TECHNICAL NOTE: PREDICTION OF THE THRESHOLD OF GLOBAL SURF-RIDING BY AN EXTENDED MELNIKOV METHOD

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ABSTRACT

An advanced version of Melnikov's method is discussed for identifying efficiently the condition of complete disappearance of the overtaking wave periodic motion of ships (upper threshold of surf-riding), in an environment of steep following seas. The key advantage of this method is that, it overcomes the constraint of small damping and small forcing that is essential when the ordinary Melnikov method is applied. The method was applied for a reference ITTC ship and the result was compared to that obtained on the basis of ordinary Melnikov analysis. Moreover, it was evaluated against direct numerical predictions of the threshold, obtained through simulation.

Keywords: *surf-riding, broaching, Melnikov, homoclinic, heteroclinic, ship, stability*

1. INTRODUCTION

The surf-riding behavior is realised by a ship when she is forced to move with speed that is equal to wave celerity, in an environment of steep following seas (Kan 1990). It is known that this phenomenon may work as precursor of broaching. Over the years surf-riding has been studied analytically, numerically and experimentally (for a review see for example Spyrou 2006). Accruing from the fact that, dynamically, the condition of global capture into surf-riding ("upper threshold") corresponds to a typical *homoclinic saddle connection* event, one of the methods that one could applied for predicting it is the so-called Melnikov method (see for example Guckenheimer & Holmes 2002). A key advantage of this method is that, it can produce a very simple formula that could be used in the context of assessing a ship's tendency for broaching, at the initial design stage. On the other hand, the method is mathematically valid if the terms that play the role of damping can be reasonably assumed as “small”. The same assumption is necessary for the terms that represent the external forcing.

In order to overcome the constraint set by the assumption of small damping and small forcing, in the current paper is discussed another, more advanced, version of Melnikov’s method, firstly introduced by Salam (1987) and expanded further by Endo & Chua (1989) who have applied it successfully. With the new mathematical formulation, the unperturbed sub-system inside the originally studied system could have a dissipative nature, i.e. it does not need to be assumed as a Hamiltonian one. Thus, excursions from the commonly made assumption of "near-Hamiltonian" character of the system can be comfortably handled.

Intrinsic to the implementation of Melinkov's method is the existence of a homoclinic orbit for its unperturbed sub-system. Realisation of such an orbit remains essential in the new formulation; and, if it does not arise naturally, it has to be created. In this respect it often suffices to introduce an initially unknown constant, whose value could be identified on the basis of the condition of formation of a homoclinic orbit. This constant is subsequently subtracted when the original ("perturbed") system is examined; hence it should be a "small" quantity, so that the final system remains identical to the initial one. Thereafter, the integration over the homoclinic orbit, that is intrinsic to the application of Melnikov's method, can be performed without difficulties.

Real ship data has been used in order to evaluate this new method against alternative predictions of the locus of global surf-riding as one of the key parameters, the nominal Froude number or the wave height, are varied. The results obtained were very close to the numerical values and consistently closer than the results of its "ordinary" version; at the expense however of not been able to produce a handy "closed-form" expression, like the ordinary Melnikov method does.

2. FORMULATION OF THE PROBLEM AND NEW METHOD OF SOLUTION

2.1 A basic mathematical model

We have adopted the modelling approach and the symbols used in Spyrou (2006). The nonlinear equation of surging in an exactly following and "harmonic" seaway accrues from application of Newton's second law:

$$\left(m - X_{\dot U}\right)\frac{dU}{dt} = T - R + X_w \qquad (1)$$

m and $-X_{\dot U}$ are respectively, the mass of the ship and the surge "added mass". U is the instantaneous velocity of the ship in the surge direction. Functions T and R stand, respectively, for thrust and resistance. X_w is the Froude-Krylov force in surge. For a harmonic incident wave, the Froude-Krylov force could be approximately expressed as:

$$X_w = -f \sin kx \qquad (2)$$

x is the distance from the centre of gravity of the ship to a reference wave trough (as a matter of fact, the origin moves with wave's celerity). f is a constant that contains the amplitude of wave excitation and k is the wave number. Resistance could be expressed as some function of velocity U, e.g. with the following simple polynomial form:

$$R = r_1 U + r_2 U^2 + r_3 U^3 \qquad (3)$$

$r_i (i = 1,2,3)$ are appropriate coefficients.

