Application of Statistical Extrapolation Techniques to Dynamic Stability

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ABSTRACT

This paper describes the application of some statistical extrapolation techniques to dynamic stability event (for example large roll angle or large acceleration). Two extrapolation techniques will be used in this study: extrapolation using a fitted distribution and extrapolation over wave height. We will focus mainly on extrapolation over wave height technique. These two techniques will be applied on two datasets obtained by numerical simulations. The first dataset represents parametric resonance process (which is considered as a nonlinear process) and the second dataset represents a linear process. Both processes are obtained from a very long simulation 1200 hours (3h x 400) in order to insure a better statistical convergence of the sampling. In addition, these extrapolation techniques will be validated using direct counting, and finally a ranking in term of accuracy and simulation time will be discussed.

It's demonstrated that extrapolation techniques derived in close form for linear process could be used for nonlinear process (dynamic stability process such as parametric roll) under some conditions. It's also demonstrated that extrapolation over wave height can be used with distribution using time to first event (as described in the Interim Guidelines On The Second Generation Intact Stability Criteria) as well as with other probabilistic distributions.

Keywords: *Dynamic stability, Monte Carlo, Extrapolation over Hs, GEV distribution, GPD, Bootstrap, Direct counting.*

1. INTRODUCTION

Difficulties to evaluate the probability of large event (roll angle and accelerations) are related to both the rarity of the failure and the nonlinearities of the dynamical system describing ship behavior in rough seas. These nonlinearities are introduced by stiffness, roll damping, and excitation for example. These nonlinearities are essential to properly model dynamic stability phenomena (parametric roll, pure loss of stability, broaching, …). Therefore, an accurate and realistic assessment may be limited to numerical simulations (for example using potential code for parametric roll) and model test.

The probability of stability failure is used in direct stability assessment (DSA) of Second Generation Intact Stability Criteria (SGISC) as specified in MSC.1-Circ. 1627. To this end, some form of counting of stability failure events in a given time is required, which means that such events need to be encountered in the simulations or in model experiment. This leads to the problem of rarity, i.e. when the time between events is longer than a relative time scale (roll period in the context of the SGISC). This means, the need for long simulations. In addition, a reliable estimation of the stability failure probability requires simulations where a sufficiently large number of stability failure events is encountered, which further increases the required simulation time. Practically speaking, this means that there are some conditions where the event is not observed during the simulation time or the model test run time. And there are other conditions which may lead to very few observed events so that direct counting cannot be considered as a reliable option. Therefore, in order to reduce simulation time or number of simulations, one of the solutions is a statistical extrapolation.

It's important to state that in SGISC, the use of statistical extrapolation procedures are allowed in the guidelines of DSA as described in MSC1.-Circ. 1627. Moreover, statistical extrapolation is widely used for prediction of extreme events which utilizes extreme value theory (Gumbel, 1958). This type of methodology is based on the extreme value distribution to be fitted to the measured or simulated statistical data; then the distribution can be used to predict an extreme value that can occur with a given probability. Another extrapolation procedure which can reduce significantly the simulation time is the extrapolation over wave height.

Application of some statistical extrapolation techniques to probabilistic assessment of dynamic stability of ships is the main scope of this work. The next sections of this paper will describe the application and validation of two extrapolation procedures namely extrapolation with a fitted distribution and extrapolation over wave height. These procedures will be applied on two datasets representing a linear process and a nonlinear process.

2. EXAMPLE CASE

The roll motion time series has been obtained by performing a time domain simulation on C11 containership. The main characteristics of this vessel are contained in Table 1 and a body plan is shown in Figure 1.

Table 1: Main characteristics of C11 containership

Parameter	Value	Unit
Length between perpendiculars	262.0	m
Breath	40.0	m
Speed	0.0	m/s
Natural roll period	25.1	s
Metacentric height	2.75	m
Bilge keel length	76.28	m
Bilge keel breath	0.4	m

Simulations conditions

Nonlinear time domain computations using HydroStar++ (see Wandji (2018) for more details on this tool) have been performed in following, irregular and short crested seas for 5 sea states. The five sea states have the same wave period (Tp = 12.5s) and different wave heights (Hs=3m, 4m, 5m, 6m and 7m). For each sea state, 400 realizations of 3 hours have been computed. For each realization a different set of random phases, frequencies of the wave component composing the sea state is used, as described in St Denis and Pierson (1953). To ensure that this discretization does not lead to self-repeating effect, the procedure described and used in Wandji (2022) has been applied.

In some sea states, the ship experiences large roll motions. These roll motions may be caused by parametric resonance, as the natural roll period is about twice the encounter period in following seas. An example of roll motion time series obtained for one realization of 3 hours for the sea state with Hs = 6m is shown in Figure 2 (blue line). Note that this signal can be considered as a nonlinear process since parametric rolling is a known to be highly nonlinear phenomenon (Bulian, 2005).

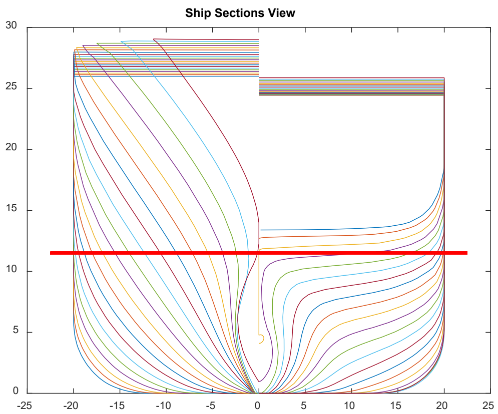

Figure 1: Body plan of C11 containership.

