Research and development of early detection algorithm of parametric resonance for ships and offshore floating structures in waves

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ABSTRACT

Parametric resonance can result in extreme motions on both ships and floating structures. Therefore, an early detection algorithm of parametric resonance is necessary for the safe operation of both ships and floating structures. In this paper, a formerly developed parametric resonance early detection algorithm based on incremental real-time Hilbert-Huang Transform (IR-HHT) technique is improved to apply on both ship and offshore floating structures. The improved detection scheme is applied on the parametric resonance experiment results of KCS containership and DDS platform in regular waves. Results show that the improved detection algorithm can successfully detect the parametric resonance of KCS containership and DDS platform at its early onset stage in regular waves. Moreover, by using the proposed optimal factor *k* in the improved detection algorithm, large fluctuation on instantaneous frequency (IF) at the beginning caused by the interference of initial pitch/roll motion on the small heave motion is overcome. Finally, the advantage of the proposed detection method is discussed through comparison with the straightforward method.

Keywords: *Parametric resonance, Early detection algorithm, Hilbert-Huang Transform, Instantaneous frequency.*

1. INTRODUCTION

In heavy sea states, extreme parametric resonant motions on ships and offshore structures may be induced by large variations of restoring characteristics. Parametric resonance is a dangerous stability failure mode which can cause unexpected large motions and even severe cargo losses. For ships, such losses were reported on containerships (France et al., 2003), small fishing vessels (Neves et al., 1999), cruise ship and PCTC (Ovegård et al., 2012). For offshore structures, parametric resonance is often observed on spar (Haslum and Faltinsen, 1999) and semi-submersible (Mao and Yang, 2016) platforms.

The study on the parametric resonance were conducted through numerical simulations (Bulian, 2005; Hashimoto and Umeda, 2010; Neves et al., 1999; Spanos and Papanikolaou, 2007) and model experiments (Hashimoto et al., 2007; Neves et al., 2002; Taguchi et al., 2011; Wei et al., 2018). In 2020, the International Maritime Organization (IMO) approved the interim guidelines on the second generation intact stability criteria for parametric roll (IMO, 2020), which povides a guideline to avoid parametric resonance in design stage. However, some ships and offshore structures can still be vulnerable to parametric resonance in real operational sea.

Therefore, study on the avoidance and stabilization of parametric resonance in the operational stage is also very important. It was pointed out by Yu et al.(2012) that the parametric resonance stabilization techniques was effective when the parametric roll amplitude was still small and anti-roll control was activated early. Thus, in order to achieve a good performance on parametric resonance warning and stabilization, an on-board real-time parametric roll early detection algorithm is needed to detect and warn the parametric resonance when the roll/pitch amplitudes are still small. For the early detection of ship parametric roll, Galeazzi et al.(2013, 2015) proposed a signal-based parametric roll detection method combining a spectral correlation detector in the frequency domain with a phase synchronization detector in the time domain.

The proposed detection schemes are fully validated to be effective and robust using the full-scale long-term voyage data. Yu et al.(2016) developed an alternative signal-based detection method using the incremental real-time Hilbert-Huang Transform (IR-HHT) technique. The detection algorithm based on the IR-HHT approach is proved by both numerical simulations and model experiments to be capable of detecting the frequency shift and amplitude growth during the initial stage of parametric rolling. Acanfora et al.(2018) proposed a straightforward method for detecting the ratio between pitch and roll period potentially leading to parametric roll motions for a ship in the seaway. The method is validated by applying on simulated time histories of ship motion, which is a container ship traversing the Pacific Ocean. However, there are still few applications of these detection algorithms on offshore floating structures, such as spar and semi-submersible platforms.

In this paper, the effectiveness of the real-time parametric resonance early detection algorithm proposed by Yu et al.(2016) on both containership parametric roll and parametric resonance of Deep Draft Semi-submersible(DDS) is validated using model experiment data. The detection scheme is further improved by adopting the optimal determination of algorithm factors. Suitable algorithm settings for different types of structures are provided accordingly.

2. EARLY DETECTION ALGORITHM

In this section, the formerly developed parametric resonance early detection algorithm (Yu et al., 2016) based on incremental real-time Hilbert-Huang Transform (IR-HHT) technique is introduced. And the optimal determination of algorithm factors is discussed for the application on both ship and offshore floating structures.

