A vulnerability criterion of ship yawing in following waves

K. J. Spyrou, *National Technical University of Athens,* k.spyrou@central.ntua.gr
V. Margari, *National Technical University of Athens,* vickyd.margari@gmail.com

ABSTRACT

A vulnerability criterion for avoiding dynamic yaw instability in following/quartering waves is proposed. This criterion can provide protection for cases of broaching-to of medium to larger size ships, where substantial unwanted yaw is developed without the ship being involved in surf-riding. Cases as these are not addressed by the recently finalised Second Generation Intact Stability Criteria of IMO. The underlying mechanism of instability discussed here is a parametric yaw phenomenon, that can be treated analytically with satisfactory accuracy on the basis of a linear manoeuvring mathematical model for regular waves. The criterion was evaluated against simulations. It could be employed as an additional vulnerability check for broaching-to during early design.

Keywords: *Ship dynamics, yaw motion, course-keeping, following seas, vulnerability criterion, parametric instability, principal resonance, broaching-to.*

1. INTRODUCTION

Although overlooked sometimes, controllability in harsh environments should be classified as an important aspect of a ship's safety envelope. The significance of this matter is expected to be further enhanced in the future, as ships gradually incorporate increased levels of autonomy in their operational control. One particular aspect of controllability is course-stability in following/ quartering waves. In this respect, advanced criteria need to be developed that could be beneficial for ship design as well as for setting ship operability limits.

The very recently finalized at IMO Second Generation Intact Stability Criteria addressed indirectly the issue of course instability in following seas; however, mainly from the perspective of the avoidance of surf-riding. Whilst the latter is often a precursor of broaching-to, course instabilities of medium or larger size vessels do not involve surf-riding. Phenomena corresponding to the so-called cumulative type of broaching-to, that is, a gradual (oscillatory) growth of yaw, have been neglected. Some insights on the mechanism of this type of instability were provided in Spyrou (1996). However, no validated criterion addressing directly this cumulative type of yaw motion instability has been available. As a matter of fact, the new IMO criteria have accounted for broaching-to indirectly and only with regard to the occurrence of surf-riding.

In this paper, earlier work of the first author on this topic is expanded, in order to fill the identified gap and arrive to a practical ship course-keeping criterion (Spyrou, 1996 & 2007). The criterion is derived from a linear sway-yaw-rudder mathematical model, which, as it is pointed out, is equivalent to a third order yaw equation having time-dependent coefficients at several places. The criterion is basically a mathematical expression of the system's principal instability region boundary. The classic harmonic balance technique has been applied on the third-order yaw equation in order to produce the expression of this boundary. From a dynamics perspective, such a criterion could be regarded as a generalization (incorporating an extra degree of freedom) of a principal resonance criterion, derived for a Mathieu-type equation. It is well-known that a Mathieu type model is commonly used for describing, qualitatively, the parametric roll behaviour of ships; an approach followed also in the vulnerability criteria of parametric roll found in the IMO Second Generation Intact Stability Criteria (IMO, 2021). This interesting unity of the fundamental dynamics governing the types of instability exhibited in the roll and yaw ship motion has already been pointed out (Spyrou, 2000). The analytical form enables easy implementation as a vulnerability check.

The proposed criterion was verified by carrying out systematic comparisons against direct numerical simulations, at two levels. Firstly, with regard to the original sway-yaw-rudder mathematical model, in order to verify that the derived analytical formula of the criterion coincides well with the principal instability boundary corresponding to the original system. Secondly, with regard to simulation results deriving from an expanded mathematical model incorporating nonlinear surge motion, so that the significance of the interplay with surging phenomena and its effect on the yaw instability boundary can be assessed.

Course stability charts based on the new criterion are presented, depending on the wave characteristics and the rudder's control, for a ship that is standard reference in broaching-to studies.