Propeller thrust should be a function of, at least, velocity and propeller rate n:

$$T = \tau_2 U^2 + \tau_1 U n + \tau_0 n^2 \qquad (4)$$

$\tau_i (i = 1,2,3)$ are suitable coefficients. Substituting (2) - (4) into (1), then expressing everything with respect to $\dot x = U - c$, where c is the wave celerity, and letting $y = kx$, the differential equation for surging in following seas can be expressed as follows:

$$y'' + p_1 y' + p_2 y'^2 + p_3 y'^3 + \sin y = r/q - by' \qquad (5)$$

The prime denotes the derivative with respect to the scaled time $\tau = \sqrt{q}t$. Also, $p_i (i = 1,2,3)$ and q are suitable functions of r_i, τ_i, c, and f and, for a given ship, these are constants. One should notice that $b\,y'$ was separated from $p_1 y'$. This is due to the fact that the coefficient b contains the unknown parameter n. The same holds for r. Therefore, if the wave had been fixed, r and b would be the unknown quantities of our problem.

2.2 Implementation of the extended Melnikov method

In the ordinary Melnikov method the unperturbed sub-system is always assumed to be Hamiltonian and thus it may not include any damping terms. As is obvious, the subsequently applied perturbation should, at best, contain damping as a small (in fact infinitesimal) quantity. However, the extended Melnikov method can accommodate damping terms that can be large. Moreover, such terms may be included even in the definition of the unperturbed sub-system, that needs no longer to be Hamiltonian.

Having said that, a homoclinic orbit for the unperturbed sub-system is still needed and, around that one should calculate the Melnikov integrals. It is often possible to obtain such an orbit, by introducing an initially unknown constant "torque" σ, whose value needs to be identified. Subsequently our unperturbed sub-system should become:

$$y''+p_1y'+p_2y'^2+p_3y'^3+\sin y+\sigma=0 \tag{6}$$

In (6) one could have assumed all linear and nonlinear terms as large, without facing any difficulty in the implementation of the method. However, in the sample ship that was investigated here for the purpose of demonstrating the potential of the method, the cubic damping term turned out to be a really small number, when the scaled velocity y' obtained realistic values. Subsequently, in this case only linear and quadratic damping terms needed to be treated as large quantities and our unperturbed sub-system could be somehow simplified:

$$y''+p_1y'+p_2y'^2+\sin y+\sigma=0 \tag{7}$$

Then the perturbation should become:

$$g(y)=\varepsilon\left(-\frac{b}{\varepsilon}y'-\frac{p_3}{\varepsilon}y'^3+\frac{1}{\varepsilon}\frac{r}{q}+\frac{1}{\varepsilon}\sigma\right) \tag{8}$$

The Melnikov function that corresponds to our system is given by the following expression (Salam 1987):

$$M(t_0)=-\frac{b}{\varepsilon}I_1-\frac{p_3}{\varepsilon}I_2+\frac{1}{\varepsilon}\left(\frac{r}{q}+\sigma\right)I_3 \tag{9}$$

I_1, I_2 and I_3 should be calculated by the following integrals:

$$\begin{aligned} I_1&=\int_{-\infty}^{\infty}y_2^2\exp(p_1t+2p_2y_1)dt; \\ I_2&=\int_{-\infty}^{\infty}y_2^4\exp(p_1t+2p_2y_1)dt; \\ I_3&=\int_{-\infty}^{\infty}y_2\exp(p_1t+2p_2y_1)dt. \end{aligned} \tag{10}$$

where (y_1, y_2) are the coordinates on the phase plane $(y,\ y')$ of the homoclinic orbit for the unperturbed system (6). The threshold of global surf-riding is found when the Melnikov function becomes zero. This should happen if:

$$\frac{r}{q}-\frac{bI_1}{I_3}=\underbrace{\frac{p_3I_2}{I_3}-\sigma}_{C} \tag{11}$$

As p_3 and q are system parameters, for a specific ship and a specific wave these are constants. I_1, I_2, I_3 and σ are constants too that can be calculated. In particular for calculating σ, two points are selected in the vicinity of the saddle of sub-system (7), one very close to its outset and the other very close to its inset (their directions are known from linear analysis around the saddle - to be noted that we refer to a cylindrical phase - plane where the two saddles of sub-system (6) coincide). Dynamically, the unstable manifold coming from the one point will end up at the other one in finite time *T*. The coordinates of these points (x_1, x_2) can be expressed in terms of σ. And coordinates of the homoclinic orbit (F_1, F_2) are functions of σ and *T*. Then there are two equations and two unknowns.

$$\begin{aligned} F_1(T,\sigma)-x_1(\sigma)&=0; \\ F_2(T,\sigma)-x_2(\sigma)&=0. \end{aligned} \tag{12}$$

The value of σ can be determined by numerically solving (12). Details of obtaining the values of σ in a general context are discussed in Endo and Chua (1989). The homoclinic orbit (y_1, y_2) needs to be determined numerically because its analytical calculation on the basis of (7) seems unlikely. Also, numerically should be carried out the integrations (10), for the same reason. The right hand side of (11) is a constant. The only truly unknown terms are r and b, which are functions of propeller rate n. Given the wave height, the critical propeller rate which may lead to global surf-riding should be determined as solution of equation (11).

