

RESEARCH OF SHIP DYNAMIC ON FOLLOWING WAVES

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ABSTRACT

This paper discusses the problems of control of ship stability changing in waves. The analysis is carried out using differential equation of roll while the righting moment is considered a nonlinear spatial function. This paper is focused on the formation of attractor sets and analysis of dynamics of a ship at various intensities of external excitation.

1. INTRODUCTION¹

Research of a ship dynamic on following waves at periodic decrease of stability is one of the important directions in a general problem of a safety of navigation [1] - [8]. The critical situations arising in these conditions are in detail enough investigated with the help of theoretical and experimental methods. Among works of such direction, it is necessary to allocate researches devoted to the analysis of physical pictures of ship dynamics on following waves [6], [7], [8] in which three are determined characteristic happen of capsizing complete loss of stability, low-frequency resonance (modes rolling in conditions of the parametrical resonance) and basic and broaching (loss of manoeuvreability and sudden evolution of a ship at absence of efficiency of rudder control.

Alongside with the marked situations, which became typical at study of stability on following waves, there is one more situation connected to continuous deterioration initial stability. The effect of stability decrease is to the greatest degree shown at courts with the Sfigurative stability diagram. The presence of such diagram at influence of large packages of irregular waves causes sharp deterioration initial of metacentric height, that results to crank of a ship. The phenomenon of crank frequently is observed in practice of operation. The cases of heavy failures connected to sharp deterioration of stability on waves are known. So, for example, the failure of the cargo-ship «Poronaisk» on following waves is described. Occurrence of an emergency preceded strong rolling with amplitude $30 - 35^{\circ}$, caused by periodic decrease initial stability (initial metacentric height decreased up to -0.30 m and more at influence of abrupt waves).

In experiments with model of the cargoship «Poronaisk» on irregular waves [6] the sharp deterioration of stability, resulting to crank of a ship was established during passage of top of a wave through it middle. It promoted occurrence rolling of the large amplitude (up to 30° and more), resonant amplitude, considerably exceeding size, (about 23°) at a situation of a ship board to waves. From a fragment of record of change in time of initial metacentric height of the cargo-ship «Poronaisk» at 8-ball waves (fig. 1) is obvious,

¹ This paper was translated from Russian with computer software and may be difficult to read in English. As the National Committee did not have resources for professional translation of this paper beyond the abstract, Russian text was used for peer review and included in the electronic version of the proceedings for further references.



as far as the strong fluctuations are undergone by size in these conditions.

It is necessary to note, that the excessive roll on following waves was marked and on other sea ships, for which the sharp reduction of initial metacentric height is characteristic, which has caused crank at passage of top of a wave through middle of a ship [6].

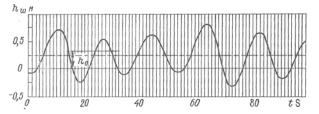


Figure 1. Change of initial metacentric height at movement of a ship on irregular passing excitement.

In the report the results of mathematical modeling of ship dynamics are discussed at periodic decrease of stability on following waves. The analysis is carried out for a ship with the S-figurative stability diagram on the basis of the differential equation rolling, including nonlinear spatial function, on waves. The basic attention is given to the analysis of dynamic pictures of ship behavior at a various level external disturbance.

2. MATHEMATICAL MODEL

Ship dynamics on waves is described nonlinear differential equations [2],[6],[7]:

$$(J_x + \lambda_{44})\ddot{\theta} + M_R(\dot{\theta}) + M(\theta, \varphi, t) = M_x(t) \quad (1)$$

where components are functions, describing a ship as dynamic system (inertial, damper, righting and disturbance components).

The most complex function in (1) is righting component which is included in the differential equation rolling. Righting component differs by essential nonlinearity, complexity and multivalence. Continuously changing in time and space, this function substantially determines result of integration of system (1) at study of physical pictures of interaction of a vessel with external environment.