Time domain detection algorithm

When parametric resonance occurs, the frequency of linear wave induced motion f_θ is about twice of the nonlinear resonant motion frequency f_{roll}. Thus it is possible to detect parametric resonance through the detection of frequency difference between f_θ and f_{roll}. In the parametric resonance early detection method developed by Yu et al. (2016), the signal-based time-frequency dependent analysis method, Hilbert-Huang Transform (HHT) firstly proposed by (Huang et al., 1998), is applied to acquire the frequency information of the heave, roll and pitch motion signals in time domain. Then the proposed time domain detection scheme is applied on the frequency information to detect the frequency difference between f_θ and f_{roll}. Details of the detection method are described as follows.

Firstly, the Hilbert-Huang Transform is used to obtain the instantaneous frequency (IF) of combined motion signal $x(t)$:

$$x(t) = x_N(t) + kx_L(t) \tag{1}$$

Where, $x_N(t)$ and $x_L(t)$ represent the motion time signals of the nonlinear resonant motion and the linear wave induced motion. k is the factor to amplify the otherwise but not signifcant linear wave induced motion. For ship parametric roll, the linear wave induced motion and the nonlinear resonant motion are normally referred as pitch motion and roll motion respectively. For parametric resonance of offshore platform, the linear wave induced motion and the nonlinear resonant motion are normally referred as heave motion and pitch/roll motion respectively.

In the HHT, the multi-component combined motion signals are decomposed into a set of nearly mono-component signals through empirical mode decomposition (EMD). These nearly mono-component signals are so called intrinsic mode functions (IMF). Once the IMFs are extracted, the Hilbert transform is applied to each of these IMFs to obtain the instantaneous frequency. Moreover, an incremental real-time HHT(IR-HHT) algorithm is developed for the on-board real-time detection of parametric roll during the model experiment. Based on this algorithm, the HHT technique can operate incrementally on the data flows of real-time motions during the model experiment.

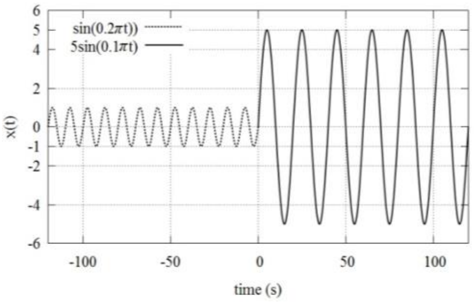

Figure 1 Artificial parametric roll time series x(t) (Yu et al., 2016)

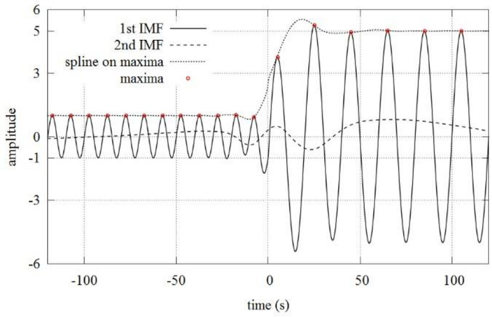

Figure 2 IMFs separated from x(t) (Yu et al., 2016)

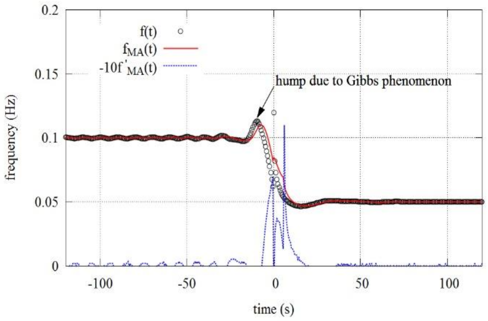

Figure 3 instantaneous frequency $f_{MA}(t)$ (Yu et al., 2016)

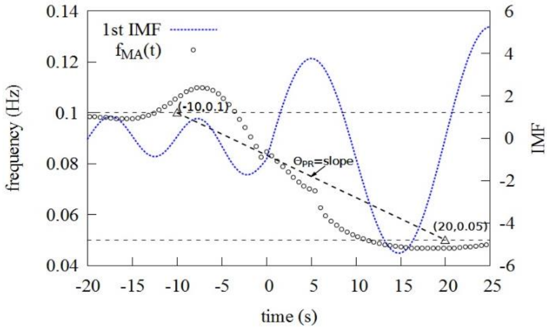

Figure 4 1st IMF and IF around t=0 (Yu et al., 2016)

