Split-time Algorithm Implementation in Advanced Hydrodynamic Codes

Kenneth Weems, *Naval Surface Warfare Center Carderock Division*
Vadim Belenky, *Naval Surface Warfare Center Carderock Division*

ABSTRACT

The paper describes the current state of numerical implementation of the split-time method for the estimation of probability of capsizing in irregular waves using an advanced numerical code – Large Amplitude Motion Program (LAMP). The split-time method resolves the probability of capsizing into two steps. The first step or "non-rare" problem is the statistical estimation of crossing rates over an intermediate threshold; the second step or "rare" problem is the calculation of the probability of capsizing after crossing. Motion perturbations are used to estimate the latter. The value of the perturbation of the roll rate at the instant of crossing which would lead to capsize is used as a metric of danger. Metric values from all crossings are extrapolated using the generalized Pareto distribution to determine a rate of capsize after crossing. The implementation is based on 3 degrees-of-freedom model (heave-roll-pitch), in which the body nonlinear formulation is used for hydrostatic and Froude-Krylov forces while all other hydrodynamic forces are modeled with empirical coefficients. The paper describes the initial testing of the algorithm, problems that were encountered and ongoing development including introduction of the hydrodynamic memory effects in the simulation of perturbed motions

Keywords: *Probability of capsizing, Numerical Simulations, split-time method, motion perturbation*

1. INTRODUCTION

The objective of the split-time method is to use the capability of advanced numerical codes for the estimation of the probability of rare event such as capsizing in waves. As capsizing in realistic conditions is too rare to be observed with a practical set of numerical simulation, the split-time method proposes the separation of the problem into "non-rare" and "rare" problems based on what is observable and non-observable in "normal" numerical simulations in random irregular seas.

The solution of the "non-rare" problem consists of computing a set of simulations in pseudo-random realizations of the irregular sea conditions and identifying crossings of an intermediate threshold roll angle. In this context, crossings consist of up-crossings of the positive threshold roll angle and down-crossings of the negative threshold roll angle. Crossings of this threshold should be observable in these "normal" numerical simulations in a statistically representative quantity. The choice of the threshold is arbitrary, but only independent crossing events can be used for the further calculations. As a result, the selection of the intermediate threshold is a mostly an issue of calculation efficiency – too low of a threshold will result in a large number dependent crossings, many of which would need to be discarded, while too high of a threshold will result in too small a number of crossings.

The "rare" problem focuses on the estimation of the conditional probability of capsizing when crossing has occurred. A metric of the danger of capsizing is calculated at the instant of each crossing using a motion perturbation approach. A series of perturbation simulations are performed in the same waves as the non-rare simulation, starting from the crossing point but with the roll rate increased until capsizing is observed. The smallest roll rate perturbation which leads to immediate capsizing is the metric of capsizing danger as it measures how close the ship was to capsizing, even though capsizing or even an extreme roll angle may not have been observed.

Once the sufficient size of metric value sample (sufficient number of crossings) has been collected, the tail of its distribution can be modeled and used to estimate the conditional probability of capsizing at the instant of crossing, as illustrated in Figure 1.

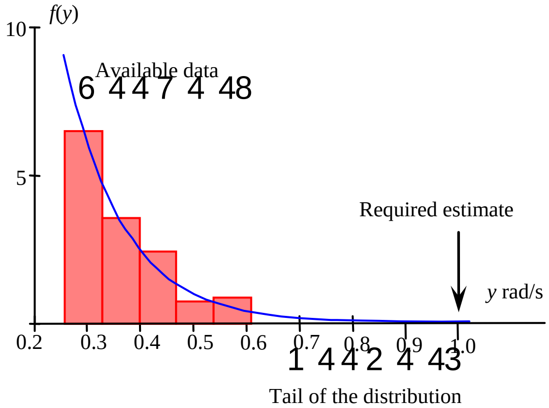

Figure 1 Calculation of the conditional probability of capsizing after crossing

In order to facilitate the modeling of the tail, the metric is calculated as:

$$y_i = 1 - \dot{\phi}_{Ui} + \dot{\phi}_{Cri} \; ; \quad i = 1, \ldots, N_U \qquad (1)$$

where $\dot{\phi}_{Ui}$ is the value of rate roll observed at the *i*-th crossing, $\dot{\phi}_{Cri}$ is the value of perturbed roll rate at that crossing which lead to capsizing, and N_U is the number of crossing observed. The probability of capsizing after crossing is calculated by extrapolating this distribution to a value of 1.0.