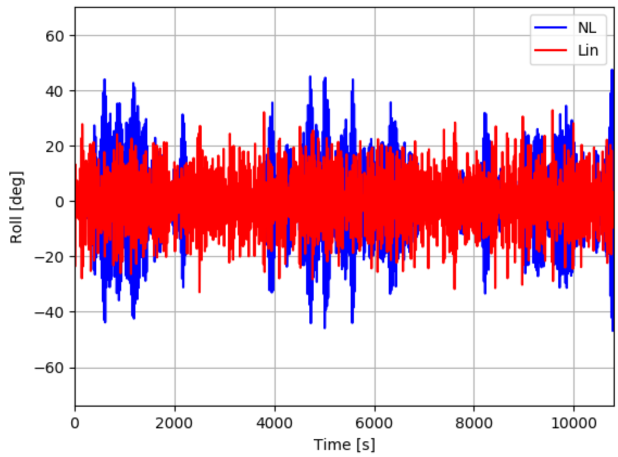

Figure 2: 3h time series of nonlinear (parametric resonance, blue line) and linear processes (red line) obtained for Hs=6m & Tp=12.5s.

Construction of the linear process

The same technique utilized in Wandji (2022) to build the linear process is used here. This technique consist to estimate the power spectral density (PSD) over the sample of nonlinear roll motions and then used this PSD to generate a linear stochastic process. For each sea state the linear process was generated for 400 records, 3 hours each. Thus the nonlinear and linear processes have the same energy content. Figure 3 shows the two spectrums derived from the two processes, they are identical. An example of 3h time series of the linear process for sea state with Hs=6m is shown in Figure 2 (red line).

Using the two processes (linear and nonlinear) defined above, we will apply some extrapolation techniques on these two datasets. In this paper, if not otherwise specified, all results for the linear process will be represented in red and the results for the nonlinear process (or parametric resonance process) in blue.

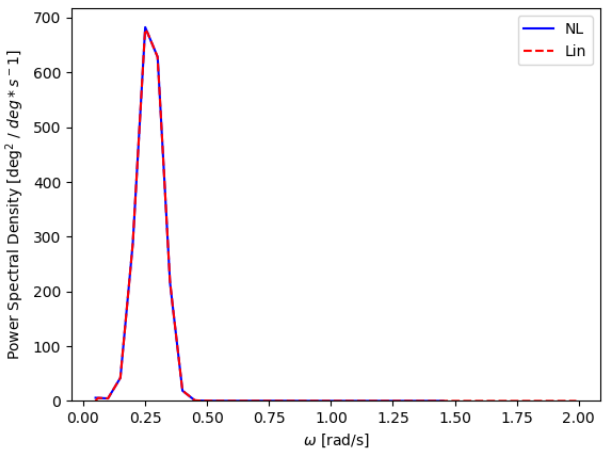

Figure 3: Power spectral density for nonlinear (blue) and linear (red) processes for Hs=6m.

3. EXTRAPOLATION TECHNIQUE USING FITTED DISTRIBUTIONS

This technique is able to characterize probability of events that are too rare to observe in model test or numerical simulation. A distribution is used to fit the observed data, and using the fitted distribution, the probability is assessed for the level of interest.

Block maxima and fitted distribution

In block maxima, the extreme value is built by determining the maximum of the signal for different time windows of the same length (also called block i.e. determining the maximum value of each block). Moreover, this distribution is also strongly connected to maximum over a duration distribution as shown in Wandji (2022). The distribution of extreme values is a particular case of order statistics (Gumbel, 1958), and considering a set of independent identically distributed variables, the cumulative distribution has been shown to be the so called Generalized Extreme Value (GEV) distribution that holds for the maximum value regardless on how the process is distributed.

For a normal process (i.e. linear process) x, with standard deviation σ_x, it has been shown that the extreme value distribution follows the 1st expression in the formula (1) (see Wandji, 2022) and can be approximated by a Gumbel distribution (2nd expression in formula (1)) which is the first type of the GEV distribution. In formula (1), T represents the length or duration of each block and T_z is the upcrossing period of the process.

$$F_T(x) = \exp\left[-\frac{T}{T_z} * exp\left(-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2\right)\right]$$
$$\approx \exp\left[-exp\left(\frac{\sqrt{2Ln(T/T_z)} - \frac{x}{\sigma_x}}{\frac{1}{\sqrt{2Ln(T/T_z)}}}\right)\right] \quad (1)$$

Using a time windows corresponding to the simulation length of 3 hours, the extreme values distribution has been fitted for the linear and nonlinear processes. The GEV distribution is defined by 3 numbers: a shape parameter, a scale parameter and the location parameter. The parameters of an extreme value distribution can be determined using many methods. In this work, the method of Maximum Likelihood Estimation (MLE) has been used. The idea behind the MLE method is to find the values of the parameter that are “more likely” to fit the data (Coles, 2001). The results for the sea state with Hs = 6.0m for both linear and nonlinear processes are shown in Figure 4.

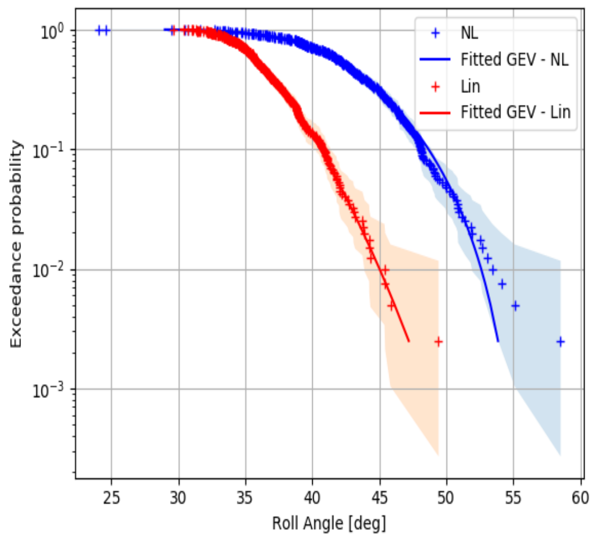

Figure 4: Block maxima fitted with GEV distribution for linear and nonlinear processes – Hs=6m & Tp=12.5s

The linear (Lin) and nonlinear (NL) process data have been fitted with the GEV distribution as shown in Figure 4. Moreover, the observed data are plotted with their confidence interval (CI) for 95% confidence level. One could observe that the fitted distribution remains always in the CI for linear process; while for the nonlinear process, the fitted distribution tends to leave the CI at the queue of the distribution where the data are statistically not converged. The confidence intervals are built using the binomial distribution as described in Brown et al. (1999) with Jeffreys interval. Jeffreys interval has a Bayessian derivation. Jeffreys interval has the advantage of being equal-tailed i.e. for a 95% confidence level, the probabilities of the interval lying above or below the true value are both close to 2.5% (Jeffreys, 1961).

