NEW GENERATION INTACT STABILITY CRITERIA: A STEP FORWARD

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ABSTRACT

For contributing towards the development of new generation intact stability criteria at the International Maritime Organization (IMO), the authors provide draft vulnerability and performance-based criteria with their sample calculations for harmonic resonance under dead ship condition and broaching. Here the draft performance-based criteria are based on first principle approach using capsizing probability as their index and the draft vulnerability criteria using simplified but theoretical approach are adjusted to realise consistent safety level with the result of the draft performance-based criteria. For parametric rolling, numerical results of comparative calculation of several prediction methods are reported with guides for vulnerability criteria and performance-based criteria.

Keywords: *capsizing probability, broaching, harmonic resonance, parametric rolling*

1. INTRODUCTION

At the International Maritime Organization (IMO), new-generation intact stability criteria are now under development for allowing the use of first-principle tools for three major capsizing modes: harmonic resonance under dead ship condition, manoeuvring-related problem such as broaching and stability variation problems such as parametric rolling. It was already agreed that the new criteria should consist of vulnerability criteria and performance-based criteria and be supplemented with operational guidance. (IMO, 2008a) Now the intersessional correspondence group is requested to collect draft criteria together with sample calculation results for specified sample ships. Prior to this requirement, the delegation of Japan (2008) provided an example of the new criteria at the 51st session of Sub-Committee on Stability, Load Line and on Fishing Vessel Safety (SLF). This paper describes its technical background and further research progress after the document submission.

For dead ship stability, the weather criterion was originally developed and has been widely used within the framework of the current intact stability code. However, its limitation due to empirical estimation of some coefficients sometimes induces doubt of its applicability to unconventional ships. Thus, simplified formulae for effective wave slope coefficient and nonlinear roll damping are expected as alternatives to current empirical formulae. If the safety level of modified weather criterion is properly adjusted, it can be utilised as a vulnerability criterion. As a performance-based criterion, capsizing probability calculation based on piece-wise linear approximation of restoring arm curve with standard wind-wave statistics could be recommended. In this paper, for passenger and cargo ships, firstly capsizing probabilities are calculated using the piece-wise linear approach and then the critical wind speed of the modified weather criterion is adjusted to realise consistent safety level.

For manoeuvring-related problems such as broaching, estimation of surf-riding threshold in regular following waves could be used as a vulnerability criterion. This is because the surf-riding is a prerequisite for broaching. As a performance-based criterion, probability calculation of capsizing due to broaching in irregular waves seems to be appropriate. Here the probability can be calculated by integrating a joint probability density of local wave height and wavelength within the deterministic zone for capsizing due to broaching. The remained problem is a way to specify the wave steepness and wave length for deterministic vulnerability criteria so that comparison studies are carried out for two sample ships.

For stability variation problems such as parametric rolling, simple analytical formulae for predicting the occurrence and the magnitude of parametric rolling in regular longitudinal waves were already established within the uncoupled roll model. These formulae seem to be suitable for the vulnerability criteria for this phenomenon but the mutual relationship with more rigorous model has not yet been established particularly in head waves. Thus, comparison results are reported in this paper.

