Validation of Split-time Method with Volume-Based Numerical Simulation

Kenneth Weems, *Naval Surface Warfare Center Carderock Division*

Vadim Belenky, *Naval Surface Warfare Center Carderock Division*

Bradley Campbell, *Naval Surface Warfare Center Carderock Division*

ABSTRACT

The paper describes the results of a statistical validation of the calculation of the probability of capsizing in irregular waves with the split-time method. The objective of the validation is to demonstrate that the split-time method correctly estimates probability of capsizing without necessarily observing it. Very large data sets of motion simulations were produced for severe sea conditions using a very fast but qualitatively realistic volume-based code, and a "true" rate of capsizing was determined by collecting the observed capsizes in this data. A series of small subsets of these data sets were then used with the split-time estimation, which was compared to the observed rate. In order to validate the evaluation of the confidence interval, the comparison was performed many times and the percentage of successful estimations was counted. If this percentage tends to the confidence probability, the statistical validation is successful. The paper contains results for 14 different conditions, varying significant wave height, modal period and relative heading. For the 95% confidence probability, the percentages of successes were between 80% and 100% for 50 sets; between 87% and 99% for 150 sets and finally converged to the theoretical 95% when all the sets were averaged.

Keywords: *Statistical validation, Probability of capsizing*

1. INTRODUCTION

The probabilistic assessment of capsizing in irregular waves with advanced hydrodynamic codes leads to the solution of an extrapolation problem. Capsizing is too rare to be observed in realistic sea conditions within a reasonable simulation time. The split-time method is a technique of extrapolation that is specifically intended for the estimation of capsizing probability; its development is reviewed in Belenky, et al. (2016). The cited reference reported a successful statistical validation for a single condition (significant wave height, modal period, speed and heading). The objective of the present study is to check the robustness and repeatability of that success by carrying out additional validation calculations for different conditions.

The development of extrapolation methods for probabilistic assessment of seakeeping in extreme condition (Anastopoulos, et al. 2016, Belenky, et al. 2016, Campbell, et al. 2016) poses the problem of statistical validation. The result of simulation-based extrapolation is a random number that is estimated with a confidence interval. If a true value is known, the extrapolation can be regarded as successful if this true value falls within the confidence interval. However, due to the very same random nature, a single successful extrapolation result is hardly convincing. How would one know if this was not just a coincidence?

To ensure that the result is stable relative to the environmental conditions, Smith and Campbell (2013) and Smith, et al. (2014) introduced a multi-tier concept of statistical validation, which was originally proposed by Smith (2012) for general ship motion validation. The first tier is elemental – it is successful if the extrapolation result contains a "true" value within its confidence interval (the methodology of obtaining the true value is considered in the next section). The extrapolation procedure is then repeated several times for exactly the same condition, but using independent data sets – this is second tier. A successful validation for a given condition produces a certain percentage of successes, referred to as a "passing rate." Smith and Campbell (2013) proposed 90% as a level for acceptance, based on practical considerations.

The third tier of statistical validation includes consideration of several conditions reflecting the expected operations. It is not yet clear how many of those conditions need be successful for an extrapolation method to pass. Examples of the application of the procedure for the EPOT (Envelope Peak over Threshold) method (Campbell, et al. 2016) are considered in Smith (2014) and Smith and Zuzick (2015).

The calculation of the confidence interval of the extrapolated estimate is a key element for the statistical calculation and should be validated separately. The Generalized Pareto distribution (GPD) was used to approximate a tail for both split-time method and EPOT, from which one can create a set of GPD distributed data and apply the calculation of confidence interval. If these calculations are correct, the passing rate must tend to the confidence probability used in those calculations, see Glotzer, et al. (2016) for details.

This paper applies this multi-tiered procedure (Smith and Zuzick, 2015) to the evaluation of the probability of capsizing in irregular waves with the split-time method.

2. EVALUATION OF "TRUE VALUE"

The extrapolation validation procedure reviewed in the previous section requires *a priori* knowledge of the probability of capsizing. Theoretical solutions for probability of capsizing are available for piecewise linear models (Belenky, et al, 2016), but while these models do describe capsizing qualitatively, i.e. as a transition between two stable equilibria, they are too simplistic to be considered as realistic ship motions. In particular they cannot describe the realistic change of stability in waves as well as the fact that the hydrostatic restoring is inseparable from wave excitation for large-amplitude ship motions.

These effects are naturally included in advanced hydrodynamic codes (Reed, et al. 2014) such as LAMP (Lin and Yu 1990). However, these high-fidelity codes are not fast enough to produce samples of sufficient size that a statistically relevant number of capsizes can be observed in relevant wave conditions, as millions of hours may be required (Campbell, et al. 2016).

The solution was proposed by Weems and Wundrow (2013). The idea is to compute instantaneous submerged volume and calculate the inseparable hydrostatic and Froude-Krylov forces from this volume. The rest of the forces are approximated as coefficients. This approach yields reasonable results for relatively long waves, as the wave curvature is not resolved over the ship breadth but is resolved over the ship length, see Figure 1. Weems and Belenky (2015) show the qualitative adequacy of the approach by comparing shape of distributions of roll motion between the volume-based calculation and LAMP.