The mathematical description nonlinear spatial function of the restoring moment on excitement is represented by the formula [1],[6],[7]:

$$M_{W} = M(\theta, \varphi, t) =$$

$$D [l(\theta, \varphi) + \Delta l(\theta, \varphi)] \cos(\sigma_{k} t - \varepsilon)];$$

$$l(\theta, \varphi) = 0.5 [l(\theta, \varphi)_{max} + l(\theta, \varphi)_{min}],$$

$$\Delta l(\theta, \varphi) = 0.5 [\Delta l(\theta, \varphi)_{max} + \Delta l(\theta, \varphi)_{min}];$$

$$M_{W} = \Phi(\theta, \varphi_{k}, t) = D l(\theta, \varphi, t),$$
(2)

where $\Delta l(\theta, \varphi)_{max}$ and $\Delta l(\theta, \varphi)_{min}$ – extreme meanings increment of lever stability, appropriate to a situation of a ship on a foot and top of a wave at various of angle course φ ; $l(\theta, \varphi, t)$ – lever of the righting moment determined for the various moments of time, ε phase: $\varepsilon=0$ and 2π – ship on a sole of a wave; $\varepsilon=\pi/2$ – on a forward slope; $\varepsilon=\pi$ – at top of a wave; $\varepsilon=3/2\pi$ – on a back slope.

General expression of increment function of a lever stability $\Delta l_w(h_w/l_\lambda,\theta,\phi)$, determining influence of excitement at various parameters of the form of the case and Froude, in a range of corners of a roll up to 60° inclusive and $\phi = \pm 0 \div 45^\circ$ looks like:

$$\Delta I_{W}(h_{W} / \lambda, \theta, \varphi) = B\left[\Phi\left(\frac{h_{W}}{\lambda}, \theta, \varphi_{k}\right) + \frac{\sum_{m=1}^{6} A_{m} f_{m}(\theta, \varphi_{k}) + \sum_{p=1}^{3} C_{p} E_{p}(\theta, \varphi_{k})\right], \qquad (3)$$

$$\Phi(h_{W} / \lambda, \theta, \varphi_{k}), \sum_{m=1}^{6} A_{m} f_{m}(\theta, \varphi_{k}), \sum_{n=1}^{8} B_{n} F_{n}(\theta, \varphi_{k}), \sum_{p=1}^{3} C_{p} E_{p}(\theta, \varphi_{k})$$



Included in model (2) functions, characterize the data of standard models and sums of the amendments on influence of the linear, square-law and cubic members of decomposition of a Taylor number on size increment of a lever stability, determined with the account diffraction and interference components at movement of a ship with an any angle course on waves. The geometrical interpretation of function $M(\theta, \varphi, t)$ as a complex spatial surface is given in a fig. 2.

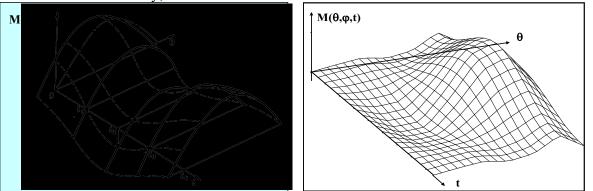


Figure 2. Nonlinear function, describe of righting moment on waves: all-round curves – the instant stability diagrams; a dotted line – temporary curve (section of a surface at θ = const); a dot-dash line – stability diagram on silent water.

Having calculated of increment Δl_w in a necessary range of corners of a roll, it is uneasy to establish lever stability l_w and to construct the appropriate diagrams for a situation of a ship at top and sole of a wave under the formula $l_w = l + \Delta l_w$, where l – lever stability, received by usual account on silent water [3], [4]. For calculation on computer formula to l_w have is

$$l_{w} = l + B \left(\Phi_{jk} + \sum_{i=1}^{17} A_{i} f_{ijk} \right),$$
(4)

where A_i for base model

$$A_{1}^{*} = L/B - 4.82, A_{2}^{*} = B/T - 2.67,$$

$$A_{3}^{*} = H/T - 1.3, A_{4}^{*} = \kappa - 0.7, A_{5}^{*} = \varphi - 0.692,$$