2. MATHEMATICAL MODEL

In the first instance, a standard sway-yaw-rudder model has been selected, with terms corresponding to Froude-Krylov harmonic wave excitation appearing at the right-hand-sides (see Figure 1).

$$(m - Y_{\dot{v}})\dot{v} - Y_v v + (mx_G - Y_{\dot{r}})\dot{r} + (mu - Y_r)r = Y_\delta \delta + Y_W \quad (1)$$

$$(mx_G - N_{\dot{v}})\dot{v} - N_v v + (I_Z - N_{\dot{r}})\dot{r} + (mx_G u - N_r)r = N_\delta \delta + N_W \quad (2)$$

where u, v and r are the surge and sway velocity and yaw rate respectively, m the ship's mass, I_Z the yaw moment of inertia and x_G is the longitudinal distance of ship's centre of gravity from the moving axes' origin, O. The wave forces are expressed assuming small yaw angles:

$$Y_W = \bar{Y}_W \sin\psi \cos(\omega_e t - \theta_1) \approx \bar{Y}_W \, \psi \cos(\omega_e t - \theta_1) \quad (3)$$

$$N_W = \bar{N}_W \sin\psi \cos(\omega_e t - \theta_2) \approx \bar{N}_W \, \psi \cos(\omega_e t - \theta_2) \quad (4)$$

while the frequency of encounter is calculate, in the first instance, with the additional assumption of constant forward surge velocity (i.e. $u \approx U$).

$$\omega_e = \sqrt{gk} - kU\cos\psi \approx \sqrt{gk} - kU \quad (5)$$

The rudder angle δ is assumed to follow a very simple control law without delay (Lewis, 1989):

$$\delta = -k_1(\psi - \psi_r) - k_2\dot{\psi} \quad (6)$$

where, k_1 and k_2 are the proportional and differential gain of the rudder, respectively and ψ_r is the desired heading.

After the standard calculations and replacements, a 3rd order equation for the yaw is derived:

$$\begin{aligned} &T_1T_2\dddot{\psi} + [T_1 + T_2 + k_2KT_3\ ]\ddot{\psi} \\ &+[1 + k_1KT_3 + k_2K + A_1\cos(\omega_e t - \theta_1) + A_2\cos(\omega_e t - \theta_2)]\dot{\psi} \\ &+\{k_1K - [A_1\omega_e\sin(\omega_e t - \theta_1) + A_2\omega_e\sin(\omega_e t - \theta_2) + A_3\cos(\omega_e t - \theta_1) + A_4\cos(\omega_e t - \theta_2)]\}\psi \\ &= Kk_1\psi_r \end{aligned} \quad (7)$$

where

$$T_1T_2 = [(m - Y_{\dot{v}})(I_Z - N_{\dot{r}}) - (mx_G - Y_{\dot{r}})(mx_G - N_{\dot{v}})]/B_o \quad (8)$$

$$T_1 + T_2 = [(mx_G u - N_r)(m - Y_{\dot{v}}) - Y_v(I_Z - N_{\dot{r}}) + N_v(mx_G - Y_{\dot{r}}) - (mu - Y_r)(mx_G - N_{\dot{v}})]/B_o \quad (9)$$

$$KT_3 = [(m - Y_{\dot{v}})N_\delta - (mx_G - N_{\dot{v}})Y_\delta]/B_o \quad (10)$$

$$K = (N_vY_\delta - Y_vN_\delta)/B_o \quad (11)$$

$$B_o = N_v(mu - Y_r) - Y_v(mx_G u - N_r) \quad (12)$$

$$A_1 = [(mx_G - N_{\dot{v}})\bar{Y}_W]/B_o \quad (13)$$

$$A_2 = -[(m - Y_{\dot{v}})\bar{N}_W]/B_o \quad (14)$$

$$A_3 = N_v\bar{Y}_W/B_o \quad (15)$$

$$A_4 = -Y_v\bar{N}_W/B_o \quad (16)$$

The proposed dynamic stability criterion was calculated based on equation 7, as it is presented in the next Section, but it was additionally evaluated accounting for surging effects. Hence, simulations were carried out with an expanded system, complemented with the following equation of surge motion:

$$(m - X_{\dot{u}})\dot{u} = T(u) - R(u) + X_W \quad (17)$$

where

$$X_W = \bar{X}_W \cos\psi \sin(\omega_e t - \theta_3) \approx \bar{X}_W \sin(\omega_e t - \theta_3) \quad (18)$$