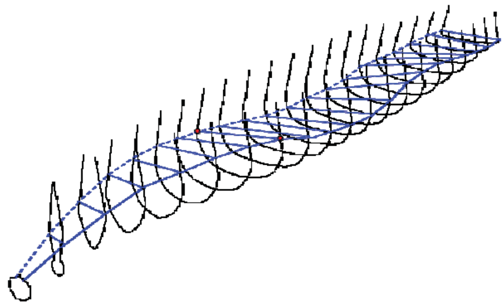

Figure 1 Station/incident wave intersection for volume based hydrostatic and Froude-Krylov forces for the ONR Tumblehome hull in stern oblique seas (Weems and Wundrow, 2013)

The use of the volume-based calculation instead of surface pressure integration for hydrostatic and Froude-Krylov forces makes the model almost as fast as models based on ordinary differential equations. Weems and Belenky (2015) reported that 10 hours data was generated in 7 seconds on a single processor of a laptop computer, allowing millions of hours of simulation data to be practically computed on a standard workstation or modest sized cluster.

3. ESSENCE OF THE SPLIT-TIME METHOD

The objective of the split-time method is to provide a means to use an advanced numerical code for estimating the probability of rare event without actually observing it in simulations. Its principal idea is to separate the estimation procedure into an observable or "non-rare" problem and a non-observable or "rare" problem. The "non-rare" problem is an estimation of the crossing rate of an intermediate threshold. It has to be low enough to observe a statistically significant number of upcrossing events in, say 100 hrs, but high enough so that most of these upcrossings can be treated as independent events.

The "rare" problem is solved for each upcrossing with a motion perturbation scheme shown in Figure 2. The roll rate is perturbed at the instant of upcrossing until capsizing is observed. The minimum value of roll rate perturbation leading to capsizing is a metric of the danger of capsizing danger at the instant of upcrossing.

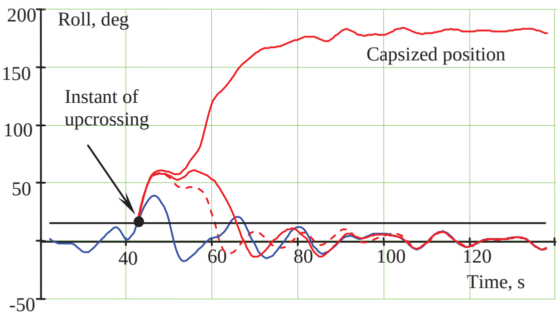

Figure 2 Illustration of motion perturbations

Given a sufficient number of upcrossings, the tail of the distribution of the metric value can be modeled with Generalized Pareto distribution (GPD), from which the estimate for the probability of capsizing can be evaluated. The most up-to-date description of the procedure can be found in Belenky, et al. (2016).

4. RESULTS

A typical example of the tier-two validation set is shown in Figure 3. A Bretschneider spectrum was used to simulated long-crested waves with a significant wave height of 9.0 m and a modal period of 14 s.

The subject ship is the ONR tumblehome topside configuration (Bishop, et al. 2005), speed was 6 knots and heading 60 degrees relative to wave propagation. The "true" value of the capsizing rate was estimated from 176 capsizing cases observed during 200,000 hours of the volume-based simulations.

The tier-two validation data set consists of 50 independent extrapolations shown in Figure 3. Each extrapolation estimate uses 100 hours of volume-based simulations, with no capsizing cases observed during those times. The extrapolation result is presented with a confidence interval for the 0.95 confidence probability. Besides these boundaries, each extrapolation has the most probable value (x in Figure 3) and the mean value (circle in Figure 3). The calculation of the mean and most probable value is discussed in details in Belenky, et al. (2016). The tier-one validation is successful if the confidence interval contains the "true" value. The case shown in Figure 3 has 45 individual extrapolations that contain the "true" value in its confidence interval. The tier-two validation is successful when a percentage of the underlining tier-one validation successes is close to the accepted confidence level. This number is 0.90 for the considered case, which would be considered a successful "passing rate" by Smith and Campbell (2013).

The environmental conditions for the entire validation campaign described in this paper are presented in Table 1, while the results are summarized in Table 2. The tier-two validation procedure was repeated three times on independent data to check the variability of the results.