It should be reminded that, the ordinary Melnikov method produces the following simple, "closed-form", expression for the unknown quantity r (Spyrou 2006):

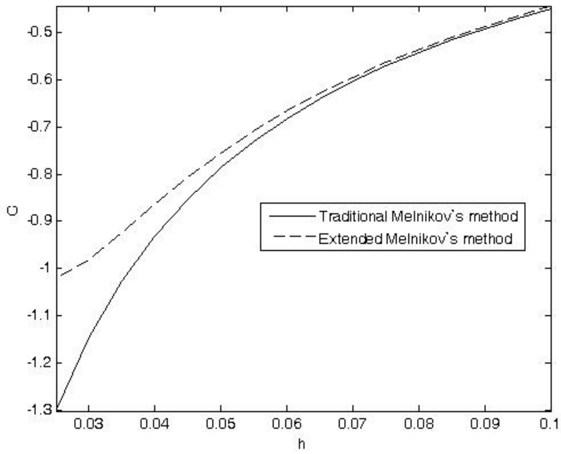
$$\frac{r}{q}+b\frac{4}{\pi}=\underbrace{-\frac{4}{\pi}p_1+2p_2-\frac{32}{3\pi}p_3}_{C} \qquad (13)$$

As a matter of fact, one is curious to observe how close to each other stand the results produced by the two methods.

3. PRELIMINARY RESULTS

Data from the well-known ITTC purse-seiner with length $L=34.5m$ has been used in order to evaluate the method. The wave length λ was fixed at twice the ship length. The wave steepness $h=H/\lambda$ could be varied, depending on the nominal Froude number (H is the wave height). The Froude-Krylov force was calculated in the standard way. Then the value of σ was determined for each wave steepness scenario. Finally, the critical values of r/q were obtained. Repetitive application of these steps produced, after overlapping the graph deriving from expression (13), the diagram of Fig. 1 where, for convenience, as independent parameter is used the wave steepness h. This is plotted versus the quantity named as "C" in equations (11) and (13). One observes that, as the wave steepness is increased, the predictions from these two methods come closer to each other. Higher steepness is indeed the area of practical interest.

Since r and b are functions of propeller rate, the critical values of propeller rate n (and thus nominal Froude number) can also be calculated as the wave steepness is varied. Moreover, numerical simulations can be carried out on the basis of the original system given by Eq. (5) in order to assess the differences. The comparison of the three methods is shown in Figure 2. The extended Melnikov method follows consistently the numerical predictions based on the original system even in the lower wave steepness region



where the ordinary Melnikov diverges.

Figure 1. Comparison of the constant values between the two Melnikov methods.

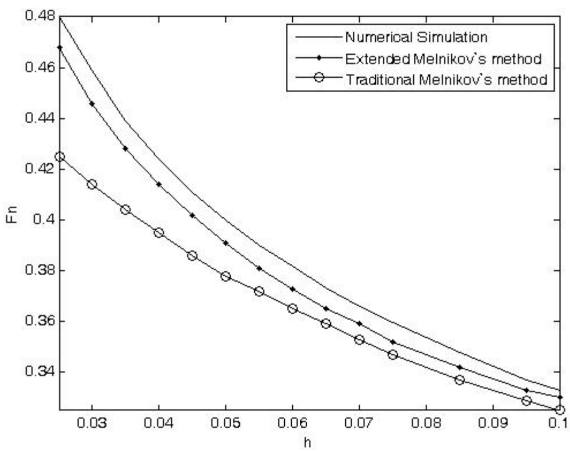


Figure 2. Comparison of boundary curves of global surf-riding according to the three different methods.

4. CONCLUSIONS

An advanced version of Melnikov's method was implemented for analyzing the surf-riding problem, for the purpose of overcoming the assumption of small damping that is intrinsic to the standard Melnikov. Indeed, the new method seems to be producing a result of consistently good accuracy; however the console of having a simple analytical formula is not catered. The possibility of a semi-empirical correction of the standard Melnikov formula in the light of the new result could be worthy of investigation.

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