$$A_{6}^{*} = Fr - 0.28, A_{7} = B_{1} = A_{1}^{2}, A_{8} = B_{2} = A_{2}^{2},$$

$$A_{9} = B_{3} = A_{3}^{2}, A_{10} = B_{4} = A_{5}^{2}, A_{11} = B_{5} = A_{6}^{2},$$

$$A_{12} = B_{6} = A_{1}A_{2}, A_{13} = B_{7} = A_{2}A_{4},$$

$$A_{14} = B_{8} = A_{1}A_{6}, A_{15} = C_{1} = A_{2}^{3}, A_{16} = C_{2} = A_{3}^{3},$$

$$A_{17} = C_{3} = A_{5}^{3}$$
(5)

Here *i*, *j*, *k* - counters of cycles: *i* determines number of the amendment on a deviation of the characteristics of the form of the case and Froude number from given to standard model, θ – angle of heel, φ – angle of course.

3. OBJECTS OF MODELING AND INTERPRETATION OF DYNAMIC INTERACTION

As object of modeling the cargo ship having by the following the characteristics is accepted: main dimensions (length, width, draught, height of a board) – L = 96 m, B = 14,6 m, T = 6,0 m, H = 8 m: factors of completeness (block coefficient, coefficient of waterline completeness and middle-frame) $\delta=0,680$, $\alpha=0,790$, $\beta=0,975$; initial metacentric height h=0,33m; co-ordinate of a centre of gravity ZG=5,75m; ship speed 13,5 kn.

The characteristics of the diagram of static stability on silent water: shoulders static stability (through 10 degrees) $- l_{10}=0,05m$, $l_{20}=0,17m$, $l_{30}=0,33m$, $l_{40}=0,44m$, $l_{50}=0,38m$, $l_{60}=0,28m$; the maximal lever of static stability $l_{max}=0,44m$, angle of a maximum of the diagram $\theta_m = 40^\circ$, angle of decline the diagram $\theta_V = 72^\circ$.

Dynamics of waves was simulated with use of experimental data about group structure of a wave field. From all set of groups in structure



of irregular waves the greatest interest is represented by packages of waves including highest waves, which length is close to length of a ship. The group structure appears various depending on width of a spectrum of excitement [6]. So, for example, the waves of swell at width of a spectrum 0,2-0,3 can contain packages including up to 5-8 waves, close to regular (fig. 3A). At enough wide spectra more often the packages consisting of 3-4 irregular waves are marked. Among them it is possible to allocate short groups reminding packages of swell (fig. 3B), and structure containing almost regular waves, close on the sizes, (fig. 3C).

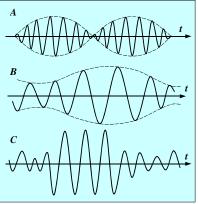


Figure 3. Waves packet in composition of irregular waves: A – packet of swell; B, C – packet almost regular waves.

The approximation reflecting features of an irregular wave field with the account inside of group structure and group properties of waves was represented by two-frequency spectrum [6]:

$$S(\sigma,\Sigma) = \frac{1}{4\pi^2} \int_{-\infty-\infty}^{\infty} R^*(t,\tau) e^{-i(\sigma\tau+\Sigma t)} dt d\tau \quad (6)$$

where $R^*(t,\tau)=M^*[\zeta(t) \ \zeta(t+\tau)]$ – correlation function of process $(M^*$ -operator of mathematical expectation); $\zeta_0(t)=\zeta(t)-M^*\zeta(t)$.

Equation (6) permits to interpret wave as probability process received by cyclic modulation of initial stationary process. The approached estimation of a spectrum for the established wave can be made on the basis of his representation as product [6]:

$$S(\sigma, \Sigma) \approx S(\sigma)S(\Sigma),$$
 (7)

where $S(\sigma)$ – frequency a spectrum which is taking into account inside group structure of wave; $S(\Sigma)$ – spectrum bending around of casual process taking into account group property of waves.