A review of the background theory of the split-time method for the probability of capsizing in wave is available from Belenky, et al. (2016).

2. NUMERICAL CODE

The initial implementation of the split-time method is carried out using the Large Amplitude Motion Program (LAMP) as a platform. LAMP is a mature all-purpose numerical code for ship motions and loads; its theoretical background is described by Lin and Yue (1990). Hydrostatic and Froude-Krylov forces are calculated with the full 3D body-nonlinear formulation. The diffraction and radiation forces are computed using a 3-D potential flow panel model using either a body-linear or body-nonlinear formulations. Other forces (roll damping, maneuvering forces, control systems, etc.) are included using a variety of time-domain models.

The LAMP system consists of a number of modules providing tools for the preparation and verification of input data and the post-processing of simulation results.

3. CALCULATION SCHEME

The overall sequence of calculations is illustrated in Figure 2. After setting up the LAMP model, a number of independent records, each corresponding to different realizations of the same irregular sea spectrum, are computed. A typical set of simulations contains 200 records of 30 minutes each. The 30 minute record length is long enough for the initial transition to be considered small portion of the record, but short enough to require a moderate number of wave components (usually 250-300) wave components to avoid self-repeating effect. Presenting the 100 hours sample in 200 independent records also facilitates parallel calculations, so a cluster or High Performance Computing (HPC) can be used in its full effect and mitigates potential non-ergodicity effects.

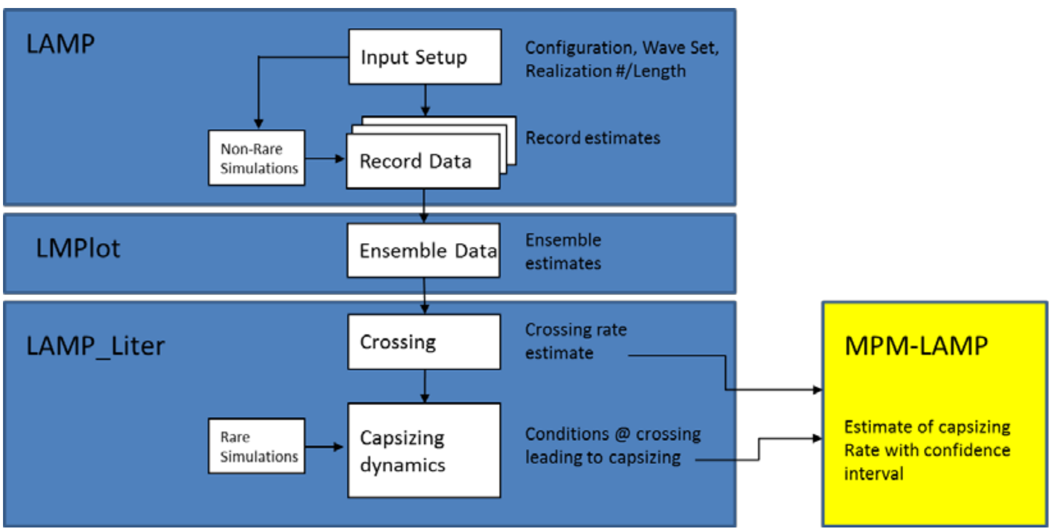

Figure 2 General scheme of split-time method implementation with LAMP

The set of the time history records computed for exactly the same set of conditions (wave spectrum, ship speed and relative wave heading) represent an ensemble. Statistical estimates of the ensemble are computed using LMPlot, which is the principal LAMP-system module for post-processing and plotting.

The LAMP_Liter module reads the non-rare simulation histories, identifies crossings of one or more specified threshold levels, calculates the estimated crossing rate and runs the perturbation simulations to find the value of the metric at each crossing. The MPM-LAMP module fits the GPD to the metric values, extrapolates to find the probability of capsizing after crossing and calculates the overall capsizing rate.

The initial implementation and testing of the split-time method in LAMP considers 3-DOF motions (heave-roll-pitch) and uses the 3-D body nonlinear formulation for hydrostatic and Froude Krylov forces, while diffraction and radiation are modeled using empirical coefficients rather than the full potential flow solution of the wave-body interaction problem. This configuration of the LAMP solver is known as LAMP-0. For these calculations, the same options are used for both the non-rare and rare simulations, though this is not required by either the theory or its implementation.