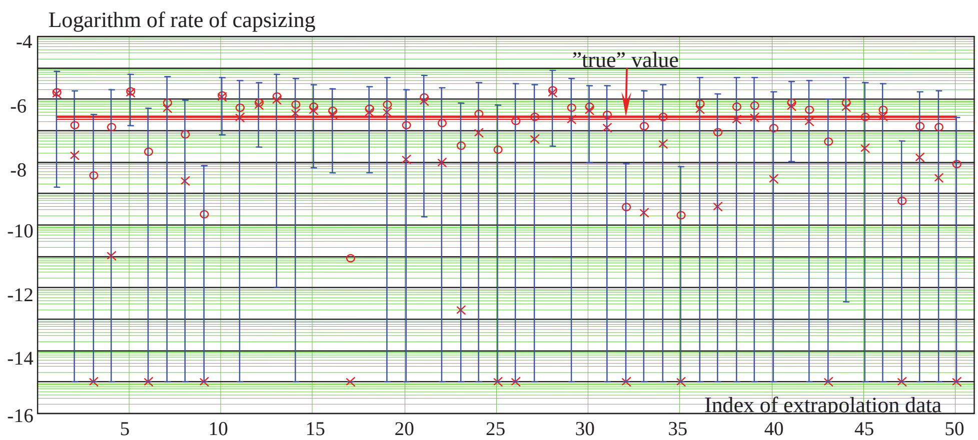

Figure 3 Example of validation tier-two case; significant wave height 9.0, modal period 14s, heading 60 deg, passing rate 0.90

Table 1 Summary validation conditions and "true" value estimates

Significant wave height, m	Modal Period, s	Heading, degrees	Exposure, hr	Number of Capsizes	Estimate of rate 1/s	Low boundary of rate	Upper boundary of rate
8.5	14	45	200,000	8	1.13E-08	4.24E-09	1.98E-08
8.5	14	60	200,000	31	4.38E-08	2.97E-08	5.93E-08
9	14	35	720,000	12	4.71E-09	2.04E-09	7.37E-09
9	14	40	200,000	12	1.70E-08	8.48E-09	2.68E-08
9	14	45	200,000	51	7.20E-08	5.37E-08	9.18E-08
9	14	50	20,000	7	9.89E-08	2.83E-08	1.84E-07
9	14	55	60,000	69	3.25E-07	2.50E-07	4.05E-07
9	14	60	200,000	176	2.49E-07	2.12E-07	2.85E-07
9	14	65	200,000	80	1.13E-07	8.90E-08	1.38E-07
9	14	70	200,000	6	8.48E-09	2.83E-09	1.55E-08
9	15	45	345,000	10	8.19E-09	3.11E-09	1.33E-08
9	15	60	300,000	11	1.04E-08	4.71E-09	1.70E-08
9.5	15	45	1,000,000	157	4.44E-08	3.74E-08	5.13E-08
9.5	15	60	1,000,000	242	6.84E-08	5.98E-08	7.70E-08

Table 2 Summary of validation results

Significant wave height, m	Modal Period, s	Heading, degrees	Subset duration, hrs	Passing rate Sample 1	Passing rate Sample 2	Passing rate Sample 3	Averaged passing rate
8.5	14	45	2,000	1.00	0.98	0.90	0.96
8.5	14	60	2,000	0.92	0.96	0.94	0.94
9	14	35	2,000	1.00	0.98	0.98	0.99
9	14	40	2,000	1.00	0.98	1.00	0.99
9	14	45	2,000	0.98	0.98	0.96	0.97
9	14	50	2,000	0.98	0.92	0.94	0.95
9	14	55	2,000	0.90	0.80	0.92	0.87
9	14	60	2,000	0.90	0.86	0.94	0.90
9	14	65	2,000	0.94	0.92	0.94	0.93
9	14	70	2,000	0.92	1.00	0.90	0.94
9	15	45	2,000	0.98	0.96	0.96	0.97
9	15	60	2,000	0.96	0.98	0.98	0.97
9.5	15	45	2,000	0.96	0.94	0.96	0.95
9.5	15	60	2,000	0.98	0.94	0.96	0.96

There were two cases when the passing rate fell below 0.9: for headings 55 and 60 degrees at 9 m waves. In general, the variability of the passing rate within the same environment condition is not small. The last column in Table 2 shows averaged passing rate per condition, which is equivalent to 150 extrapolation data sets. The averaging passing rate fell below 0.9 only once, for 55 degree heading, indicating favorable tendency with the increase of sample size.

Finally, if one averages the passing rate over all the conditions tested, the theoretical 0.95 is obtained. This is yet another indication of the statistical correctness of the split-time method.

5. CONCLUSIONS AND FUTURE WORK

The split-time method for estimating probability of capsizing caused by pure loss of stability has been subjected to statistical validation for 14 environmental conditions. The true values were obtained by a very fast volume based numerical simulation with a time of exposure of up to one million hours full-scale. The rare problem solution is based on single degree-of-freedom perturbations. The average passing rate per condition varied from 0.87 to 0.99, falling below 0.90 for a single condition. The passing rate averaged over all the tested condition was 0.95, while the confidence probability was 0.95. These results are encouraging.

At the same time, the described validation campaign shows the necessity to refine the acceptance criteria, in particular what passing rate should be expected depending on how many extrapolation data sets were used. The acceptance criteria are needed for the tier-three validation level which addresses overall acceptance.

6. ACKNOWLEDGEMENTS

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