2. STABILITY ASSESSMENT FOR DEAD SHIP CONDITION

2.1 Capsizing probability calculation method as draft performance-based criterion

If we calculate capsizing probability in beam wind and waves, it can be a conservative measure for stability under dead ship condition, which should cover all possible combination of wind and wave directions. (Umeda et al., 2007a) It is also noteworthy here that the reliable estimates based on the Monte Carlo simulation using nonlinear time domain code requires prohibitively large computational efforts because capsizing probability of a practical ship is small. Thus the authors propose to use a capsizing probability calculation method based on piece-wise linear approximation of nonlinear restoring arm in beam wind and waves (Belenky, 1993 and Paroka et al., 2006) as the way to treat the problem of rarity (Japan, et al 2007). This piece-wise linear approximation allows us to utilise analytical solutions for minimising its computational load. Although Paroka et al. (2006) assumes roll angular velocity at the out-crossing a roll angle as the absolute value of a Gaussian process, experimental observation rather suggests that it has the Rayleigh distribution as shown in Fig. 1 (Maeda, 2009) and recently theoretical explanation has been published. (Belenky, 2008) Based on this fact, capsizing probability within the time duration of T is calculated with the formulae shown in Appendix 1. Here we assume that the forced roll in the range having a negative restoring slope can be ignored because of absence of resonance. (Belenky, 1993) In addition, the roll damping is estimated with Ikeda’s semi-empirical method (Ikeda, 2004) and the effective wave slope coefficient is estimated with a strip theory. The wind heeling lever is estimated with a formula of the IMO weather criterion (IMO, 2008b). Waves are assumed to have the ITTC spectrum and its significant wave height and mean period are determined from the wave statistics of the North Atlantic. Wind velocity is assumed to have the Davenport spectrum and its mean wind velocity is done to be fully correlated with the significant wave height. The justification of the latter assumption can be found in Ogawa (2009). Finally, the annual capsizing probability is calculated by using the capsizing probability within the time duration of T in stationary wind and waves and long-term statistics of wind and waves.

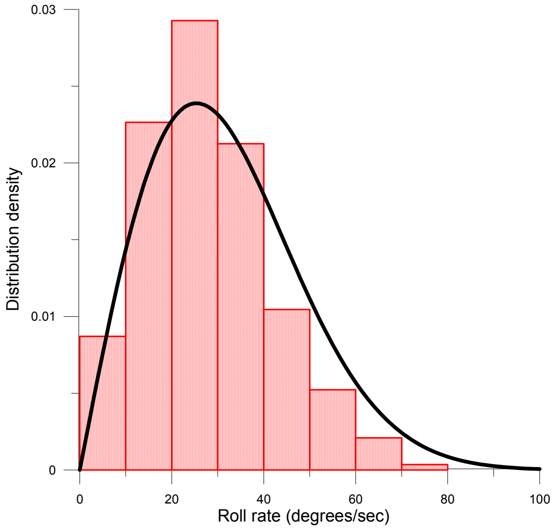

Figure 1. Probability of density function of roll angular velocity at the up-crossing roll angle of 10 degrees obtained from model experiments of the ONR tumblehome vessel.

2.2 Modified weather criterion

Weather criterion has been widely applied since the IMO adopted it as Res. A. 562 in 1985. This criterion was originally developed as Japan’s domestic standard and then the roll equation used in the USSR’s standard was partly incorporated into it. (IMO, 2008b) The applicability of this formula however, is questioned because recent ships having large breadth-draught ratios are outside the database used for development of the weather criterion. Recently the IMO (2006) allows us to alternatively use model experiments for estimating the roll angle as well as wind heeling moment and decided to keep the framework of the weather criterion as it is at least in the short-term revision of the Intact Stability (IS) Code. Since the introduction of the new generation intact stability criteria is categorised as the long-term revision, we may attempt to review the weather criterion not only its coefficient estimation methods but also its safety level specified as the wind velocity.

For updating the weather criterion, the authors propose to use a simplified formula of the effective wave slope coefficient based on the Froude-Krylov assumption (Umeda & Tsukamoto, 2008) and Ikeda’s simplified formula for roll damping (Kawahara et al., 2008). The former can be justified for large wave length to ship breadth ratio (Tasai, 1965) and the latter is based on Ikeda’s empirical method (Ikeda, 2004) but does not require to solve a boundary value problem. Other parameters of the IMO weather criterion, except for the wind velocity, are kept as it is.

2.3 Sample calculations and obtained safety level

For determining the mean wind velocity of the modified weather criterion, sample calculations were executed for a car carrier, a RoPax ship and a large passenger ship. The principal particulars of these ships are shown in Table 1. Here the RoPax ship is assumed not to be operated when the significant wave height is 10 metres or over. The annual capsizing probabilities and the critical wind velocities of the modified weather criterion are shown in Fig. 2-7.