Then, the time domain detection scheme is proposed to detect the frequency shift between f_θ and f_{roll} from the instantaneous frequency (IF) obtained by the IR-HHT technique. Fig.1-4 from the authors' previous work (Yu et al.(2016)) are the artificial parametric resonance time series $x(t)$, IMFs extracted from $x(t)$, moving average of the IF $f_{MA}(t)$ and IF around t=0. The frequency shift before and after the onset of parametric roll can be observed from the moving average of the IF $f_{MA}(t)$ in Fig.3. Based on the $f_{MA}(t)$ curve, the time domain detection scheme is designed. It includes two hypotheses: frequency discontinuity condition Γ_1 and transition rate condition Γ_2.

The frequency discontinuity condition Γ_1 is used to detect the frequency shift between f_θ and f_{roll} and is formulated as:

$$\begin{cases} f'_{MA}(t_h - \triangle t) > 0,\ f'_{MA}(t_h) < 0 \\ \forall t \in [0, \mu_1 T_{roll}],\ f'_{MA}(t_h + t) \le 0 \\ \alpha = \dfrac{f_{MA}(t_h + \mu_1 T_{roll})}{f_{Average}(t_h)} \le \alpha_{cr} \end{cases} \tag{2}$$

Where the first line represents the hump due to the Gibbs phenomenon which is one of the maxima of the IF. t_h is the time of the hump. In the second line, frequency drops within the time domain $[0, \mu_1 T_{roll}]$ after t_h due to frequency difference. μ_1 is the ratio factor and is set as 0.9 in the experiment. The frequency descending ratio α indicates the amount that frequency has dropped from the average frequency $f_{Average}$ (t_h) between $[0,\ t_h]$. The critical frequency descending ratio α_{cr} is set to be 0.62 in the experiment.

The transition rate condition Γ_2 is designed to get an earlier detection and sort out the slow frequency shift caused by the changing of sea states. It requires the changing rate of IF $f'_{MA}(t)$ to be larger than a threshold Θ_{PR}. The definition of Θ_{PR} is shown in Fig. 4 and formulated as:

$$\Theta_{PR} = \frac{f_\theta - f_{roll}}{t_{tran}} = \frac{\frac{2}{T_{roll}} - \frac{1}{T_{roll}}}{T_{roll} + T_{roll}/2} = \frac{2}{3T_{roll}^2} \tag{3}$$

Where the period of parametric resonance is assumed to be natural period T_{roll}, while f_θ is about twice of natural frequency. The transition time t_{tran} is set as one nonlinear resonant motion period plus one linear motion period.

The transition rate condition Γ_2 is defined as:

$$\exists t[t_h, t_h + \mu_1 T_{roll}],$$

$$\text{such that: } -f'_{MA}(t) > \Theta_{PR} = \frac{2}{3T_{roll}^2} \tag{4}$$

When the frequency discontinuity condition Γ_1 and transition rate condition Γ_2 are all satisfied, the time when parametric resonance is detected t_p is derived as:

$$t_p = t_h + \mu_1 T_{roll} \tag{5}$$

Optimal determination of algorithm factors

In the authors' previous work (Yu et al.(2016)), the optimal selection of the ratio factor λ, the critical frequency descending ratio α_{cr} and the factor μ_1 are conducted. The optimal α_{cr}, μ_1 and λ are set to be 0.62, 0.9 and 20%, which can get a robust detection of parametric resonance. However, the optimal determination of factor k in Eq.(1) is not discussed. k is introduced to amplify the otherwise but not significant linear wave induced motion. With the amplified linear wave induced motion, its frequency f_θ can be steady and dominant in the beginning of IF, which is enssiential for the frequency drop. The influence of the factor k on IF can be quite significant. Thus, in this paper, the optimal factor k is determined by:

$$k = \frac{\overline{|x_N|}}{A_L} \tag{6}$$

Where, A_L is the amplitude of the linear wave induced motion, $\overline{|x_N|}$ is the average of the absolute value of nonlinear resonant motion at the beginning.

For ship parametric roll, the influence of the factor k on IF is not significant. Because the linear wave induced motion, i.e. the pitch motion, is identical to initial small roll motion. Therefore in the original detection scheme, k is set to be 3 which is enough for robust detection of ship parametric roll.