The two u dependent terms (T: thrust and R: ship resistance) are based on still-water condition (usual approximations in surf-riding and broaching-to calculations) and they are expressed in simple polynomial form (see Spyrou, 2006). In a Froude-Krylov context, reference values for the phases of the wave forces are (θ_1, θ_2, θ_3)=(π/2, 0, -π/2) and they will be used for the calculations throughout this paper. It is noted that in order to include diffraction effects, the wave load amplitudes would need to be adapted as also the phases. Therefore, the structure of the model is not changed and the analysis that follows is still applicable.

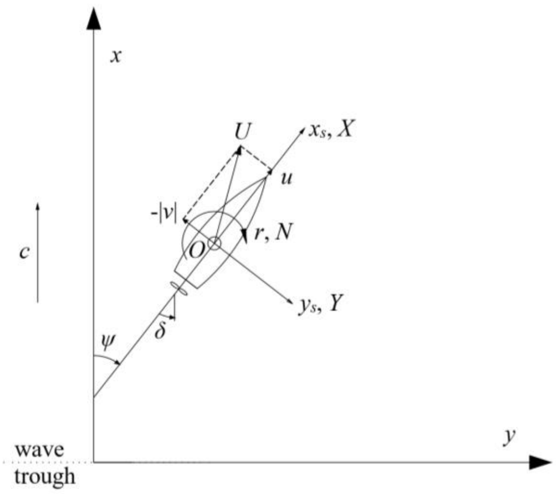

Figure 1: System of coordinates

3. CONDITION OF DYNAMIC INSTABILITY

In this section, the principal instability's region boundary of the described system is estimated analytically by applying the harmonic balanced method on the uncoupled yaw equation 7. Since the motion in the targeted area of instability is expected to be an oscillation of increasing amplitude, a solution of the form $\psi \approx \psi^{02} e^{\mu\tau} cos(\tau + \theta)$ is assumed. A scaled time parameter, τ, defined as $\omega_e t = 2\tau$ is also introduced, changing the equation accordingly:

$$\frac{d^3\psi}{d\tau^3} + \underbrace{\frac{2}{\omega_e}\frac{(T_1 + T_2 + k_2 K T_3)}{T_1 T_2}}_{a_1}\frac{d^2\psi}{d\tau^2} + \Big[\underbrace{\frac{4}{\omega_e^2}\frac{(1 + k_1 K T_3 + k_2 K)}{T_1 T_2}}_{a_2} + \underbrace{\frac{4}{\omega_e^2}\frac{A_1}{T_1 T_2}}_{a_3}\cos(2\tau - \theta_1) + \underbrace{\frac{4}{\omega_e^2}\frac{A_2}{T_1 T_2}}_{a_4}\cos(2\tau - \theta_2)\Big]\frac{d\psi}{d\tau} +$$
$$\Big[\underbrace{\frac{8}{\omega_e^3}\frac{k_1 K}{T_1 T_2}}_{a_5} - \underbrace{\frac{8}{\omega_e^2}\frac{A_1}{T_1 T_2}}_{a_6}\sin(2\tau - \theta_1) - \underbrace{\frac{8}{\omega_e^2}\frac{A_2}{T_1 T_2}}_{a_7}\sin(2\tau - \theta_2) - \underbrace{\frac{8}{\omega_e^3}\frac{A_3}{T_1 T_2}}_{a_8}\cos(2\tau - \theta_1) - \underbrace{\frac{8}{\omega_e^3}\frac{A_4}{T_1 T_2}}_{a_9}\cos(2\tau - \theta_2)\Big]\psi = \underbrace{\frac{8}{\omega_e^3}\frac{K k_1}{T_1 T_2}}_{b}\psi_r \quad (19)$$

or

$$\frac{d^3\psi}{d\tau^3} + a_1\frac{d^2\psi}{d\tau^2} + [a_2 + a_3\cos(2\tau - \theta_1) + a_4\cos(2\tau - \theta_2)]\frac{d\psi}{d\tau} + [a_5 + a_6\sin(2\tau - \theta_1) + a_7\sin(2\tau - \theta_2) + a_8\sin(2\tau - \theta_1) + a_9\cos(2\tau - \theta_2)]\psi = b\,\psi_r \quad (20)$$