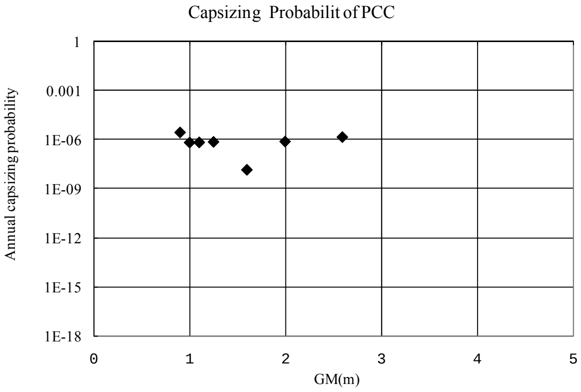

Figure 2. Capsizing probability of the car carrier in irregular beam wind and waves.

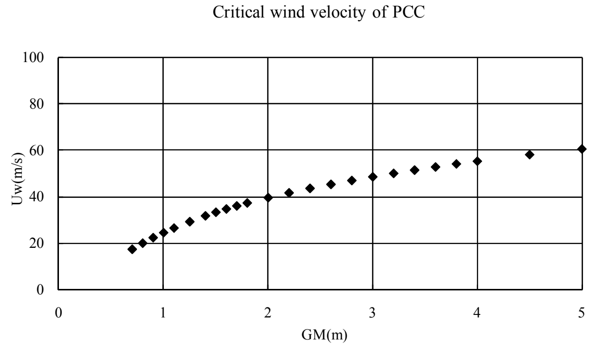

Figure 3. Critical wind velocity of the modified weather criterion for the car carrier.

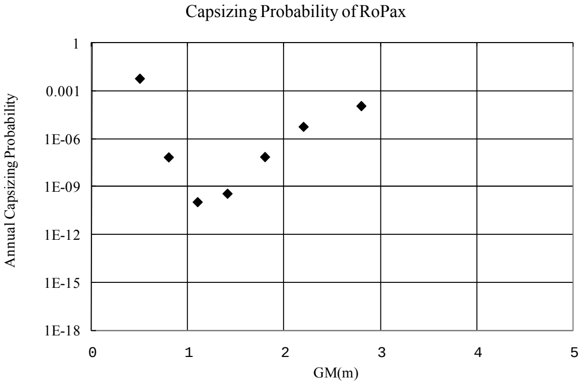

Figure 4. Capsizing probability of the RoPax ship in irregular beam wind and waves.

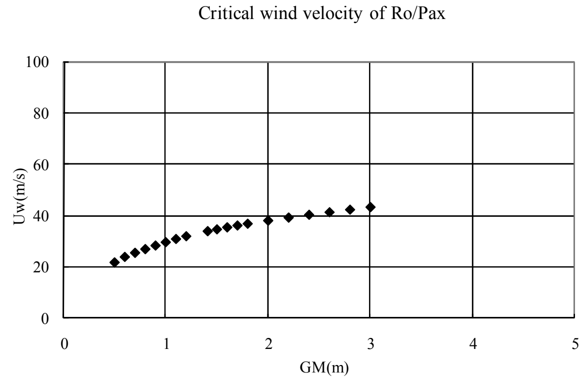

Figure 5. Critical wind velocity of the modified weather criterion for the RoPax ship.

For the car carrier, if we assume that the required safety level in annual capsizing probability is 10^{-6}, the critical wind velocity is determined as 25 m/s. Similarly, the critical wind velocities of the RoPax ship and the large passenger ship are 26 m/s and 32 m/s, respectively. Since difference in the critical wind velocity of these three ships is not large, the mean wind velocity of 32 m/s can be tentatively recommended at least for these ships. Further sample calculations are required. It is noteworthy here that metacentric height required by the current IMO weather criterion is slightly higher than the modified weather criterion here.