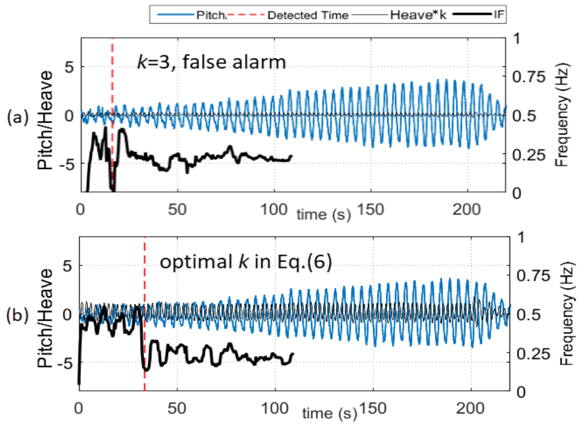

Figure 5 IFs under different factors *k*, (a): *k*=3, (b): optimal *k* by Eq. (6)

However, for parametric resonance of offshore platform, the linear wave induced motion is heave motion which is quite small and not identical to initial pitch/roll motion. This can cause large fluctuation on IF at the beginning, as shown in Fig.5(a). Fig.5 shows the detection results on parametric resonance experimental data of an offshore platform. The IFs under different factors k are plotted as thick black line. The nonlinear resonant pitch motion and the amplified linear heave motion signals (heave*k) are plotted as thick blue line and thin black line. The detection time t_p is shown as red dashed vertical line. (a) and (b) are the results of k=3 and optimal k by Eq.(6). From Fig.5(a), large fluctuation on IF at the beginning can be observed, because the small heave motion is not identical to initial pitch motion. Thus, a false alarm is generated. By applying Eq.(6) to get the optimal k, parametric resonance is successfully detected as shown in Fig.5(b).

In the next section, the detection algorithm described above with optimal factor k is applied to motion signals obtained from experiments of containership and semi-submersible platform.

3. DETECTION ON PARAMETRIC ROLL OF KCS CONTAINERSHIP

The free-running model experiments for containership in regular head waves are conducted in the towing tank of Yokohama National University, which is 100m long, 8m wide and 3.5m deep. Results on the model experiments are fully reported in Yu et al., (2017). The model ship used in the experiment is a 1/100 scale KCS (KRISO Container Ship) containership (Simman2008, 2008). The model ship, experimental setup and cases are presented in Fig.6, 7 and Table 1.

Figure 6 The KCS model ship

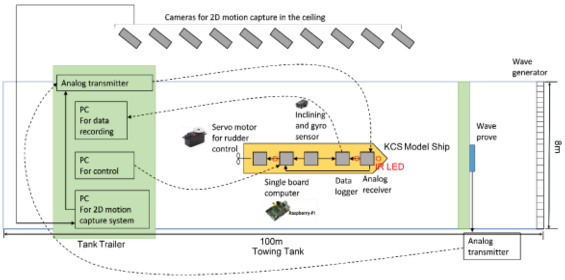

Figure 7 Setup of all the experimental equipment in the towing tank

The real-time parametric roll early detection algorithm based on IR-HHT technique is applied in the model experiments to detect parametric roll in early stage. Results are presented in Fig.8. In the figure, the instantaneous frequency(IF) [Hz] of the pitch and roll combined time series $x(t)$ obtained by the real-time detection algorithm is shown as the line with round dot. The factor k in Eq.(1) is set to be 3. The P.R. detected time t_p are shown as the dashed vertical line. The pitch frequency f_θ and the roll natural frequency f_{roll} are plotted as the thin black horizontal dash-dot line and the thick blue horizontal dash-dot line.

In the figures, at the beginning when no parametric roll occurs, the pitch motion is dominant in the pitch and roll combined time series. Thus, the instantaneous frequency of the combined time series (line with round dot) is about the pitch frequency f_θ (thin black horizontal dash-dot line) i.e., wave encounter frequency. When parametric roll occurs, the roll motion is dominant. Therefore, the instantaneous frequency (line with round dot) drops to the value around the roll natural frequency f_{roll} (thick blue horizontal dash-dot line). Based on the frequency drops, parametric roll events are detected at time t_p (dashed vertical line) when the roll amplitudes are still small for all the cases in Fig.8.

