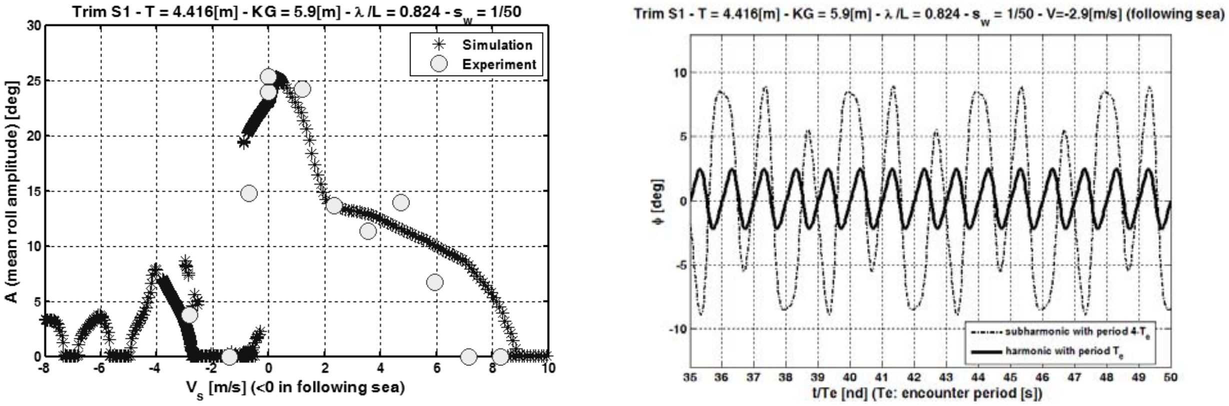

Figure 7. Experimental roll response curve and numerical simulations for $\lambda_w / L = 0.824$, $H_w / \lambda_w = 1/50$. Coexisting harmonic response and fourth order sub-harmonic response from numerical simulations are also shown for the speed 2.9m/s in following waves.

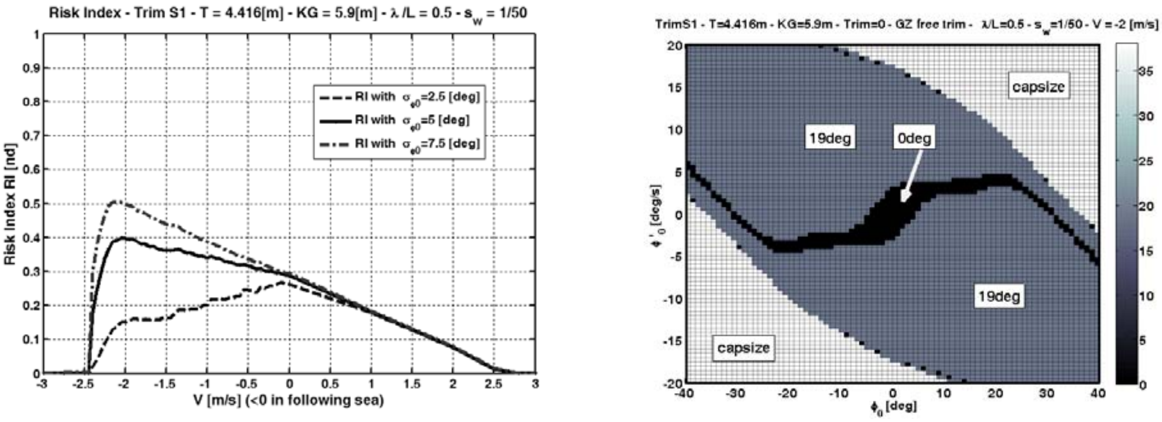

Figure 8. Example of calculation of the risk index for $\lambda_w / L = 0.5$, $H_w / \lambda_w = 1/50$ (left), and domains of attraction for a ship speed of 2m/s in following sea (right).