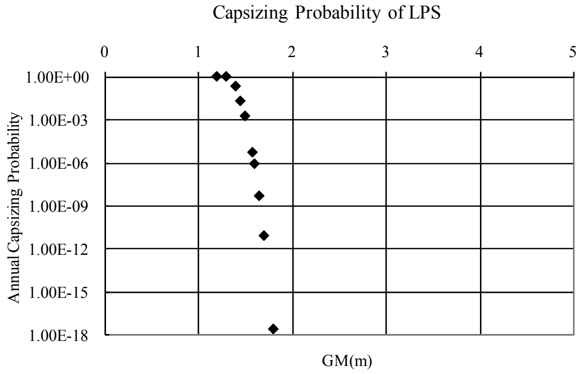

Figure 6. Capsizing probability of the large passenger ship in irregular beam wind and waves.

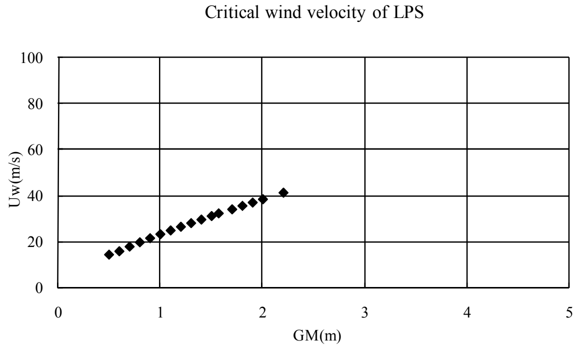

Figure 7. Critical wind velocity of the modified weather criterion for the large passenger ship.

3. STABILITY ASSESSMENT RELATING TO BROACHING

3.1 Capsizing probability calculation method as draft performance-based criterion

Broaching at occurs as a result of surf-riding at relatively large heading speed. Another way to broaching is a development of yaw instability that can occur both at low and high speed (Spyrou 1996). As the latter can be generally avoided with appropriate operational efforts such as a differential control (Spyrou, 1997) and an optimal control (Maki & Umeda, 2009), the former case should be discussed here.

The problem of rarity can be addressed by separation of the problems of dynamics and probabilistic description of broaching (Umeda et al 2007c, Themelis and Spyrou 2007).

The method proposed by Umeda et al. (2007c) for calculating broaching probability used the fact that surf-riding or broaching occurs within one or two waves and successfully validated it with the Monte Carlo simulation. Their method was extended to deal with capsizing probability due to broaching. (Umeda & Francescutto, 2008) Firstly, the deterministic critical zone of capsizing due to broaching is identified by repeating numerical simulation using a coupled surge-sway-yaw-roll model in the time domain. Secondly, the probability density of local wave steepness and wave length, which is obtained from Longuet-Higgins's theory, is integrated within the deterministic critical zone. Here the ship resistance, propeller thrust and calm-water manoeuvring coefficients should be estimated with captive model experiments or equivalent. The wave statistics of the North Atlantic is used for calculating long-term probability.

3.2 Bifurcation for surf-riding threshold as a vulnerability criterion

Since the surf-riding is a prerequisite of broaching and critical speed of surf-riding increases with heading angle from the wave direction, the surf-riding threshold in regular dead following seas is a candidate of vulnerability criterion for broaching. It is well established that this threshold can be regarded as a heteroclinic bifurcation of uncoupled surge model. (Grim, 1951; Makov, 1969) Numerical bifurcation analysis is successfully applied to this problem (e.g. Umeda et al., 2007b). Kan (1989) applied the Melnikov method and obtained an analytical formula by approximating the surge damping force as linear. While Spyrou (2006) extend it with surge damping as a cubic function, this paper provides a generalised formula allowing the surge damping as the nth order polynomial. Maki and Umeda (2008) provided formulae based on a piece-wise linear approximation of wave-induced surge force. If we use one of these simple formulae, the surf-riding threshold in dead following waves can be obtained even with a spread-sheet-type calculation. Here the ship resistance and propeller thrust should be provided in advance.

