IMPORTANCE OF MORE ACCURATE HYDRODYNAMIC MODELLING ON DETERMINING CRITICAL NONLINEAR SHIP ROLLING RESPONSE

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ABSTRACT

Critical roll motion boundaries are sensitive to accurate physical modeling of ship roll dynamics. It will be shown in this paper the importance of accurately modeling not only the important hydrostatic restoring moments but also the importance of accurately modeling the radiated wave force. It is well known in the marine hydrodynamics field that the radiated wave force is frequency dependent. However, much work in the nonlinear marine dynamical systems field has assumed frequency independence or a constant coefficients approximation. Assuming constant coefficients may be a reasonable approximation for single frequency steady state motion and even the transient response of a nonlinear system with a single frequency excitation but clearly not for multiple frequency excitation. In this work we will assess the effect of approximating the radiated wave force by constant coefficients versus the more accurate impulse response function modeling. We will apply these two types of hydrodynamic force modeling to calculate critical dynamics of ship rolling motion in regular and irregular waves. The critical dynamics are directly determined using a unique calculation method (Vishnubhotla, Falzarano, Vakakis, 2000). This method directly calculates motions on either the stable and unstable manifolds. Since the stable manifolds form the basin boundaries, the safe basin can be defined. Moreover, this method can be used as an alternative to the so-called Melnikov method by directly calculating the distance between the stable and unstable manifolds. This method is potentially more powerful than Melnikov methods since is it not dependent upon the so-called “Melnikov trick” which practically limits the Melnikov method to first order. This paper will contain results of constant coefficients (for various constant frequencies) versus impulse response function for regular wave excitation and various spectra.

Keywords: *Nonlinear, Random, Ship Rolling ,Capsizing*

1. PHYSICAL SYSTEM MODELING

In this work we study the single degree of freedom roll equation of motion with roll uncoupled from the other five degrees of freedom. The single degree roll equation of motion is as follows:

$$(I_{44} + A_{44}(\omega))\,\ddot{\varphi} + B_{44}(\omega)\,\dot{\varphi} + B_{44q}(\omega)\dot{\varphi}\,|\,\dot{\varphi}\,| + \Delta GZ(\varphi) = F_4(t) \qquad (1)$$

In the above equation it can be seen that the added mass A_{44} and the radiated wave damping B_{44} are functions of frequency and are constants only if the external excitation is harmonic. Due to the softening nonlinearity of the roll restoring moment, the roll motion may be stable and bounded or unstable and unbounded. The focus of this work is to determine the basin boundary curve which separates these two types of motions. The so-called safe basin is the region in the phase space where initial conditions located in this region will remain bounded while initial conditions outside this safe region will not remain bounded. An alternative representation of the above equation which considers the frequency dependence of the hydrodynamic reaction force (see e.g., Cummins, 1962) is as follows:

$$(I_{44} + A_{44}(\infty))\ddot{\varphi} + \int_{-\infty}^{t} K_{44}(t-\tau)\dot{\varphi}(\tau)\,d\tau$$
$$+B_{44q}(\omega)\dot{\varphi}\,|\dot{\varphi}| + \Delta GZ(\varphi) = F_4(t) \tag{2}$$

In the above equation, the integral is the so-called convolution integral and represents the hydrodynamic force due to an arbitrary excitation. If the external excitation and response is not harmonic then a more accurate modeling of the linear radiated wave force is needed. In this paper we analyze the critical motion response or basin boundaries as they are affected by the more accurate hydrodynamic modeling represented by the impulse response function. In this study we consider the external excitation to be a multi-frequency summation or realistic representation of random sea waves and we study how the basin boundary is affected by this approximation.