The nonlinear determined model describing a dynamic heel of a ship under influence of wave groups is developed on the basis of the differential equation (1). Thus the most meeting waves groups are described by:

$$\zeta = 2r_{W}(t)\sin\left(k_{1}^{*}\eta - \sigma_{1}t\right) \tag{8}$$

where $r_W(t)=r_W\cos(k_2^*\eta-\sigma_2 t)$ – amplitude, slowly varying in time, bending around, varying with frequency σ_2 and phase $k_2^*\eta$; $\sigma_1 t$ $\mu k_1^*\eta$ – frequency and phase of waves making group.

Such representation of group of waves allows to consider change of the righting moment on following waves (($\varphi=0$ or $\varphi=2\pi$) as the modulated fluctuation of his regular part determined by expression $D\Delta l(\theta,t)\cos(\sigma_k t+\varepsilon_0)$. As show calculations, it is possible also to consider size in this case slowly varying in time and approximate the function $M(\theta,t)$ as dependence:

$$M(\theta, t) = D \left[l(\theta) + \Delta l(\theta) \right] \sin \left(\sigma_k^* t + \varepsilon_0^* \right) \cos \left(\sigma_k t + \varepsilon_0 \right)$$
(9)

where σ_k^* and ε_0^* – frequency and phase bending around peak meaning(importance) $D\Delta l(\theta)$ of the restoring moment on passing excitement.

Dynamics of ship interaction to environment modeling on the base scenarios of development an extreme situation connected to sharp reduction of metacentric height. The developed model is based on construction of graph-structure of the scenarios [2]. As a result



of the analysis the scenarios described final graph was formulated

$$G_C = (S_C, P_S) \tag{10}$$

where S_C - strategy: P_S - transitions between them.

The elements column (8) allow to decide the following tasks:

- to present S_C as association of strategy $S_C^{\ y}$ and moments of management tj;
- to present P_S as structure

$$P_S \le S_C \times S_C \tag{11}$$

describing transitions between strategy with the help of displays of set S_C .

4. RESULTS OF MODELING

The mathematical modeling was carried out according to the developed plan of experiment for two scenarios of storm development submitted on a fig. 4. The first scenarios (fig. 4, curve 2) assumed gradual increase and attenuation of storm, second (fig. 4 a curve 1) – slower increase passing in brightly expressed peak with the subsequent intensive attenuation.

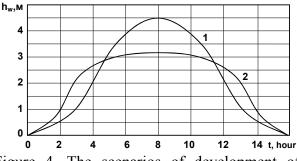


Figure 4. The scenarios of development of storm having monotonous character (2) and brightly expressed peak (1).

In process of modeling the influence of parameters of excitement on change of instant ordinates of the stability diagram (fig. 5) in conditions of development of storm was investigated. The scenarios of researched situations provided also various cases of ship behavior at displaced of a cargo owing to sharp decrease of initial stability. The special attention addressed on transformation of the stability diagram at passage of top of a wave through middle of a vessel. On the basis of results of modeling is formulated criteria basis prevention of occurrence ensuring of dangerous situations owing to sharp reduction of initial metacentric height at movement of a ship on following waves. The stability criteria are used by development of knowledge base of the on-board intelligence monitoring system of ship dynamics on waves.

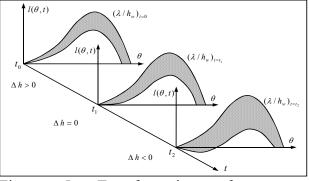


Figure 5. Transformation of extreme significance of ordinate of stability diagram owing to intensity waves.