Furthermore, the amplitude detected Φ_p for all the cases are summarized in Fig.9 (solid line with round dot and rectangle). In order to fully validate the parametric roll early detection algorithm, some cases with bilge keel are repeated by 6 to 8 times and all the amplitudes detected Φ_p are plotted as round dot in Fig.9. The rectangle is the statistical plot of repeated runs under the same case where the upper limit of the rectangle is the maximum Φ_p, the lower limit is the minimum Φ_p and the solid line is the average Φ_p. For the cases with only one run, the amplitude detected Φ_p is represented as a round dot.

According to the amplitude detected Φ_p presented in Fig.9, it can be found that parametric roll is successfully detected in all the cases and no false alarm is generated. Parametric roll events in almost all the cases and runs are detected when Φ_p is less than 6 deg. Thus, it is concluded that the detection algorithm can successfully detect parametric roll at its early stage in regular waves.

Table 1 Experimental cases for KCS parametric roll in regular head waves (model scale)

No.	Planned speed V_m[m/s]	Wave Period T[s]	Wave Length λ/L_{pp}	Wave Freq. ω_0[rad/s]	Encounter Freq. ω_e[rad/s]	T_e/T_{roll}
1	0.100	1.214	1.000	5.176	5.449	0.534
2	0.200				5.722	0.508
3	0.300				5.995	0.485
4	0.400				6.268	0.464
5	0.500				6.541	0.445
6	0.600				6.814	0.427
7	0.200	1.302	1.150	4.826	5.301	0.549
8	0.300				5.538	0.525
9	0.400				5.775	0.504
10	0.500				6.013	0.484
11	0.600				6.250	0.465
12	0.700				6.488	0.448
13	0.800				6.725	0.433
14	0.900				6.962	0.418
15	0.400	1.384	1.300	4.540	5.380	0.541
16	0.500				5.590	0.520
17	0.600				5.800	0.501
18	0.700				6.011	0.484
19	0.800				6.221	0.468
20	0.900				6.431	0.452

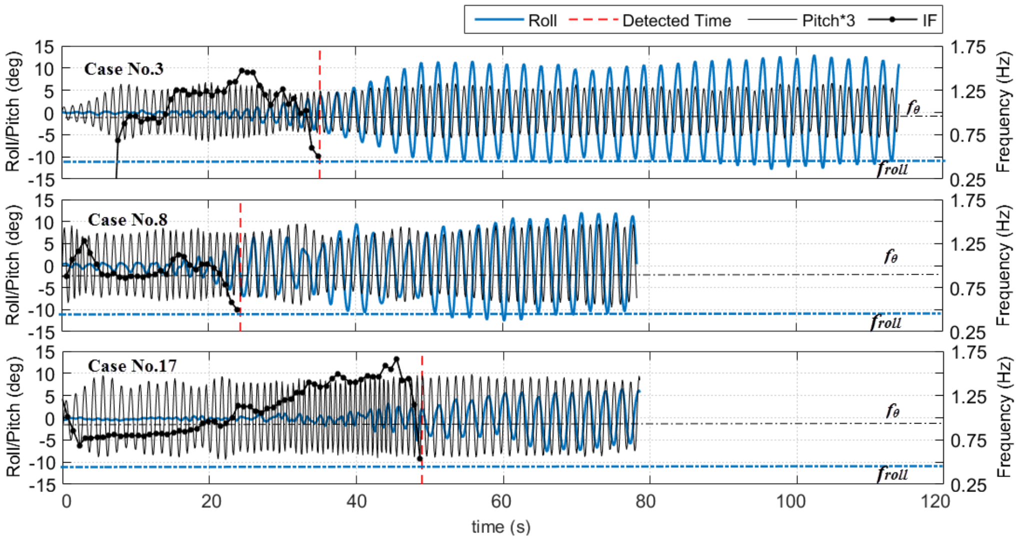

Figure 8 Time series of KCS parametric roll early detection in regular waves with BK