Peak over threshold (POT) and fitted distribution

POT is based on a statistical extrapolation using the probabilistic properties of the peaks that exceed a given threshold. For general stochastic nonlinear process, the distribution of amplitudes and conditional distribution of peaks above the threshold are unknown. Therefore, it needs to be fitted with some "approximate distribution" using the available data. Thus, the basic idea behind peak over threshold is to fit a distribution (usually a Generalized Pareto Distribution (GPD)) to the observed data above the threshold. The mathematical background of the method is the 2nd extreme value theorem, which states that the tail of an extreme value distribution can be approximated with a GPD. The tail of any distribution can be approximated by a GPD above a sufficiently large threshold (Coles, 2001).

An example of POT fitting is shown in Figure 5 for both processes for the sea state with Hs=6m and for threshold value of 35 degrees. GPD and GEV distribution were tested and both provided good results. In Figure 5, the results are shown for GEV distribution fitting using MLE method. The observed data are plotted with their CI for 95% confidence level. The confidence intervals are built using the CI of the binomial distribution as described in Brown et al. (1999) with Jeffreys interval.

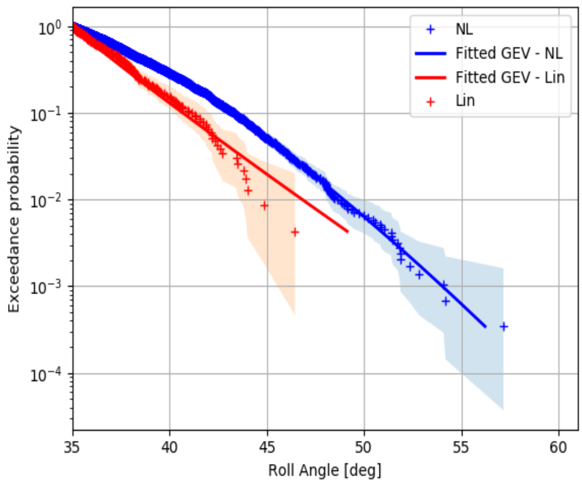

Figure 5: POT for linear and nonlinear processes with a threshold level of 35 degrees – Hs=6m & Tp=12.5s

One can observe that the fitted distribution always remain inside the CI area. Looking into the results of the linear process, the estimated shape parameter is negative meaning that the GEV distribution is a Weibull distribution. Keeping in mind that Rayleigh distribution is a particular case of the Weibull distribution, the quality of the fitting obtained for the linear process is not surprising. On the other hand it's known that for a normal distribution, the distribution of the peaks over a given threshold is a truncated Rayleigh distribution. Note that the fitting is sensitive to the threshold level. In addition, at the threshold value of 35 degrees, the independence of peaks is guaranteed. A Pearson chi-square goodness of fit tests confirmed also the validity of the fitted distribution with the score of 0.92 (>0.05) for linear process and 0.77 (>0.05) for parametric resonance process.

4. EXTRAPOLATION OVER WAVE HEIGHT

The idea behind the extrapolation over wave height is to perform model test or to simulate the ship motions with an increased value of significant wave height in order to obtain several stability failure events within acceptable computing time and to estimate the probability of failure (or the mean failure rate) for this seaway. Afterwards the probability of failure (or the mean failure rate) in a smaller seaway is determine by means of an extrapolation over wave height (Soding and Tonguc, 1986). Extrapolation over wave height is computed for different wave height but for a fixed wave period, wave direction, ship's speed and loading condition.

The linear response is characterized by the response spectrum and its first spectral moments. The root mean square of the response σ_{HS}, is given by (Volker, 2000):

$$\sigma_{HS} = \sqrt{m_0} = H_S \cdot \sigma_1 \qquad (2)$$

where m_0 is the variance of the linear response, H_S the significant wave height and σ_1 a constant.

It has been discussed and demonstrated in Wandji (2022) that under some conditions different statistical estimates are related as shown in Figure 6. Using formula (2), the statistical distribution discussed in Wandji (2022) and shown in Figure 6 can be rewritten in function of H_S. Some of these distributions will be briefly presented in this section with their application on both processes. For this application, 5 significant wave heights are used (3m, 4m, 5m, 6m and 7m).

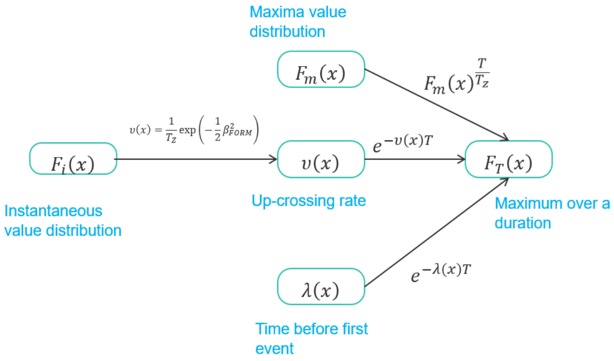

Figure 6: Relation between different statistical distributions

Distribution of maxima

The distribution of maxima (or cycle amplitude) for a linear process is known to be a Rayleigh distribution. The cumulative density function (F_m) in term of H_S can be written as:

$$F_m(x) = 1 - \exp\left[-\frac{1}{2}\left(\frac{x}{H_S\sigma_1}\right)^2\right]$$
$$\Rightarrow \ln\big(1 - F_m(x)\big) = -\frac{x^2}{2H_S^2\sigma_1^2} \tag{3}$$

From formula (3), it can be observed that, the logarithm of the exceedance probability function is linear with respect to $1/H_S^2 = H_S^{-2}$.