The bias term is omitted at this stage, i.e. the desired angle is set to ψ_r=0. Substitution of the assumed solution and of its derivatives to equation 20 leads to the following:

$$\psi_{02}e^{\mu\tau}[\mu^3\cos(\tau+\theta)-3\mu^2\sin(\tau+\theta) \quad (21)$$

$$-3\mu\cos(\tau+\theta)+\sin(\tau+\theta)]+$$

$$a_1\psi_{02}e^{\mu\tau}[\mu^2\cos(\tau+\theta)$$

$$-2\mu\sin(\tau+\theta)-\cos(\tau+\theta)]+$$

$$\psi_{02}e^{\mu\tau}[a_2+a_3\cos(2\tau-\theta_1)$$

$$+a_4\cos(2\tau$$

$$-\theta_2)][\mu\cos(\tau+\theta)$$

$$-\sin(\tau+\theta)]+$$

$$[a_5+a_6\,sin(2\tau-\theta_1)+a_7\sin(2\tau-\theta_2)$$

$$+a_8\cos(2\tau-\theta_1)$$

$$+a_9\cos(2\tau-\theta_2)]\psi_{02}e^{\mu\tau}\cos(\tau+\theta)$$
$$=0$$

Separating $\sin\tau$ and $\cos\tau$ terms and neglecting sines and cosines of 3τ leads to having to satisfy an equation of the form $Acos\tau+Bsin\tau=0$, which in order to be valid for every τ we demand A and B to be equal to zero. This results to a system of two homogenous equations with $cos\theta$ and $sin\theta$ as the unknowns, which has solutions only if its determinant, given by equation 22, is equal to zero.

$$(\frac{a_3\mu+a_8}{2}\cos\theta_1 \quad (22)$$
$$+\frac{a_4\mu+a_8+a_9}{2}\cos\ \theta_2$$
$$-\frac{a_3+a_6}{2}\sin\theta_1-\frac{a_4+a_7}{2}\sin\theta_2)^2$$
$$(\frac{a_3+a_6}{2}\cos\theta_1+\frac{a_4+a_7}{2}\cos\theta_2+$$
$$\frac{a_3\mu+a_8}{2}\sin\theta_1$$
$$+\frac{a_4\mu+a_8+a_9}{2}\sin\theta_2)^2$$
$$=(a_1\mu^2+a_5-a_1+a_2\mu+\mu^3-3\mu)^2$$
$$+(1-2a_1\mu-a_2-3\mu^2)^2$$

The boundary of stability is met when $\mu=0$ (i.e. when the amplitude of the solution is marginally steady). After the appropriate calculations and replacements, a closed form mathematical expression for the boundary is acquired:

$$16(A_3^2+A_4^2)+4(A_1^2+A_2^2)\omega_e^2+ \quad (23)$$
$$16(A_2A_3-A_1A_4)\omega_e\sin(\theta_1-\theta_2)+$$
$$+8(4A_3A_4+A_1A_2\omega_e^2)\cos(\theta_1-\theta_2)=$$
$$=4[4k_1K-(T_1+T_2+k_2KT_3)\omega_e^2]^2$$
$$+[T_1T_2\omega_e^3-4(1+k_1KT_3+k_2K)\omega_e]^2$$

This expression defines the system's *dynamic stability boundary* (*DSB*).