Ship dynamics for the various scenarios of a cargo displaced is submitted in a fig. 6 and 7. Here dark curve shows process rolling of a ship, and light line - change of speed change of heel. The first scenarios (fig. 5) corresponds to occurrence constant of the heel moment. Under his influence there is a change attractor of dynamic system. Thus of a ship rolling occurs about some of a angle static heel. If in these conditions to apply the heel moment from a wind, overturning a ship is possible. If the wind moment will have an opposite mark, the transition of a ship in another attractor is possible and the situation will be more dangerous, than shown on a fig. 6. Other scenarios (fig. 7) - even more dangerous. In these conditions the continuous increase of a roll is marked in process of increase of the heel moment from displaced of a cargo. Even without the account wind of the heel moment the situation appears menacing. The continuous increase of a roll from displaced of a cargo at the end will result in capsizing a ship.



The results of modeling of not regular situations at movement of a ship on irregular waves are submitted in a fig. 8. Four characteristic situations here are selected which can be observed of a under operating conditions researched ship. They represent non-stationary regime of fluctuations of metacentric height under influence casual disturbance (passage of waves packages such as (A,C) with a continuously increased steepness). The shading in these figures specifies area of negative meanings of metacentric height. The limiting case of change of metacentric height (fig. 8D) to complete stability loss of a ship on waves (on classification [4], [5], [8]).

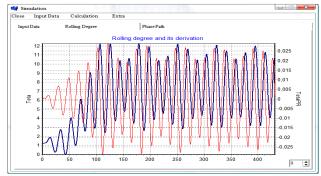


Figure 6. Replacement of attractor research dynamic system for influence of momet from

displace cargo and decrease of initial metacentric height.

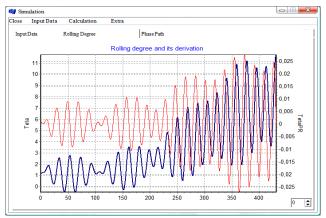
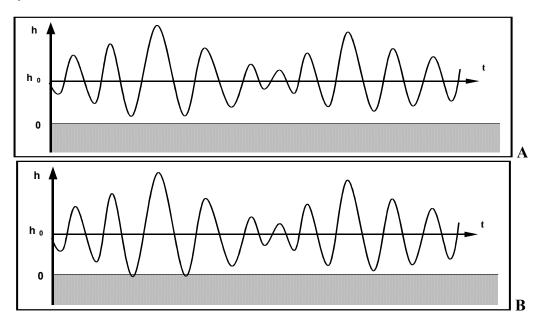


Figure 7. Increase of a heel at continuous increase of the heel moment from displace cargo and negative initial metacentric height.

The results of modeling have shown, that the occurrence «swing effect», caused of a ship crank, is found out in situations when size of initial metacentric height of a ship and the ratio of waves parameters in a package achieve the following meanings:

$$h_0 \le 0.3 \text{ m}, \lambda/L = 0.8 \div 1.3, h_W/\lambda \ge 0,$$
 (12)



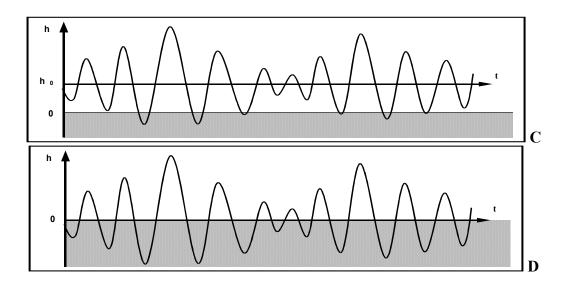


Figure 8. Curve changes of metacentric height for a various level external disturbance: A – the case of normal operation (phenomenon of crank is not shown); B – situation, when the separate meanings of metacentric height reach zero meanings; C – typical situation, when at influence of a package of waves there are separate meanings zero; D - limiting case appropriate to complete deterioration of initial stability.

The parities(ratio) (10) can be specified for a concrete vessel and are fixed in a basis of construction of indistinct logic models of processing of knowledge at functioning onboard IS of seaworthiness control [2]:

$$X \to Y(Z), \tag{13}$$

where X, Y, Z – fuzzy sets determined on universal multitude U, V, W, Z – alternative decision.