Using the linear and nonlinear processes of the example case, formula (3) has been applied and the results are presented in Figure 7. The extrapolation has been computed for three roll angle levels: 10, 20 and 30 degrees.

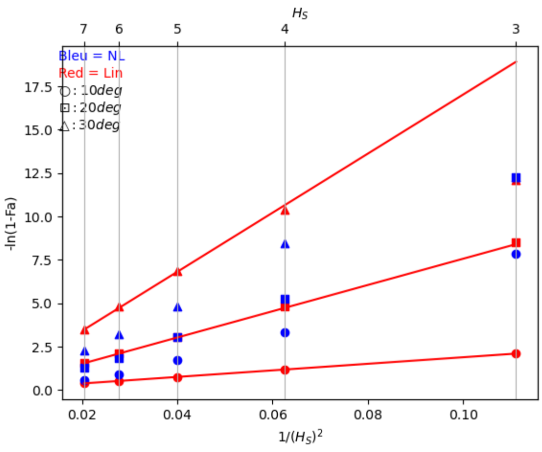

Figure 7: Extrapolation over wave height for the linear and nonlinear processes using distribution of maxima.

The linear process results follow very well a line (see Figure 7), in fact they are on the theoretical line (red line). The point out of the line (for e.g. at the roll level of 30 degrees and H_S=3m), the number of peaks are very small (less than 5) and therefore the probability computed is not reliable because the data are not statistically converged. In addition, it is interesting to note that the nonlinear process results (blue points) seem to follow a line.

In general, the formula (3) can be written as:

$$\ln\big(1 - F_m(x)\big) = A(x) + \frac{B(x)}{H_S^2} \tag{4}$$

where A and B are constant coefficients for a given roll angle, independent from significant wave height but dependent on the ship loading condition, ship's speed, wave period and wave direction.

Using the dataset obtained by time domain simulations (TDS) for Hs=6m, the distribution of maxima of the linear process for Hs=4m has been computed by extrapolation. The results are presented in Figure 8, and one can observe a good agreement between the distributions computed obtained by direct counting using Hs=4m and the one obtained by extrapolation over wave height of 6m.

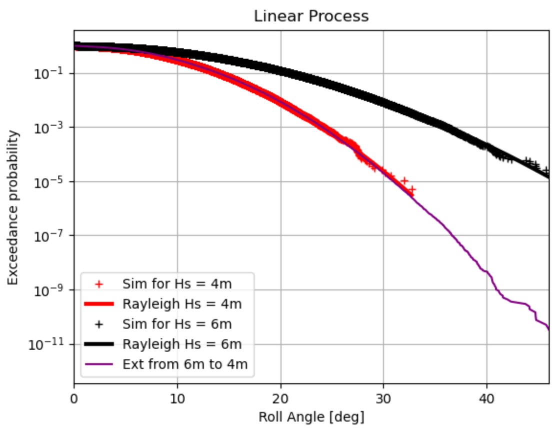

Figure 8: Linear Process - Extrapolation over wave height for distribution of maxima – From Hs=6m to Hs=4m.

For the parametric resonance process, two variants of extrapolation over wave height have been tested. The first variant consists to use the same intercept of linear case (from formula (3), one can see that the intercept is zero). The second one consist to find both the intercept A(x) and the slope B(x) using formula (4). To illustrate these variants, Hs=5m has been used for the first variant and for the second variant Hs=5m and Hs=6 have been used. Both variants have been used to extrapolate to Hs=4m. The results are shown in Figure 9. The results of the extrapolation using the second variant (purple line) are close to direct counting results computed for Hs=4m. In addition, we can see that the difference between the two variants is for roll angle smaller than 35 degrees. For roll angle above 35 degrees, both variants provide almost the same results in this case.

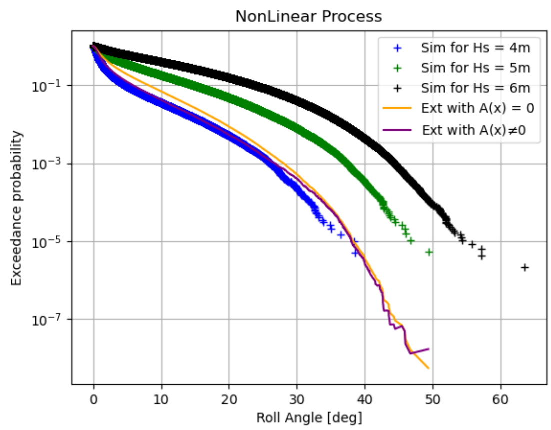

Figure 9: Nonlinear process – Extrapolation over wave height for the distribution of maxima

To estimate the accuracy of the extrapolation for the nonlinear process, confidence interval (CI) with a confidence level of 95% has been computed for two sets of extrapolations (Hs=5m and Hs=6m for the first set and Hs=6m and Hs=7m for the second set).

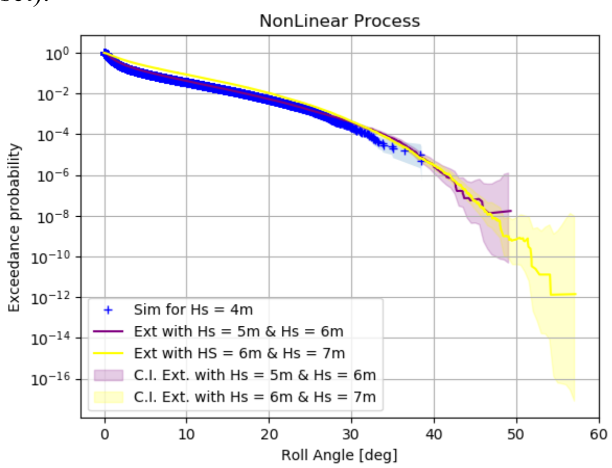

Figure 10: Nonlinear process – Extrapolation over wave height for the distribution of maxima with their CI.