It is noteworthy here that the wave steepness and the wavelength used for surf-riding threshold estimation in regular following waves should be specified preferably with comparisons with the results from a probabilistic method. Furthermore, it could be practical to develop additional criterion for judging vulnerability of broaching of a surf-ridden ship. Broaching could occur when a ship is surf-ridded and the wave-induced yaw moment exceeds the rudder-induced yaw moment. (Motora et al., 1982: Renilson, 1982) Thus an additional vulnerability criterion could be added to request the amplitude of wave-induced yaw moment smaller than the rudder-induced yaw moment at the rudder angle limit

3.3 Sample calculations and obtained wave condition

For determining the wave steepness and wavelength for the surf-riding threshold estimation, sample calculations were executed for a fishing vessel known as the ITTC Ship A2 and the ONR tumblehome vessel. The principal particulars of these ships are shown in Table 2. Here the down-flooding angle of the ONR tumble home is assumed to be 90 degrees. The capsizing probabilities in irregular waves together with the surf-riding threshold in regular following waves, are shown in Figs. 8-9.

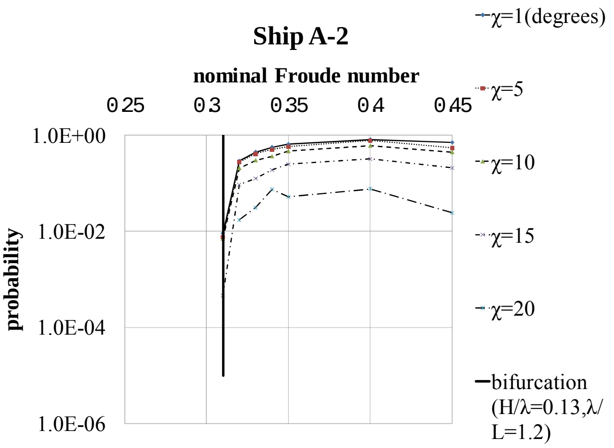

Figure 8. Hourly Probability of capsizing due to broaching for the ITTC A2 Ship in the Northern Atlantic and its surf-riding threshold in regular waves.

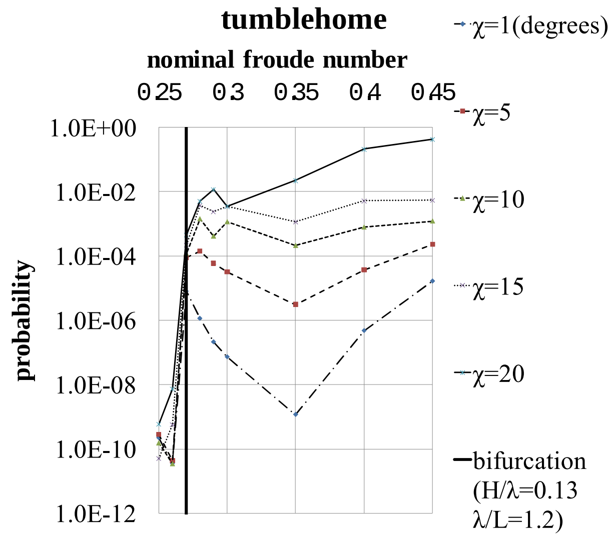

Figure 9. Hourly Probability of capsizing due to broaching for the ONR tumblehome vessel in the Northern Atlantic and its surf-riding threshold in regular waves.

For these two ships if we assume that the required safety level in hourly capsizing probability is 10^{-6}, the relevant critical speed for surf-riding can be obtained with the wave steepness of 0.13 and the wave length to ship length ratio of 1.2. Further sample calculations are required to obtain a final proposal.