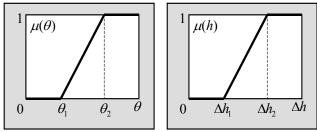


Figure 9. Membership function for antecedent of logic rules

Antecedent of implication X represents check of performance of conditions formulated on the basis of ratio (12). Consequent of implication determines occurrence of an extreme situation and development of the appropriate practical recommendations, including performance of operations on modeling and visualization of a situation. The membership functions describing conditions of occurrence of an extreme situation are submitted in a fig. 9.

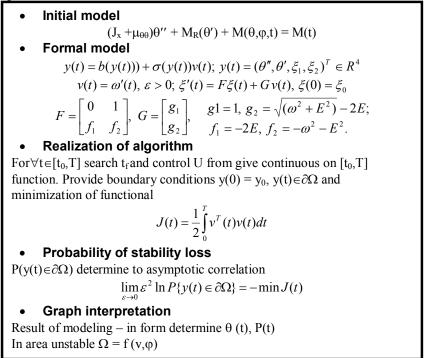
Here meanings θ_1 and θ_2 characterize a range of transition to complete loss of initial stability, Δh_1 and Δh_2 are appropriate meanings decrease of metacentric height at the moment of passage of top of a crest wave through of a ship middle.

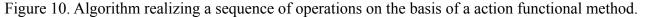
5. ESTIMATION OF STABILITY OF OSCILLATORY MOVEMENT OF A SHIP ON WAVES

The estimation of stability of oscillatory movement of a ship in conditions of periodic deterioration initial stability was carried out on the basis of a functional action method. The developed algorithm allows to calculate probability of capsizing in process of development of a ship fluctuations caused by negative occurrence of meanings of metacentric height. The stochastic algorithm realizing a sequence of operations at



performance of procedures of a functional action method is submitted in a fig. 10.





Here $(J_x + \mu_{\theta\theta})\theta''$, $M_R(\theta')$, $M(\theta, \phi, t)$, M(t) – inertia and dampfer components, righting and revolting components of mathematical model; ξ – of a component of Gauss casual process $\xi(t) \in R_2$, forming which filter is determined in parameters of dominant frequency w and width of a disturbance spectrum E; v(t) – «white noise» of small intensity; w – Winer process; e – small parameter; W – area of unstable.

The analysis of stability of oscillatory rolling regime of the ship on waves in conditions of periodic decrease of initial metacentric initial height has allowed to establish a number of characteristic laws of researched nonlinear dynamic system. These laws are determined by features of group structure of waves and intensity of oscillatory regimes. The results of computing experiments allow to construct and to analyze trajectories of dynamic system on a phase plane [4], [5]. Most typical by oscillatory regime are the cases of occurrence attractors, describing dynamics onestable, bi-stable and three-stable systems (fig. 11), and also characteristic temporary curve and one-dimensional displays from points

appropriate to the maximal meanings of amplitude for (n+1) and *n*-th of intervals of time for attractor of sets [3].

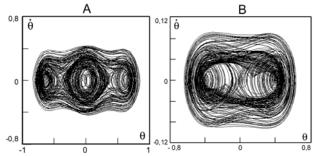


Figure 11. Phase portraits three-stable (A) and bi-stable (B) of dynamic systems describing behavior of a ship at periodic change of metacentric height in an extreme situation.

The first oscillatory regime (fig. 11A) is characteristic by continuous transition between three stable condition of dynamic system (three-stable system). This regime arises in a case, when the periodic deterioration of initial metacentric height (situation C in a fig. 8) is marked. In this case ship can be or in an initial condition of steady balance ($\theta = 0$), or with a heel on the right or left board ($\theta = \theta_1$ or $\theta = \theta_2$). The second oscillatory regime – transition to



bi-stable system (fig. 11B). In this case because of sharp deterioration of initial metacenter height at influence of packages of large waves (situation D in a fig. 8) the ship can not be in a direct situation and periodically receives a heel on right or on the left board ($\theta = (\theta_1)_{max} =$ $(-\theta_1)_{max}$). The typical picture of attractor change at formation of bi-stable system is shown in a fig. 6.