The extrapolated CI are computed from CI of wave heights used for extrapolation plus Monte Carlo simulations. The two sets have been used to extrapolate roll maxima exceedance probability for Hs=4m. We can observe from Figure 10 that the extrapolation using Hs=5m and Hs=6m provides better results than the one using Hs=6m and Hs=7m. In fact, the extrapolated distribution obtained for Hs=5m and Hs=6m (purple in Figure 10) is within the CI obtained from direct counting for Hs=4m (blue curve in Figure 10), and the estimate obtained by direct counting is within the extrapolated CI.

Upcrossing rate and time to failure

For a linear and independent process, the mean upcrossing rate according to Wandji (2022) could written using the upcrossing period T_Z as:

$$\lambda(x) = \frac{1}{T_Z} \cdot \exp\left[-\frac{1}{2}\left(\frac{x}{H_S\sigma_1}\right)^2\right]$$
$$\Rightarrow \ln(\lambda(x)) = -\ln(T_Z) - \frac{x^2}{2H_S^2\sigma_1^2} \quad (5)$$

Under the assumption of independence of events and narrow band process, the failure rate (obtained from time to first event or time between events) and upcrossing rate are similar as discussed in Wandji (2022). From formula (5) we can notice that the logarithm of the mean failure rate in function of H_S^{-2} is a line for the linear process. Using the linear and nonlinear processes of the example case, the logarithm of the failure rate have been computed for 5 significant wave heights and the results are shown in Figure 11.

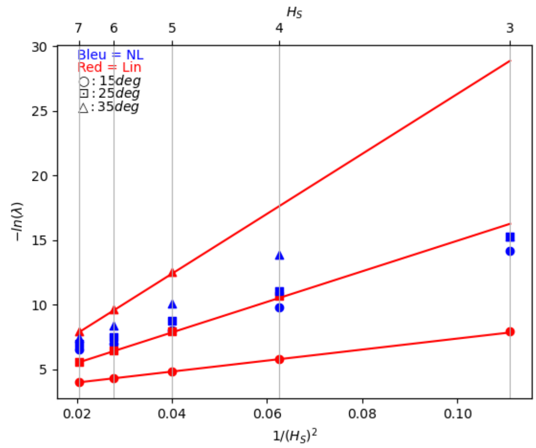

Figure 11: Extrapolation over wave height for the linear and nonlinear processes using time to first event/upcrossing rate.

Figure 11 shows the results of extrapolation over wave height computed for three roll angle levels. We can observe that also in this case the linear process follow very well a line, in fact there are on the theoretical line. Some points are missing in Figure 11 for both processes, especially at 25 and 35 degrees of roll angle. This is due to the fact that there were no upcrossing for these roll angle level. In addition, it is interesting to note that the nonlinear process results (blue points) seem to follow a line.

Using the data obtained for Hs=6m by TDS for the linear process, the failure rate for Hs=4m has been computed by extrapolation over wave height. The results in Figure 12 show a very good agreement between the failure rate obtained by extrapolation and those obtained by direct counting using time to first event.

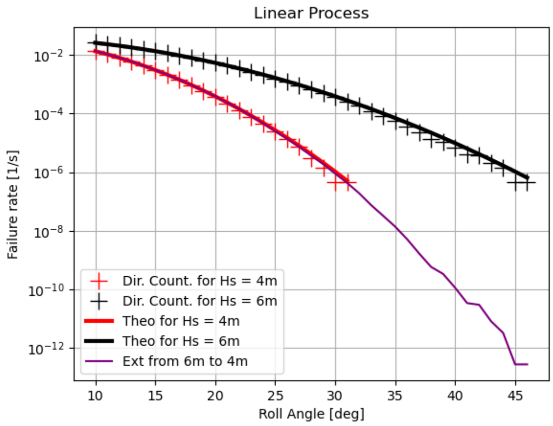

Figure 12: Linear process – Extrapolation of failure rate obtained by time to first event from Hs=6m to Hs=4m.

For the nonlinear process, two variants have been tested for extrapolation. The first variant consists of using the same intercept as the one of linear case i.e. ln(T_z) (see formula (5)). In this case, only one wave height is needed to compute the slope. The second variant consists to find the intercept and slope by using equation (6):

$$-\ln(\lambda(x)) = \ln(T_m(x)) = A(x) + \frac{B(x)}{H_S^2} \qquad (6)$$

Note that $T_m(x)$ is the mean time to failure. These two variants have been applied using Hs=5m for the first variant, and for the second variant Hs=5m and Hs=6m. The extrapolation have been performed to obtain a failure rate for Hs=4m. The results in Figure 13 show that the failure rate obtained using the second variant is close to the failure rate obtained by direct counting.

To estimate the accuracy of the extrapolation for the nonlinear process, the CI with a confidence level of 95% has been computed for two sets of wave heights using the second variant. These two sets (on one side Hs=5m and Hs=6m, another side Hs=6m and Hs=7m) have been used to obtain the failure rate for Hs=4m. The CI of the mean failure rate obtained by direct counting is built using the chi-square distribution as described in the draft Explanatory Notes of SGISC (IMO SDC 8/WP.4 and its different addendum). The extrapolated CI are computed from CI of wave heights used for extrapolation plus Monte Carlo simulations. The results presented in Figure 14 show that the extrapolated failure rate using Hs=5m and Hs=6m provides very good results, since the CI is almost completely included in the CI of the failure rate for Hs=4m obtained by direct counting.