For given wave length (λ), reference frame (θ_1, θ_2) and controller gains (k_1, k_2) this equation marks the boundary of the instability region in a wave steepness (H/λ) and Froude number (Fn) plane. In this context, there is a lower value of wave steepness at which this kind of instability occurs (the vertex of the instability region), that can be analytically calculated using equation 23. This value of wave steepness for given parameter values (λ, θ_1, θ_2, k_1,k_2) defines the proposed vulnerability criterion; the *dynamic stability limit* (*DSL*).

4. RESULTS AND EVALUATION

A series of simulations was executed for comparison, using the surge-sway-yaw-rudder model. An extensively studied purse-seiner fishing vessel of main characteristics L=34.5m, B=7.6m and T=2.99m was selected for this application (Umeda et al, 1995). The desired (ψ_r) and initial (ψ_o) heading were set to 0 and 0.075 respectively. A certain scenario was deemed unstable if the yaw angle exceeded a predetermined threshold value (here $\pm 5\psi_o$). In addition, the initial surge velocity was set equal to the nominal in every case.

Selected results for wave length equal to L and 1.25L, for different sets of controller gains, are provided in Figure 2 and Figure 3 respectively in the form of stability diagrams, with nominal Froude number (Fn) and the wave steepness (H/λ) as the variables of the two axes. Figure 4, provides results for the same scenarios as Figure 3 using the sway-yaw-rudder model for comparison.

Stable scenarios are represented in the diagrams with white colour, while dark grey corresponds to broaching-to cases and light grey to the surf-riding ones (i.e. $u = c$). The black line represents the analytically derived DSB. It is noted that the dark grey area resembling a tongue, is the targeted one corresponding to cases of cumulative broaching-to, while the spike-like region contains the broaching-to scenarios where surf-riding is involved.

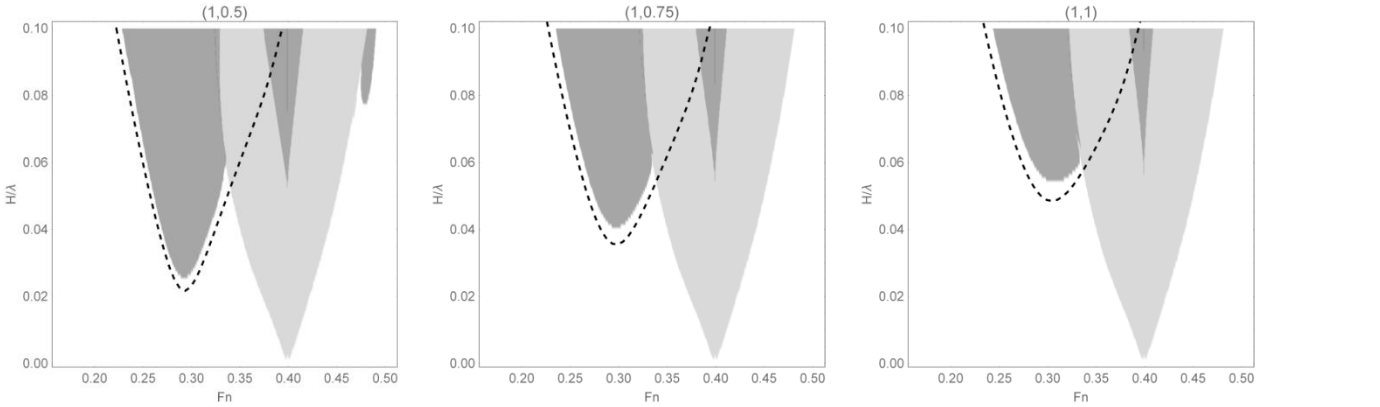

Figure 2: Stability diagrams for *λ*=*L* and different $(k_1, k_2(L/U))$ using a 3DoF system.