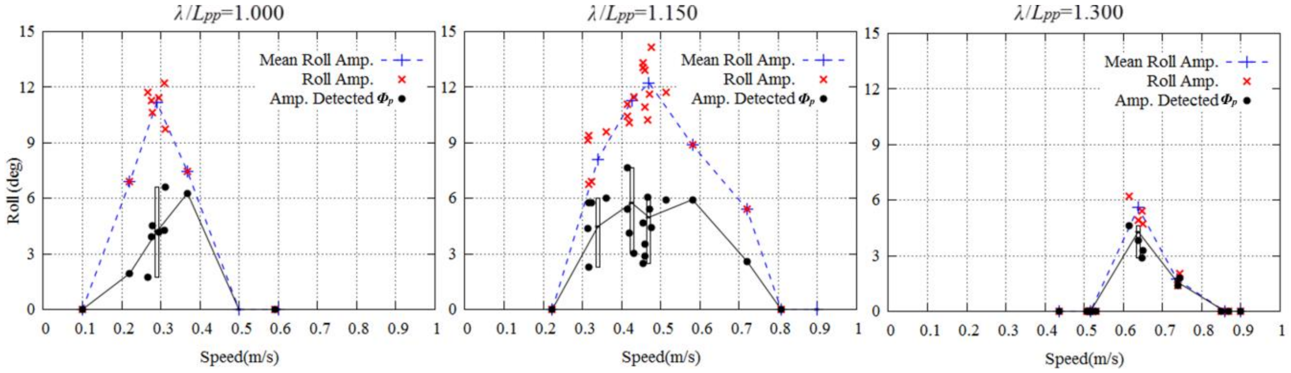

Figure 9 Occurrence and detected amplitude Φ_p of KCS parametric roll in regular head waves with Bilge Keel

4. DETECTION ON PARAMETRIC RESONANCE OF DDS PLATFORM

Model experiments and detection results

Model experiments for semi-submersible are conducted in the wave flume (60 m×3.0 m×1.5 m) of the Ocean University of China (OUC). The subject model for the experiment is a 1/100 model of a deep draft semi-submersible (DDS) prototype. Results on the model experiments are fully reported in Yu et al., (2022). The scaled model, test arrangement and test cases are shown in Fig.10 and Table 2.

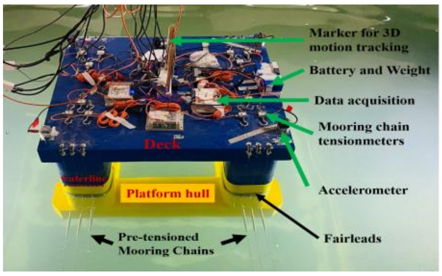

Figure 10 arrangement of the DDS model during experiments

The real-time early detection algorithm based on IR-HHT technique is applied to the motion signals obtained by model experiments to detect parametric resonance in early stage. Different from the parametric roll of ship, the parametric resonance of DDS platform is induced by the change on the longitudinal and transverse metacentric height caused by large heave motions. Thus, the parametric resonance of DDS platform can occur in both roll and pitch direction. As shown in Fig. 11 and 12, in case No.1 without mooring, parametric resonance occurs in roll direction, while parametric resonance occurs in pitch direction for case No.3 with mooring. So, in the detection algorithm, the $x_N(t)$ and $x_L(t)$ in Eq.(1) are chosen as pitch/roll motion and heave motion signals respectively.

The detection results are presented in Fig.11 and 12. In the figures, the instantaneous frequency(IF) [Hz] of the heave and roll/pitch combined time series $x(t)$ obtained by the real-time detection algorithm is shown as the thick black line. The optimal factor k is determined by Eq.(6). Detected time t_p are shown as the dashed vertical line. The heave frequency f_θ and the roll/pitch natural frequency f_{roll} are plotted as the thin black horizontal dash-dot line and the thick blue horizontal dash-dot line.

In the figures, at the beginning when no parametric resonance occurs, the heave motion is dominant in the combined time series $x(t)$. Thus, the IF of $x(t)$ (thick black line) is about the heave frequency f_θ (thin black horizontal dash-dot line) i.e., wave encounter frequency. When parametric resonance occurs, the roll/pitch motion is dominant. Therefore, the instantaneous frequency (thick black line) drops to the value around the roll/pitch natural frequency f_{roll} (thick blue horizontal dash-dot line). Based on the frequency drops, parametric resonance events are successfully detected at time t_p (dashed vertical line) when the roll amplitudes are still small for all the cases as shown in Fig.11 and 12.