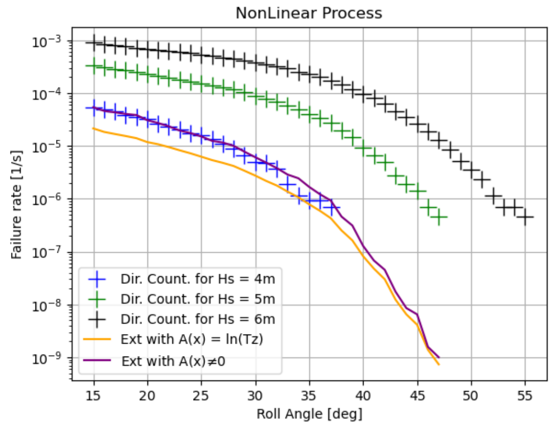

Figure 13: Nonlinear process – Extrapolation of failure rate from Hs=5m and Hs=6m to Hs=4m.

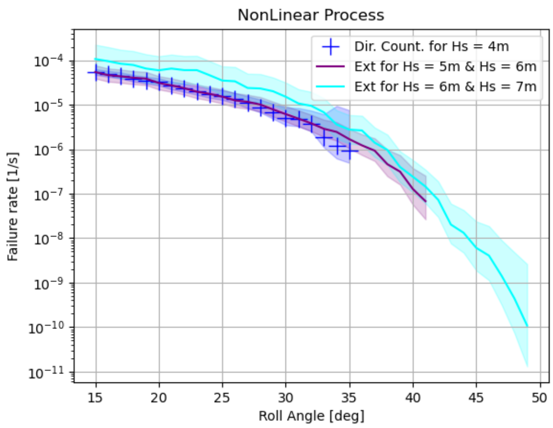

Figure 14: Nonlinear process – Extrapolation over wave height of failure rate based on time to first event with CI.

It is important to note that, the extrapolation presented in this section, especially formula (6) is one of the main statistical extrapolation procedure proposed in the Direct Stability Assessment of the SGISC (see MSC.1-Circ.1627). The condition formulated in the Interim Guidelines to avoid non-conservative extrapolation is checked. The maximum failure rate used in this section is $1.4 \cdot 10^{-3}$ (1/s), the condition is verified using the natural roll period (Troll) as $1.4 \cdot 10^{-3} < 0.05/T_{roll} = 2.0 \cdot 10^{-3}$. Thus, the stability failure rate obtained by direct counting in this work can be used for extrapolation over wave height according to IMO MSC.1-Circ 1627). The use of extrapolation over wave height using failure rate for dynamic stability problems has also been excellently discussed in Shigunov (2016 and 2017) and by Soding and Tonguc (1986). Some application can be also found in SDC8/WP.4.

A cut of the Figure 14 has been realized for a roll angle of 33 degrees. The results presented in Figure 15 shows that the failure rate obtained by direct counting is inside the extrapolated CI for Hs=5m and Hs=6m. While, this is not the case when Hs=6m and Hs=7m is used. This give the indication that the extrapolation is more accurate when wave heights used to extrapolate are no far to the extrapolated wave height.

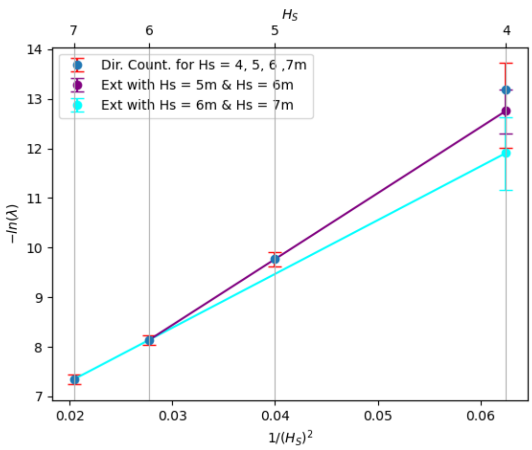

Figure 15: Nonlinear process – Extrapolation over wave height of failure rate for a roll level of 33 degrees.

Block Maxima or Maximum over a Duration

Maximum over a duration (also called block maxima) distribution for a linear process and an exposure time T is given by (Wandji, 2022):

$$F_T(x) = \exp\left[-\frac{T}{T_Z}\cdot exp\left(-\frac{1}{2}\left(\frac{x}{H_s\sigma_1}\right)^2\right)\right]$$
$$\Rightarrow \ln\left(-ln\left(F_T(x)\right)\right) = \ln\left(\frac{T}{T_Z}\right) - \frac{x^2}{2H_S^2\sigma_1^2} \quad (7)$$

From formula (7), one can notice that the logarithm of the probability is a line in function of Hs^{-2} for a linear process. Using a block of 3h for the linear and nonlinear processes of the example case, the logarithm of the probability have been computed for 5 significant wave heights. Figure 16 shows the results of extrapolation over wave height for three roll angle. We can observe that the linear process results follow very a line as expected. We can see that some points are missing, this is due to the fact that the roll angle level was not in the observed data. In addition, it is interesting to note that the nonlinear process results (blue points) seem to follow a line. The probability of exceedance for Hs=4m has been computed for the linear process by extrapolation over wave height using direct counting results for Hs=6m. The results are shown in Figure 17, and as expected the extrapolated distribution follows very well the distribution obtained by direct counting.

For the nonlinear process, two variants have been tested for extrapolation. The first variant consists to use the same intercept of the linear case (i.e. $\ln(T/T_Z)$) and compute the slope using one wave height. The second variant consist to find both the intercept and the slope from formula (8).