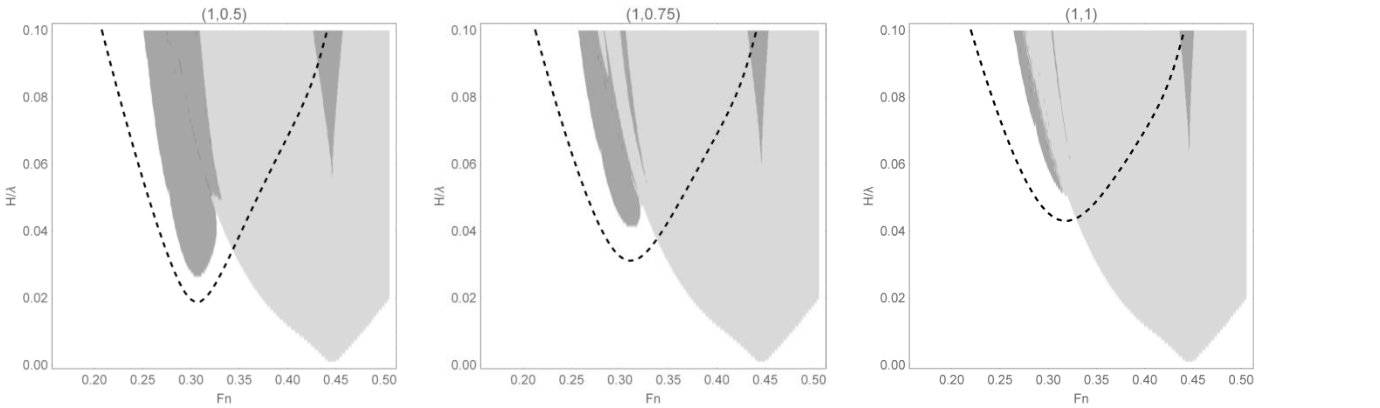

Figure 3: Stability diagrams for *λ*=1.25*L* and different gain values $[k_1, k_2(L/U)]$, using a 3DoF system.

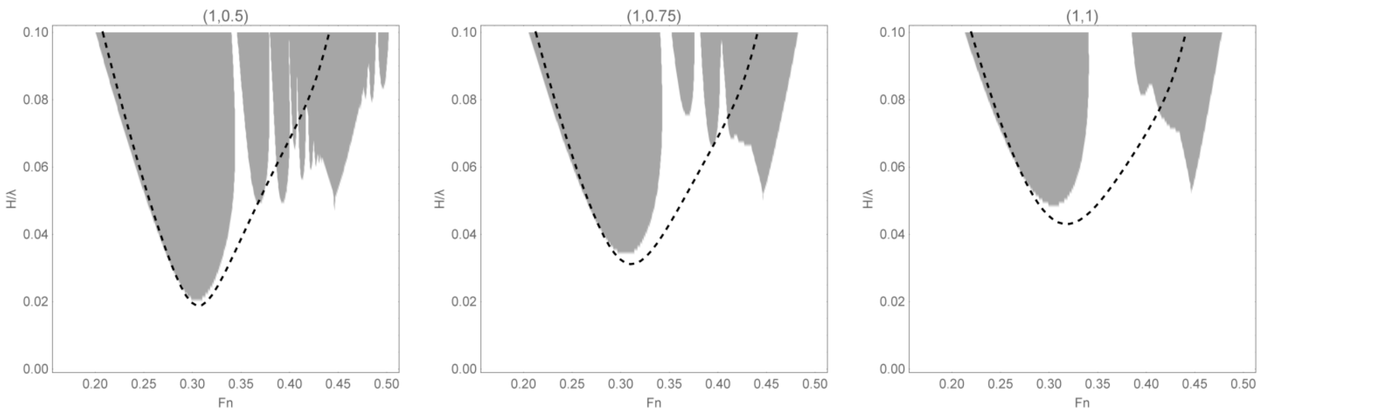

Figure 4: Stability diagrams for *λ*=1.25*L* and different gain values $[k_1, k_2(L/U)]$, using a 2DoF system.