2. DYNAMICAL SYSTEMS SOLUTION TECHNIQUE

It is well known in the nonlinear dynamics field that the safe basin boundary is simple when the excitation is small relative to the system's damping. However, as the excitation increases beyond a critical value the basin boundary or stable manifold may intersect the unstable manifold. This intersection results in a complicated fractal structure of the basin boundary. This critical amount of forcing can be approximately predicted using Melnikov methods (Falzarano, Shaw and Troesch, 1992). Although Melnikov methods are quite general (Zhang and Falzarano, 1994) and capable of analyzing e.g., multiple degrees of freedom system, they are practically limited to first order due to their use of the so-called "Melnikov trick." The Melnikov trick significantly simplifies the determination of the manifold separation since it only requires evaluation of the perturbed (with forcing and damping) differential equation along the unperturbed solution trajectory which is without forcing or damping. If the unperturbed equation is simple the solution may be known explicitly. Over the last several years we have been developing an alternative to Melnikov methods for analyzing nonlinear ship rolling motion which is based upon the theory of differential equations and was originally developed by Vakakis (1994). The method involves calculating solutions along the stable or unstable manifold (see e.g., Figure 1). Recently, we have compared our results with those obtained numerically and those obtained using Melnikov methods and found that we obtained comparable results for harmonic excitation. Specifically the numerical results differed somewhat but the Melnikov results were exactly the same. In addition we have also applied this method to consider pseudo-random excitation and in this work we investigate the effect of including a more accurate hydrodynamic model.

The dynamics solution technique can basically be summarized as follows. First determine solution to unperturbed equation without damping or forcing. For example:

$$\ddot{x} + x - k\,x^3 = 0 \tag{3}$$

gives the unperturbed solutions as follows:

$$x(\tau) = \frac{1}{\sqrt{k}}\tanh\left(\frac{\tau-\tau_0}{\sqrt{2}}\right)$$
$$\dot{x}(\tau) = \frac{1}{\sqrt{2k}}\sec h^2\left(\frac{\tau-\tau_0}{\sqrt{2}}\right) \tag{4}$$

Next express scaled equation of motion as sum of an unperturbed and an additional perturbation, i.e.,

$$\ddot{x} + x - k\,x^3 = \varepsilon\,(-\gamma\,\dot{x} - \gamma_q\,\dot{x}|\dot{x}| + F(\eta)) \qquad (5)$$

Next express the unknown solution with the additional perturbation as a series, i.e.,

$$x(\eta) = x_0(\eta) + \varepsilon\,x_1(\eta) + ... \qquad (6)$$

Using known zero[th] order solution from above, one can obtain easily the first and possibly higher order terms in the series.

$$\ddot{x} + x - 3k\,x_1\,x_0^{\,2} = G(x_0, \eta, \tau_0) \qquad (7)$$

Equation (7) is a linear differential equation with a parameter-dependent coefficient, and its general solution is obtained by using the method of variation of parameters.

$$x_{1s,u}(\eta;\tau_0) = \left[\alpha_{1s,u} - \int_0^{\eta} g(\xi)\,x_{h1}^{(2)}(\xi)\,d\xi\right] x_{h1}^{(1)}(\eta) +$$
$$\left[\beta_{1s,u} + \int_0^{\eta} g(\xi)\,x_{h1}^{(1)}(\xi)\,d\xi\right] x_{h1}^{(2)}(\eta) \qquad (8)$$

Where,

$$x_{h1}^{(1)}(\eta) = \frac{1}{\sqrt{2k}}\operatorname{sech}^2(\eta) = dx_{0s,u}(\eta)/d\eta \qquad (9)$$

$$x_{h1}^{(2)}(\eta) = 2^{1.5}k\,x_{h1}^{(1)}(\eta)\left[\frac{1}{4}\cosh^3\eta\sinh\eta + \frac{3}{16}(\sinh 2\eta + 2\eta)\right]$$

are two linearly independent homogeneous solutions, and $\alpha_{1s,u}$ and $\beta_{1s,u}$ are constants evaluated so that $x_{1s,u}(\eta;t_0)$ is bounded as $\eta \to \pm\infty$. In general for harmonic excitation it is seen that the first term in expression (8) is bounded while the second term is not. In fact the function $x_{h1}^{(2)}(\eta)$ diverges whereas the definite integral reaches a finite limit as $\eta \to \pm\infty$.

More details of this method in general are contained in Vakakis (1994) and this method applied to this specific problem in Vishnubhotla, Falzarano, and Vakakis (2000). We have since applied this method to more general perturbations including impulse response function modeling of the hydrodynamic reaction forces and pseudo-random external forcing and those results are the focus of this paper.