4. NON-RARE PROBLEM

The non-rare problem is solved by searching for crossings of one or more prescribed threshold roll angles. Once a crossing has been found, the value of the roll rate at the instant of crossing is determined by interpolation, see Figure 3.

The rate of crossing is estimated over the ensemble of records:

$$\hat{\lambda} = \frac{N_U}{N_T \Delta t} \qquad (2)$$

where N_U is the observed number of crossings, N_T is total number of data points in all records, and Δt is the time increment (data sampling rate), which is assumed to be the same for all records. The boundaries of the confidence interval of the crossing rate are calculated with the assumption of binomial distribution (Belenky, et al. 2016).

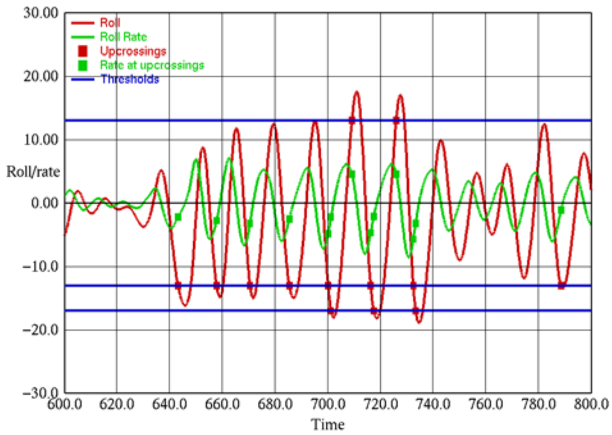

Figure 3 Non-rare problem: search for crossings and calculation of the roll rate values at the instants of crossing

5. RARE PROBLEM

The calculation of the critical roll rate is carried out using the motion perturbation method (MPM) as illustrated in Figure 4. The MPM is essentially a series of short simulations, starting from the instant of crossing, in the same waves as the non-rare simulation and with initial conditions other than roll rate set to ship's position and velocity at the crossing. The initial roll rate is systematically changed until capsizing is observed. Note that when the perturbed simulation does not capsize, the motion returns to its original time history. The critical roll rate is the smallest roll rate leading to capsizing.

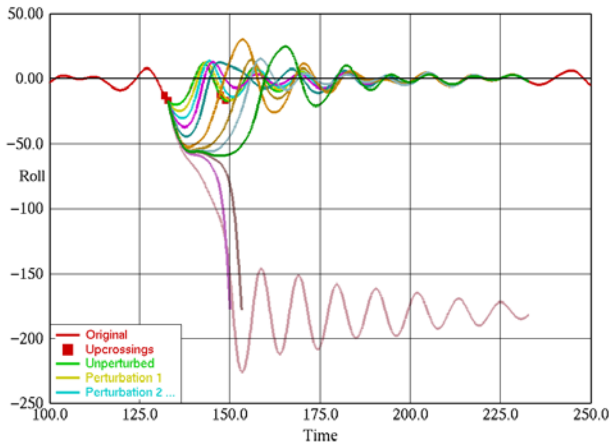

Figure 4 Calculation of critical roll rate with the motion perturbation method

As it can be seen from Figure 4, some of the time histories, while obviously bound to capsize, did not actually reach the motion about the capsized equilibrium. The reason is that LAMP calculations sometimes exhibit numerical instability when roll angle passes 90 degrees. This numerical instability is caused by the way in which the 3-DOF motion constrains have been implemented in the LAMP's 6-DOF dynamic solver.

The split-time, however, does not require simulations to be carried so far – it is simply necessary to determine whether capsizing would occur. In fact, to reduce the computational effort, the perturbation simulations are usually truncated as soon as a roll angle of 90 degrees is reached or the motion converges to the unperturbed solution.

After the calculation of the capsizing likelihood metric (1), the results must be de-clustered, as the fitting of the GPD requires independent data points. As can be seen from Figure 3, crossings are observed in clusters and are likely to not be independent events. To produce independent data points, the metric data (1) is de-clustered. An estimate of the auto-correlation function for the roll response is calculated from the non-rare motion data and a de-correlation time is found by looking for the point where the envelope of the peaks of the auto-correlation falls below 0.05, see Figure 5.

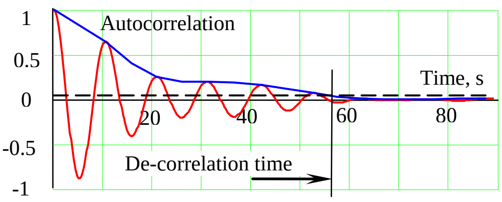

Figure 5 Calculating de-correlation time from the auto-correlation of the roll response

Crossing events which are separated by the de-correlation time are assumed to be independent while events closer than that are assumed to be part of a cluster. The largest metric value in each cluster is selected to provide only independent data for the GPD fit.