4. COMPARISON OF SEVERAL ESTIMATION METHODS OF PARAMETRIC ROLLING

4.1 ITTC recommended formulae for the amplitude of parametric rolling

It is desirable to use an analytical formula for determining the amplitude of parametric rolling. For this purpose, based on the work of Spyrou (2005) the International Towing Tank Conference (ITTC) recommended the two formulae as a guideline. (ITTC, 2005) These are derived with a standard technique of nonlinear dynamics, such as an averaging method, from uncoupled roll equation with time-varying metacentric height and linear roll damping. One of these uses a cubic fitting of calm-water restoring arm and the other does a quintic fitting. These formulae are recommended primarily for following waves but their applicability to head waves is examined in this paper.

4.2 Numerical simulation in the time domain for parametric rolling

Several numerical codes are available for this mode. In this work a numerical simulation code of uncoupled roll model (Hashimoto et al., 2007) is used with restoring arm variation where heave and pitch are implicitly taken into account. Here the restoring arm variation is estimated as the sum of the Froude-Krylov component using heave and pitch motion calculated by a strip theory, radiation and diffraction effects calculated by a strip theory for a heeled hull.

4.3 Sample calculations

The comparison study among the ITTC formulae, the numerical simulation in the time domain and the model experiments are executed for a containership known as the ITTC A1 ship and the car carrier used in 2.3. For the ITTC formulae, the metacentric variation is calculated with the Froude-Krylov calculation with the static balance in heave and pitch and its simplified formula for the containership and car carrier, respectively. The linear roll damping is estimated with Ikeda's simplified formula (Kawahara et al., 2008) with Takahashi's empirical correction of its forward speed effect. For the numerical simulation in the time domain, the roll damping is estimated with two different methods; Ikeda's method and its simplified version. The model experiments for the containership and the car carrier are cited from Umeda et al. (2008) and Hashimoto et al. (2007), respectively.

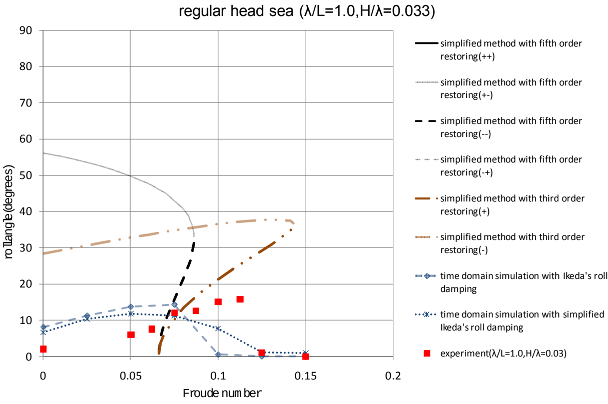

Figure 10. Comparison in parametric rolling amplitude among the several prediction methods and the experiment for the ITTC A1 containership in regular head waves.

The comparisons among these in regular head waves are shown in Figs. 10-11. The ITTC formulae significantly overestimate in amplitude of parametric rolling both the experiment and the numerical simulation. The occurrence zones, however, are reasonably predicted. This is because the ITTC formulae can capture bifurcation structure of parametric rolling. The selection of curve-fitting of calm-water restoring arm changes their qualitative and quantitative natures so that this could be one of the major drawbacks of these formulae. The numerical simulation with Ikeda's simplified formula of roll damping shows acceptable agreements with the model experiment. Forward speed effect based on Takahashi's method provides better results of parametric rolling than Ikeda's together with other assumptions of mathematical modelling here.

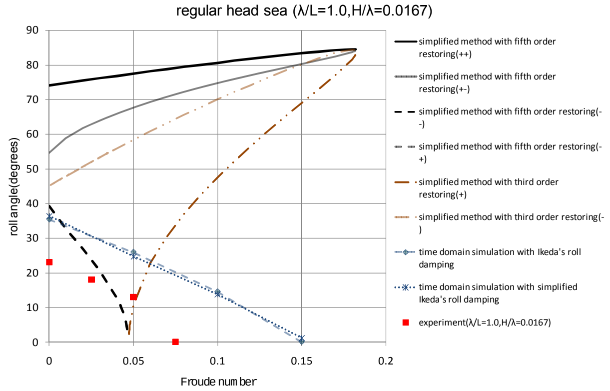

Figure 11. Comparison in parametric rolling amplitude among the several prediction methods and the experiment for the car carrier in regular head waves.