Furthermore, the steady parametric roll/pitch amplitude and the amplitude detected Φ_p for all the cases are summarized in Fig.13 (solid line with square dots and crosses). According to the amplitude detected Φ_p presented in Fig.13, it can be found that parametric resonance is successfully detected in all the cases and no false alarm is generated. Parametric resonance4 events in almost all the cases and runs are detected when Φ_p is less than 4 deg. Thus, it is concluded that the detection algorithm can successfully detect parametric resonance of DDS platform at its early stage in regular waves. The effectiveness of the parametric resonance early detection algorithm based on the IR-HHT technique in regular waves is verified through model experiments.

Table 2: Test cases for parametric resonance of DDS platform in regular waves

Case No.	*Mooring*	H(cm)	T(s)	χ(deg)
1-*-#	No	2、10	2.0 to 2.5 interval 0.1	180(heading)
2-*-#	No	4、10	2.0 to 2.5 interval 0.1	90(beam)
3-*-#	Yes (4 mooring chains)	10、14	1.0 to 2.6 interval 0.05	90(beam)

Notes: H, T and χ are wave height, wave period and heading angle respectively. The * represents wave height, # represents wave period.

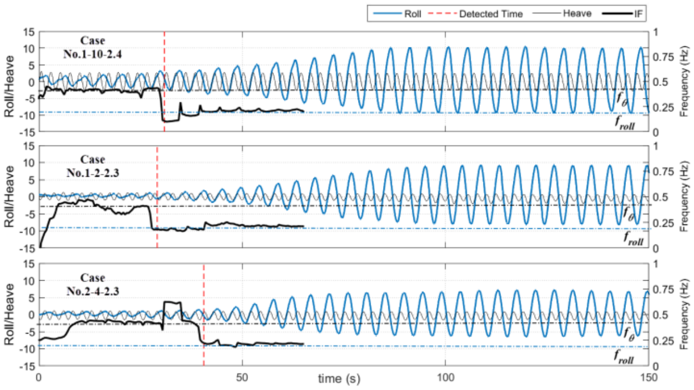

Figure 11 Time series of DDS parametric resonance early detection in regular waves without mooring

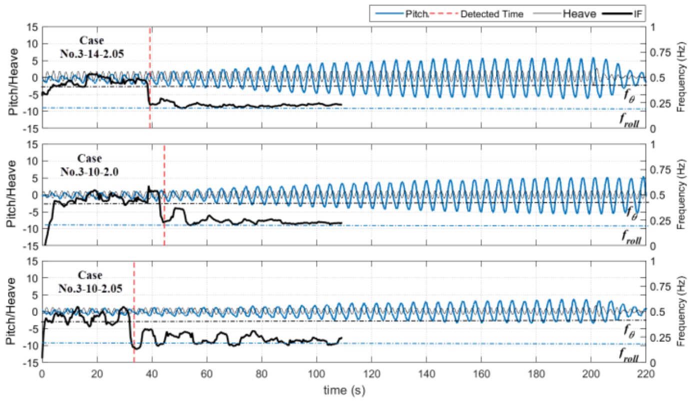

Figure 12 Time series of DDS parametric resonance early detection in regular waves with mooring

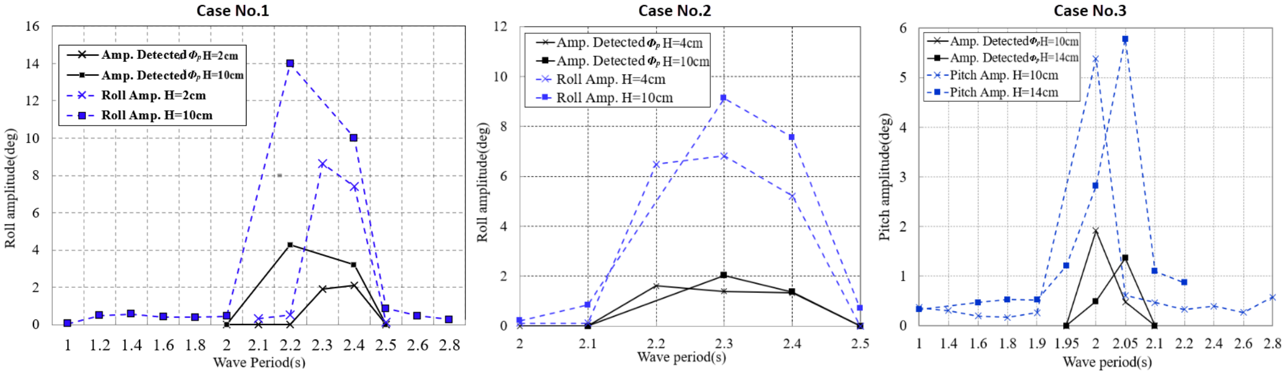

Figure 13 Occurrence and detected amplitude Φ_p of DDS parametric resonance in regular head waves