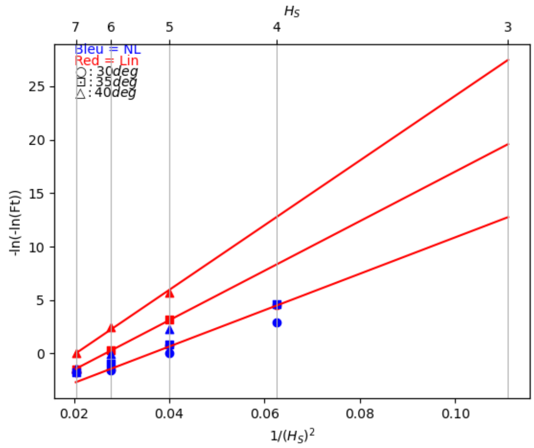

Figure 16: Extrapolation over wave height for the linear and nonlinear process using block maxima distribution.

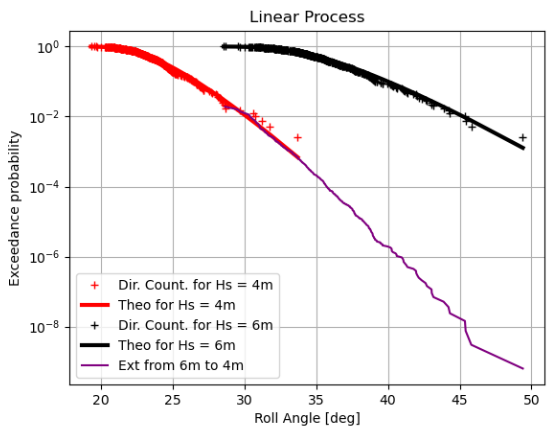

Figure 17: Linear process – Extrapolation over wave height for block maxima distribution.

$$\ln\left(-\ln\left(F_T(x)\right) = A(x) + \frac{B(x)}{H_S^2} \quad (8)$$

To illustrate these two variants, one wave height (Hs=5m) was used for the first variant and two wave heights (Hs=5m and Hs=6m) were used for the second variant. These two variants have been used to extrapolate at Hs=4m and the results are presented in Figure 18. From the results in Figure 18, we can see that the results of the extrapolation using the second variant (purple line) are close to the direct counting results for Hs=4m. To estimate the accuracy of the extrapolation for the nonlinear process, CI for a confidence level of 95% has been computed for two sets of extrapolations (Hs=5m and Hs=6m for the first set and Hs=6m and Hs=7m for the second set) using the second variant.).

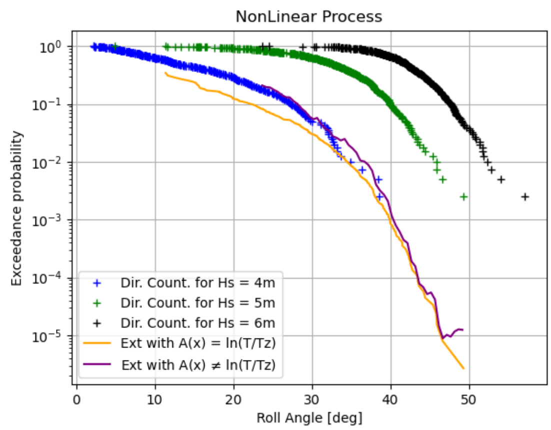

Figure 18: Nonlinear process – Extrapolation over wave height for the block maxima distribution.

The extrapolated CI are computed from the CI of wave heights used for extrapolation plus Monte Carlo simulations. The results are shown in Figure 19, and an analysis of these results shows that the extrapolation is good for the two sets since the exceedance probability assessed by direct counting is contained in both extrapolated CI.

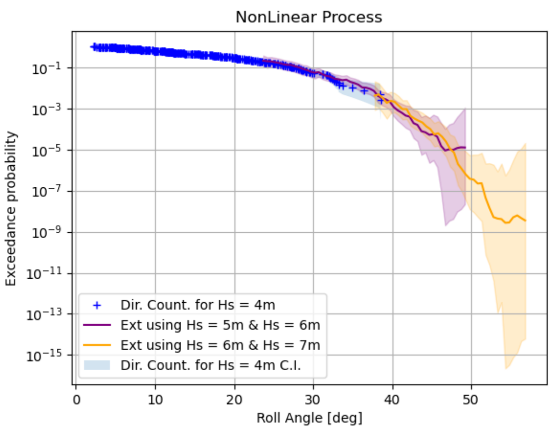

Figure 19: Nonlinear process – Extrapolation over wave height of block maxima distribution with their CI.

To further understand the results of Figure 19, a cut is performed at 38 degrees roll angle. The results are shown in Figure 20, and one can notice that both sets of extrapolated can capture the direct counting results. Thus, in this case extrapolation over wave height and extrapolation using a fitted distribution provide a comparable precision.

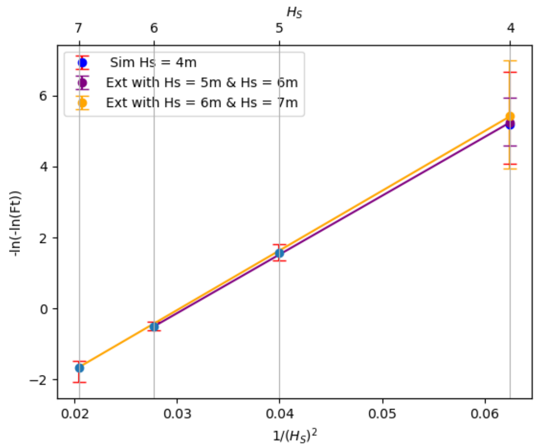

Figure 20: Nonlinear process – Extrapolation over wave height of block maxima for 38 degrees roll angle.

5. ACCURACY AND SIMULATION TIME

In this section we will compare the precision of some statistical extrapolation techniques presented in sections 3 and 4 with the computation time. The extrapolated Hs used in this section is Hs=4m.

Figure 21 shows results regarding block maxima distribution extrapolated using a fitted distribution (GEV distribution in this case) on one side and another side using extrapolation over wave height. The CI for block maxima with a fitted distribution is assessed using a bootstrap statistic procedure (Davison and Hinkley, 1997).