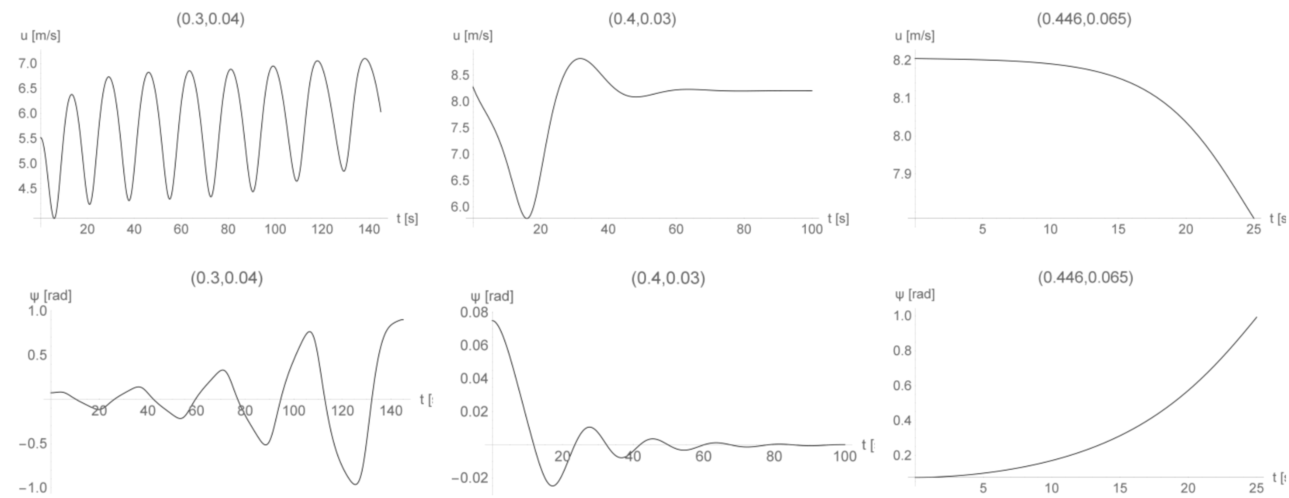

Figure 5: Heading and surge velocity for λ=1.25L, (k_1, $k_2(L/U)$)=(1,0.75) for different (Fn, H/λ) values using a 3DoF system.

Time histories of the heading and the surge velocity for scenarios belonging to these dynamically different areas are provided in Figure 5, with reference to the first chart of Figure 3 [i.e. λ=1.25L and (k_1,k'_2)=(1,0.5)].

As can be seen from Figure 2 and Figure 3 the DSB encloses the targeted instability region. As regards to the vertex of the instability region, it provides a fairly accurate, but always conservative, estimation.

A comparison between Figure 3 and Figure 4 illustrates the effect of the surge component to the dynamic behaviour of the system; areas of higher order instability give their place to surf riding and the targeted area shrinks. It is noted that in the case of wave length equal to L the area of interest remains mostly unchanged.

Application

The developed criterion was used for creating the stability diagrams provided in Figure 6 for the case of following waves. On these diagrams, with reference to a certain ship, the stability limit H/λ values are easily available, as functions of the proportional gain of the controller, for different values of the (non-dimensional) differential gain and a given wave length. Thus, if the sea characteristics are available, suitable combinations of controller gains can be selected ensuring course stability. It is observed that the DSL is more sensitive to changes of the differential gain than that of the proportional, and thus its appropriate setting could be more effective in eliminating this kind of instability. This criterion is easily applicable if the particulars and the hydrodynamic characteristics of the ship are available, and simple rudder and wave forces models (expressed as in equations 3 and 4) are selected.