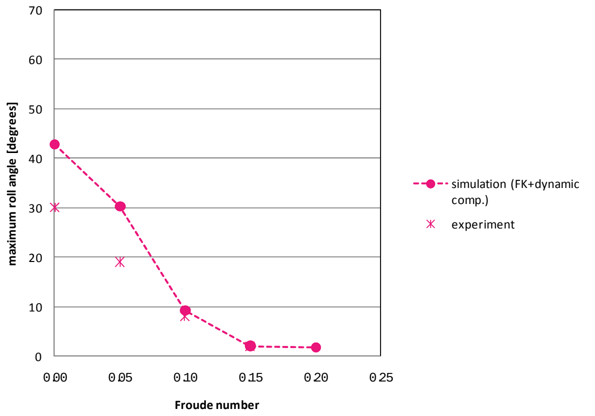

Figure 12. Comparison in parametric rolling amplitude between the calculation and the experiment for the car carrier in irregular head waves.

The final goal of performance-based criterion is to accurately predict the maximum amplitude of parametric rolling in irregular head waves or probability of exceeding critical roll angle of cargo damage. For this purpose, comparisons in parametric rolling in irregular head waves between the experiment and numerical simulation in the time domain are shown in Fig. 12 for the car carrier. Here ensemble averages of 5 realisations of calculated maximum amplitude during half an hour are plotted. This is because parametric rolling could be practically non-ergodic.

5. CONCLUDING REMARKS

1) For harmonic resonance under dead ship condition, a weather criterion using new formulae of roll damping and effective wave slope coefficient and a capsizing probability calculation method using piece-wise linear approximation of restoring arm in beam wind and waves are tabled with sample calculations as candidates of its vulnerability and performance-based criteria, respectively. The safety level of the former is adjusted with the latter.

2) For broaching, a bifurcation analysis for uncoupled surge model in regular following waves and a calculation method of capsizing probability due to broaching are provided with sample calculations as candidates of its vulnerability and performance-based criteria, respectively. The safety level of the former is adjusted with the latter.

3) For parametric rolling, applicability of the ITTC-recommended formulae and numerical simulation in the time domain are examined with sample calculations. The former could overestimate the latter.

6. ACKNOWLEDGEMENTS

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8. APPENDIX 1

The formula of capsizing probability within the duration of T in stationary seaway, *P(T)*, is as follows:

$$P(T)=1-\exp(u_{cap}\cdot T)l \qquad \text{(A1)}$$

where

$$u_{cap}=u_{capl}+u_{capw} \qquad \text{(A2)}$$

$$u_{capl}=u_{l}P_{A}(A>0) \qquad \text{(A3)}$$

$$u_{capw}=u_{w}P_{A}(A<0) \qquad \text{(A4)}$$

$$u_{l}=\frac{1}{2\pi}\sqrt{\frac{V_2}{V_1}}\exp\left[-\frac{(\phi_M-\phi_D)^2}{2V_1}\right] \qquad \text{(A5)}$$

$$u_{w}=\frac{1}{2\pi}\sqrt{\frac{V_2}{V_1}}\exp\left[-\frac{(-\phi_M-\phi_D)^2}{2V_1}\right] \qquad \text{(A6)}$$

$$P_A(A>0)=\int_0^{\infty}f(A)dA \qquad \text{(A7)}$$

$$f(A)=\frac{(\lambda_1-\lambda_2)^2(A-C_4)}{V_2}\exp[-\frac{(\lambda_1-\lambda_2)^2(A-C_4)^2}{2V_2}] \qquad \text{(A8)}$$

$$C_4 = -\frac{\lambda_2}{\lambda_1 - \lambda_2}\phi_D \tag{A9}$$

$$\lambda_{1,2} = -\alpha_E \pm \sqrt{\alpha_E{}^2 + \omega_0{}^2 k f_1} \tag{A10}$$