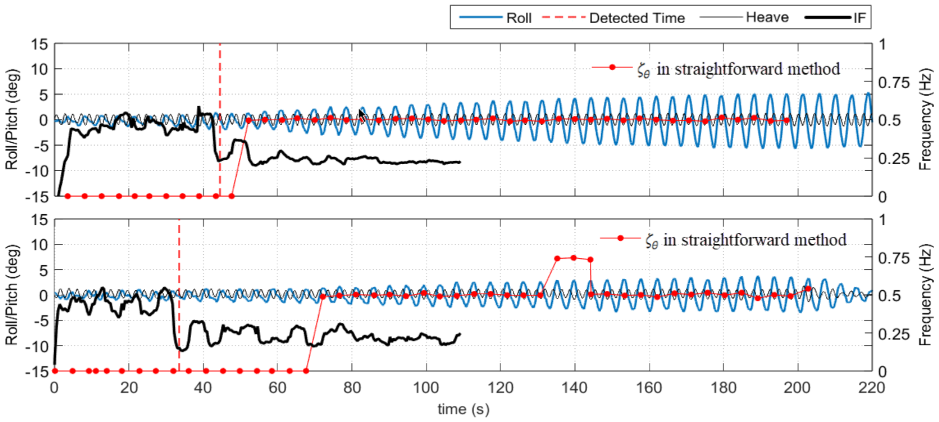

Figure 14 comparison on the detection performance of different methods

Comparison with straightforward method

A straightforward method for parametric resonance detection is to calculate the ratio between the period of the linear pitch/heave motion and nonlinear roll moiton. If the ratio is about 0.5, it will potentially lead to parametric resonance motion. Acanfora et al.(2018) proposed a parametric resonance detection method based on this simple idea. In their research, the ratio ζ_θ is defined as:

$$\zeta_\theta = \frac{\overline{T_\theta}}{\overline{T_\varphi}} \Phi_{index} \qquad (7)$$

Where $\overline{T_\theta}$ and $\overline{T_\varphi}$ are average periods of the linear pitch/heave motion and nonlinear roll moiton.

$$\Phi_{index} = \begin{cases} 1 & if \ \sqrt{\overline{\varphi_A^2}} \geq \varphi_{alert} \\ 0 & if \ \sqrt{\overline{\varphi_A^2}} < \varphi_{alert} \end{cases}$$

φ_{alert} is the arbitrarily adopted threshold of roll angle that triggers an alarm, which is set to 1.5 degrees in this research.

In order to compare the detection performance of the method developed in this paper and the traightforward method in Acanfora et al.(2018), their detection results on DDS parametric resonance are obtained and presented in Fig.14. In the figure, the ratio ζ_θ calculated using the straightforward method is plotted as red dots in each period. When ζ_θ is about 0.5, parametric resonance is detected.

Though comparison results, it is found that the method proposed in this paper can get earlier detection on parametric resonance than the straightforward method. Actually, the detection time of the straightforward method is mainly determined by φ_{alert}. If a smaller φ_{alert} is applied, an earlier detection can be obtained. However, it is difficult to decide φ_{alert} especially in real irregular sea. This is a disadvantage of the straightforward method. However, the method proposed in this paper has no such disadvantage. Because the instantaneous frequency, which is independent of motion amplitude, is used for parametric resonance detection.

5. CONCLUSION

In this paper, the effectiveness of the real-time parametric resonance early detection algorithm on both KCS containership and DDS platform are validated using model experiment data.

Given the characteristics of parametric resonance of offshore platform, the detection scheme is improved by using optimal factor k.

An optimal factor k is introduced to get a consistence performance of the detection scheme on both ship and offshore platform:

$$k = \frac{\overline{|x_N|}}{A_L}$$

With the optimal factor k, large fluctuation on IF at the beginning caused by the interference of initial pitch/roll motion on the small heave motion can be overcome.

Through validation using model experiment data, the improved detection algorithm can successfully detect parametric resonance of KCS containership and DDS platform at its early stage in regular waves.

Finally, the detection method developed in this paper is compared with the straightforward method to show advantage of the proposed method.

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