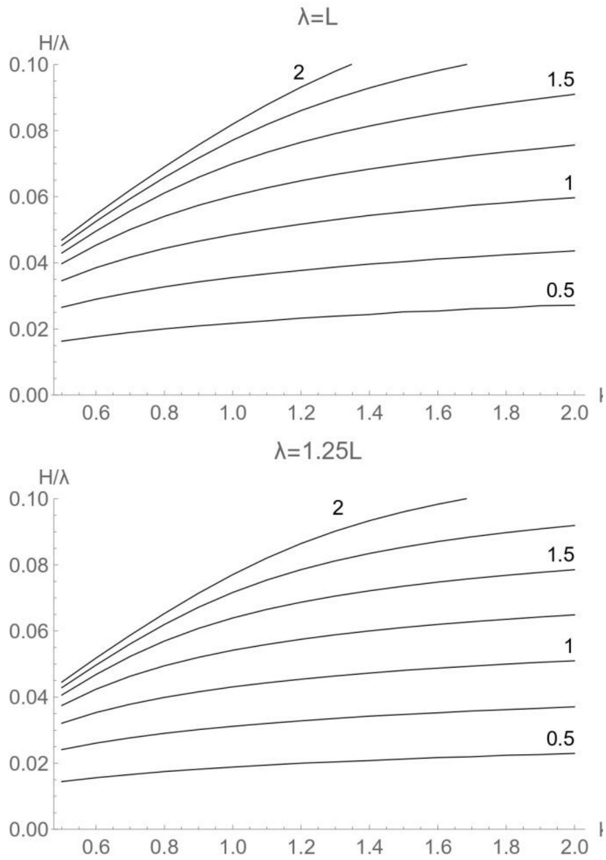

Figure 6: Dynamic stability limit as function of k_1 for a range of $k'_2 = k_2(L/U)$ values and different wave lengths.

5. CONCLCUDING REMARKS

An analytical criterion for avoiding cumulative type broaching-to has been proposed. No similar criterion has been available yet. The underlying mechanism of this dynamic instability can be explained by the time-dependence of the coefficients of the decoupled yaw equation. The proposed criterion should be applied in pair with the zero-frequency-of-encounter quasi-static criterion of yaw stability that has been known since Wahab & Swaan (1964). These combined can ensure (at vulnerability level) the avoidance of direct and cumulative broaching-to. Additional investigations (not reported here) have indicated that the comparative stringency of the requirements of these two criteria varies, depending on the control gain values.

The accuracy of estimation of the instability region's boundary, by the current analytical method, appears quite satisfactory for practical use in order to judge vulnerability at the initial design stage. In a next step, transient effects as well as the effect of the surge velocity could be incorporated in the criterion aiming to improve its accuracy.

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REFERENCES

IMO, 2021. Outcome of the Regulatory Scoping Exercise for the use of Maritime Autonomous Surface Ships (MASS), MSC. 1/Circ. 1638, 3 June 2021.

Spyrou, K. J., 1996. Dynamic Instability in Quartering Seas: The Behavior of a Ship During Broaching, Journal of Ship Research, Vol. 40, No. 1, March 1996, pp. 46-59.

Spyrou, K. J., 2000. Similarities in the yaw and roll dynamics of ships in extreme astern seas, IUTAM Symposium on Recent Developments in Nonlinear Oscillations of Mechanical Systems (edited by Nguyen Van Dao, E.J. Kreuzer, Hanoi, Solid Mechanics and Its Applications, pp. 279-290, Kluwer Academic Publishers, Netherlands, ISBN 978-94-011-4150-5.

Spyrou, K. J., 2006. Asymmetric surging of ships in following seas and its repercussions for safety, Nonlinear Dynamics, 43, 149-272.

Spyrou, K. J., 2007. A note on a simplified model of ship yawing in steep following seas, Proceedings, 12th International Congress of the International Maritime Association of the Mediterranean (IMAM 2007) Maritime Industry, Ocean Engineering and Coastal Resources, Varna, 1-5 September, pp. 85-94.

Umeda, N., Hamamoto, M., Takaishi, Y., Chiba, H., Matsuda, A., Sera, W., Suzuki, S., Spyrou, K., and Watanabe, K. (1995) Model experiments of ship capsize in astern seas, Journal of the Society of Naval Architects of Japan, 177.

Wahab, R. & Swaan, W. A., 1964. Coursekeeping and broaching of ships in following seas, Journal of Ship Research, 7(4), 1-15.