Here the first and second ranges have positive and negative slopes of restoring arm, respectively. ϕ_M: border of ranges, ϕ_D: mean heel angle, V_1: variance of roll angle in the first range, V_2: variance of roll angular velocity in the first range, α_E: equivalent linear roll damping coefficient in the first range, ω_0: natural roll frequency in the first range, k_{f1}: slope of the restoring arm in the second range.

9. APPENDIX 2

A generalised formula of surf-riding threshold using the Melnikov method is presented below. Here assuming the ship resistance, R, and the propeller thrust coefficient, T, as,

$$\begin{cases} R(u) \approx \sum_{i=1}^{n} r_i u^i = r_1 u + r_2 u^2 + \cdots \\ K_T(J) \approx \sum_{i=0}^{n} \kappa_i J^i = \kappa_0 + \kappa_1 J + \kappa_2 J^2 + \cdots \end{cases} \tag{A11),}$$

the effective thrust, T_e, becomes;

$$T_e(u;n) = \sum_{i=0}^{n} \frac{(1-t_p)(1-w_p)^i \rho \kappa_i u^i}{n^{i-2} D^{i-4}} \tag{A12).}$$

Where J is defined as follows;

$$J = \frac{(1-w_p)u}{nD} \tag{A13).}$$

Here D : propeller diameter, n : propeller revolution number, t_p : thrust deduction coefficient, u : ship forward velocity, w_p : effective propeller wake fraction and ρ: wave density. Making use of Melnikov's technique (Guckenheimer & Holmes, 1983) yields;

$$2\pi \frac{T_e(c_w;n) - R(c_w)}{f} = \sum_{i=1}^{n}\sum_{j=1}^{i} C_{ij}(-2)^j I_j \tag{A14),}$$

where

$$C_{ij} \equiv \frac{c_i}{fk^j}\binom{i}{j}\frac{(fk)^{j/2}}{(m+m_x)^{j/2}} c_w^{i-j} \tag{A15),}$$

$$c_i = -\frac{(1-t_p)(1-w_p)^i \rho \kappa_i}{n^{i-2} D^{i-4}} + r_i \tag{A16),}$$

$$I_j = \sum_{\substack{p=0 \\ p=2q,\, p\neq 2q+1}}^{m} \frac{\pi}{2^{k+p-1}}\binom{m}{p}\binom{p}{p/2} \text{ ,for } j = 2m \tag{A17).}$$

$$I_j = \sum_{p=0}^{m}\binom{m}{p}\frac{2(-1)^p}{2p+1} \text{ ,for } j = 2m+1 \tag{A18).}$$

Here c_w: wave celerity, f : amplitude of wave-induced surge force, k: wave number, m; ship mass and m_x: added mass in surge. Eq (A14) represents the condition of surf-riding threshold.

Table 1. Principal particulars of sample ships used for the dead ship condition.

Items	car carrier	RoPax ship	large passen-ger ship
L_{bp} (m)	192	170	242.5
Breadth (m)	32.26	25	36
Draught (m)	8.18	6.6	8.4
Block coefficient	0.543	0.521	0.705
GM (m)	1.25	1.41	1.579
Roll Period (s)	22.0	17.9	23.0

Table 2. Principal particulars of sample ships used for broaching and parametric rolling.

Items	ITTC Ship A1	ITTC Ship A2	ONR tumble-home
Lpp (m)	150	34.5	154
Breadth (m)	27.2	7.60	18.78
Draught (m)	8.5	265	5.494
Block coefficient	0.667	0.597	0.535
GM (m)	1.0	1.00	2.07
Roll Period (s)	20.1	7.40	11.99