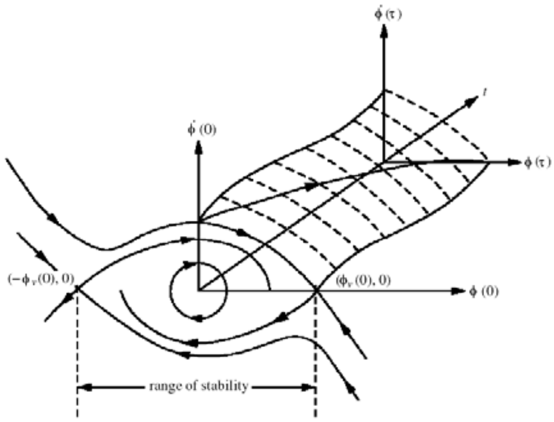

Figure 1.Determination of Solution on Stable Manifold.

3. RESULTS

Currently, the US Navy is involved in the design and construction of a new generation of destroyer hull forms, the so-called DDG-1000 Zumwalt class. This innovative hull is radically different from previous and existing destroyer hull forms, most notably in that it has tumble home sides, a wave piercer bow and a broad flat stern. The ship is designed to minimize signature not unlike the stealth aircraft. Unfortunately, due to this hull form's unique features the vessel's motion response is dramatically different from existing hull forms. In an effort to gain a better understanding of this new vessels motion unique motion response, we study the response of the US Navy's currently existing destroyer hull form a so-called traditional hull form. The traditional hull form is similar to the US Navy's Burke class DDG hull form although an earlier version of the hull. The physical characteristics of this vessel are summarized in Table 1.

Table 1. Physical Characteristics of Traditional Hull-form.

Parameter	Units	Dimensional Value
Length of the vessel, L	ft	466.00
Displacement, Δ	lb	18900000.00
Linear restoring arm, C_1	ft	6.570
Nonlinear restoring arm, C_3	ft	3.120
Wave amplitude, ζ	ft	5.70
Forcing frequency, ω	rad/s	0.90
Linear natural frequency, ω_n	rad/s	0.572
Hydrodynamic mass, $(I_{44} + A_{44}(\omega))$	slug-ft^2	380000000.00
Linear damping, B_{44} (ω)	slug-ft^2-s^{-1}	1810000.00
Nonlinear damping, B_{44q}	slug-ft^2	18400000.00
Total wave force, F_{44} (ω)	lb-ft^{-1}	4125000.00

In this paper we analyze the critical roll motion response of the traditional hull as affected by improved hydrodynamic modeling. The key result of this analysis is the comparison of the critical safe basin boundaries using the two different hydrodynamic modeling. The results are for the traditional US Navy destroyer hull-form in a pseudo-random seaway. The seaway is represented by two-parameter seaway with intensity Sea State 2, and significant wave height of 2.9 feet and a peak period of 7.5 seconds. The constant coefficients added mass and damping are calculated at the vessel's linear natural frequency. Since roll is lightly damped and highly tuned one would expect the constant coefficients and impulse response function results to be quite close. However, since the roll restoring moment curve for this vessel is highly nonlinear, the magnification curve is significantly bent to lower frequencies and the linear roll natural frequency is not indicative of the response. For a description of this phenomenon for the general see, Falzarano, Esparza and Taz Ul Mulk, (1994) and for this particular hull see Juckett, Falzarano, Vishnubholtha (2006).

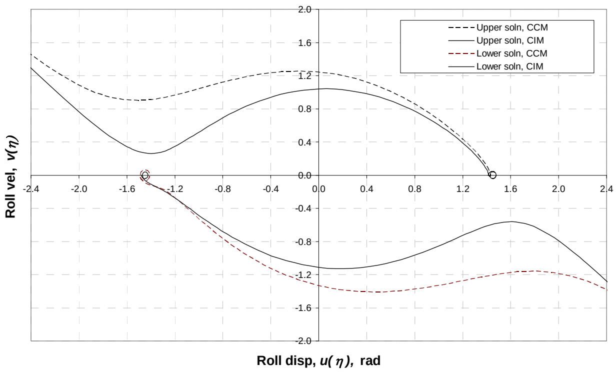

Figure 2. Comparison of Critical Roll Response CCM vs. IRF.

One can see from Figure 2 that the effect of the more accurate impulse response function hydrodynamic modeling can be significant.

4. CONCLUSION

The ultimate goal of this research is to predict when a vessel is likely to capsize in random seas over its lifetime. This research described in this paper provides a tool to answers only part of that question and much more work is needed. However, we hope that this is a valuable contribution in this area and may eventually make it possible for vessel designers to assess the safety of a proposed innovative vessel design which may be dramatically different from existing vessel designs.

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