The interesting law established during computing experiment, consists in modeling spatial- temporary behavior of a ship on the basis of the central synergetic theorem, of complex system, determining dynamics, as interrelation of growing and its fading configurations with order parameters [3]. Is that all researched established. spatialtemporary condition of system operate in order parameters (principle of submission). The transition to order parameters in complex system will carry out to compression of the information and to significant reduction of number of freedom degrees. Thus the order parameters determine movement of system elements, which, in turn, in common (cooperation) characterize action of order parameters (principle circular of cause) [10].

6. ALGORITHM OF THE CONTROL OF A CRITICAL SITUATION

The control of occurrence of a critical situation connected to periodic deterioration initial stability was carried out on the basis of the analysis of the mathematical modeling data. The given below approach is realized on the basis of the Bayes formula [5] and can be used at construction of algorithm of the control of ship dynamics in onboard IS of maintenance of ship seaworthy qualities functioning in real time regime. In this case algorithm uses the received of information from sensors measuring system. The sequence of operations of the control is represented as the following steps.

Step 1. To calculate probability that at number of events m the reason B (occurrence of negative meanings decrease of metacentric height takes place at movement of a ship on following waves):

$$P(B) = \sum_{i=1}^{m} P(B_i) = 1$$
(14)

Step 2. To accept $P(B_i)$ – a priori probability that the reason Bi took place, and $P(A|B_i)$ – conditional probability that A (the occurrence crank of a ship on waves) comes owing to the reason B_i . provided that A has taken place

Step 3. To calculate a posteriori probability $P(B_i|A)$ provided that A comes owing to the reason B_i :

$$P(B_i|A) = P(B_i)P(A|B_i) = P(A)P(B_i|A) P(B_i|A) = P(B_i)P(A|B_i) / P(A),$$
(15)

where $P(B_i)$ – a priori probability that the reason B_i took place

Step 4. To determine probability of event *A*:

$$P(A) = \sum P(B_i) P(A \mid B_i)$$
(16)

Step 5. To substitute size P(A) in expression for $P(B_i|A)$. Then under Bayes theorem we have:

$$P(B_i | A) = P(B_i)P(A | B_i) / \sum P(B_i)P(A | B_i)$$
(17)

Step 6. To generate model of a situation «number of realizations N – a priori probability P», having accepted a priori distribution on the data of dynamic measurements. In a case of absence of these data as a first approximation to accept a priori probability in each realization equal 0,5, that reflects a case, when with equal probabilities both considered situations are possible. After reception of the additional data n a posteriori probability of their occurrence to calculate under the formula:



$$(n+1) / [n+2 - (n/N)]$$
 (18)

The initial information for maintenance of functioning of algorithm is formed by data processing of dynamic measurements during IS functioning.

7. CONCLUSION

On the basis of the carried out research it is possible to make the following basic conclusions:

1. At movement on following waves of ships with S-figurative stability diagram chart at the certain meanings of initial metacentric height the phenomenon of crank resulting to occurrence of an extreme situation (opportunity of capsizing owing to displace of a cargo and other adverse factors) is marked.

2. The occurrence and development of an extreme situation owing to decrease of initial stability is possible in conditions intensive external disturbance, packages, shown at influence, of waves in structure of irregular waves. A ratio of dangerous waves in a package to determined by features of dynamics of interaction, geometrical and hydrodynamic characteristics of a ship.

3. Depending on wave structure the various regime of oscillatory movement of a ship are possible. The first regime represents three-stable system. She is formed during periodic decrease of metacentric height. The second regime – typical bi-stable system, at which formation a ship appears or with a heel on right, or on the left board. Occurrence of attractor sets in considered dynamic system represents the large danger from the point of view of stability loss on waves. The analysis of probability of overturning can be carried out with use of a functional action method.

4. On the base the data of modeling can be constructed of criteria ratio describing occurrence of a considered extreme situation, and to use these data at the control of ship dynamics on waves with the help to on-board IS, functioning on the basis of knowledge models.

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