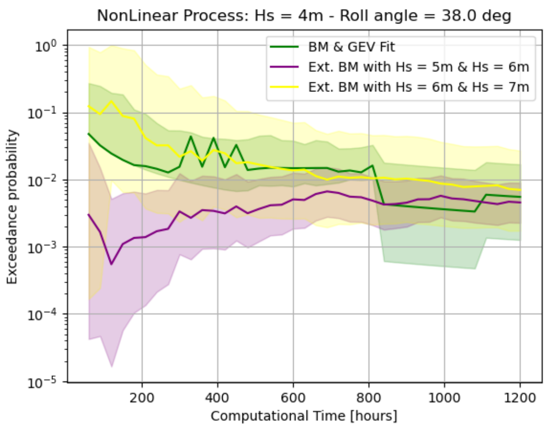

Figure 21: Accuracy vs Simulation time – Extrapolation over wave and fitted distribution on block maxima.

For extrapolation over wave height two sets of wave height have been used (Hs=5m and Hs=6m for the first set and for the second set Hs=6m and Hs=7m) and the roll angle level is set to 38 degrees. From Figure 21, we can see that extrapolation over wave height using Hs=5m and Hs=6m provide more accurate results in this particular case. It's interesting to note that after a long simulation time (800hours) the estimates obtained using the three methods are within the CI of each method.

A comparison between failure rate obtained by direct counting and the failure rate obtained by extrapolation over wave height is shown in Figure 22. One can observe that for a comparable accuracy, the extrapolation over wave height method is faster than direct counting. For long simulation, the extrapolated and the direct counting CI overlap.

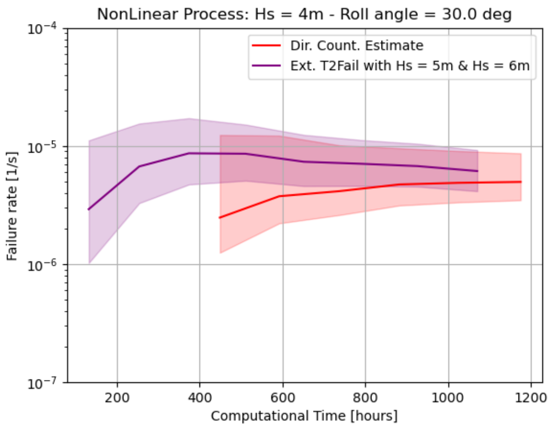

Figure 22: Accuracy vs Simulation time – Failure rate by extrapolation over wave height and direct counting.

Another comparison is carried out using block maxima extrapolated with a fitted distribution and the estimate of failure rate (assessed using time to first event) obtained by direct counting. The results are displayed in Figure 23. We can observe that for both methodologies the accuracy increases for long simulation time and also that block maxima CI is entirely included in the direct counting failure rate (obtained by time to first event) CI. Note that from failure rate, the exceedance probability is computed using and exposure time of 3hours.

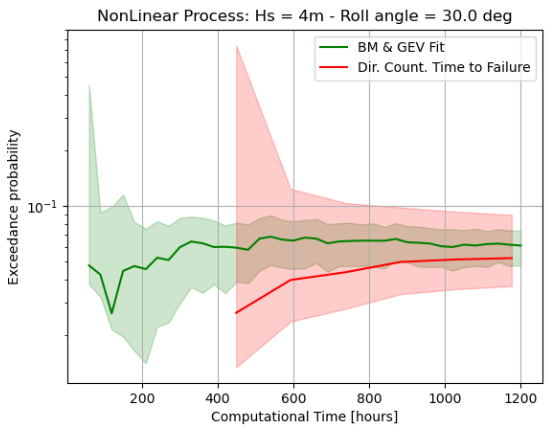

Figure 23: Accuracy vs Simulation time – time to failure and block maxima.

6. CONCLUSIONS

The aim of direct stability assessment procedure described in MSC.1/Circ.1627 is the estimation of a likelihood of a stability failure in a random seaway. Because the stability failure may be rare for the cases practically relevant for DSA, very long simulations are necessary. One solution to solve the problem of rarity is the use of statistical extrapolation methods. Therefore, extrapolation method may be applied as an alternative to direct counting procedures. Nevertheless, some caution should be exercised because uncertainty increases, as the extrapolation is associated with additional assumptions used to describe ship motions in random seaway. Consequently, the statistical uncertainty of the extrapolated value should be provided in a form of boundaries of the confidence interval evaluated with a confidence level (a 95% confidence level is used throughout this paper).

The main scope of this work was to apply some statistical extrapolation techniques to a dynamic stability case such as parametric resonance. Two big classes of extrapolation methods have been revisited. The first class is extrapolation method using a fitted distribution such as a Generalized Extreme Value distribution or a Generalized Pareto distribution. The second class is extrapolation over wave height which has been applied on failure rate, cycle amplitude distribution and block maxima distribution.

We have seen that these extrapolation methods are derived in close form for linear processes and can be used successfully with some assumptions also for nonlinear processes. In order to confirm this, the extrapolation methods have been applied to the entire distribution (many roll angle level) for a linear and nonlinear processes (having the same energy content). It has been shown that the extrapolated values for the linear process follow very well the theoretical line.

The accuracy of the extrapolation methods for the nonlinear process has been evaluated by building the confidence interval and by comparing the extrapolated results with those obtained by direct counting. We have seen that the block maxima distribution can be extrapolated using extrapolation over wave height or by extrapolation by a fitted distribution. Thus, the block maxima distribution can be used in the probabilistic methods proposed in DSA of the SGISC (MSC.1/Circ. 1627).

We have compared the simulation time and the accuracy of the extrapolation techniques, and we have seen that methodology which use extrapolation over wave height could be very fast. Nevertheless this methodology should be used with caution, since the results could have a bias depending on how far is the extrapolation wave height from the starting wave heights.

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