The procedure for fitting the GPD distribution to the LAMP-computed metric has been implemented following Campbell, et al. (2016).

6. INITIAL TESTING

Initial testing has been performed on a Windows workstation and on the NSWCCD SeaTech Linux cluster. On the SeaTech cluster, 5 cores on each of 10 nodes can be used to run 50 LAMP or LAMP-Lite simulations in parallel, resulting in a run time for the complete procedure of about 30 minutes per long-crested condition for a properly selected threshold.

Initial testing considered 10 conditions: two significant wave heights with five relative headings each. The fitted GPD distributions have shown smaller values of the shape parameter in comparison to the validation runs made with the volume-based numerical model (Weems, et al. 2016). A full investigation into the relationship between the GPD parameters and the characteristics of the hydrodynamic model and dynamical system remains for future work, though some first steps in this direction can be found in Belenky, et al. (2016a).

7. HYDRODYNAMIC MEMORY

A significant challenge of using motion perturbation methods with numerical seakeeping simulation tools is the consideration of the hydrodynamic memory effect. Hydrodynamic memory is an effect in which the flow field and forces of the wave-body hydrodynamic interaction problem are dependent on the short or medium-term history of the solution and cannot be completely quantified as functions of the state variables and their derivatives as in a model based n ordinary differential equations (ODE). In potential flow seakeeping models, this memory is associated with the unsteady disturbance wave field generated by the ship's unsteady motion (radiation waves), interaction with the incident wave (diffraction waves) and forward speed (Kelvin waves). In viscous flow solvers (e.g. RANS and LES), they will also be associated with the generation and evolution of vortical flow structures and the like.

Motion perturbation analysis requires simulations starting at crossing points of the non-rare simulations with variations to selected state variables, which will be the roll velocity for the present capsizing problem. It is relatively straightforward to save the complete state of the calculation, including the unsteady free surface disturbance, and then to restart the perturbation simulation from this point. However, large variations in the roll rate generally result in a significant transient behavior due to the impulsive change in velocity, which often lead to instability in the free surface potential flow solution.

The simplest solution to the problem is to use an ODE-like approximation for the disturbance wave forces in the perturbation simulations rather than attempting to solve the free surface potential flow problem. In its most basic form, this consists of the prescribed added mass and damping coefficients of the LAMP-0 model used in the implementation and initial testing of the MPM described abvoe and in the validation cases described in Weems, et al. (2016). As these provide an explicit calculation of the radiation and diffraction effects in terms of the state variables, they have no problem with the perturbation to the roll rate or other state variables and have the significant advantage that they result in a relatively fast calculation of the perturbation simulations. The approach is, however, approximate and the effect of the approximation will need to be quantified.

The incorporation of the regular time-domain free surface potential flow solution in the perturbation simulations comes down to introducing the perturbation of the motion while maintaining the stability and correctness of the flow solution. The most promising scheme identified to date is to begin the perturbation calculation some time, perhaps 10-20 seconds, before the crossing event, with prescribed motions during the period up to the event. The prescribed motions would be based on the motions from the non-rare simulation with the velocity perturbation feathered in over this time. An advantage of such an approach is that it could be implemented with regular check-pointing of the non-rare solution without having to identify and save crossing points during the non-rare simulations. A disadvantage of such an approach is that it will be computationally relatively expensive.

Another approach toward incorporating memory into the perturbation simulations would be to use an impulse response function (IRF) solution of the disturbance potential. The IRF-based formulation of the wave-body interaction problem uses body-linear solutions of the impulsive radiation and diffraction problems that are convoluted with the wave and motion time history to provide a very rapid approximate body-nonlinear solution. The method has long been used for constant course and speed seakeeping simulations (Weems, et al. 2000), and could be adapted to the perturbation simulations in which the ship can be assumed to have constant course and speed for the duration of the perturbation. The motion perturbation would still need to be added to the motion history but stability and speed issues would be considerably mitigated.

It is quite likely that practical considerations will drive the implementation toward an ODE-like model of the disturbance, albeit one with non-constant coefficients derived from the motion history. However, a solution with the more complete hydrodynamic memory is necessary to quantify the effect of the memory and develop the required models.

8. SUMMARY AND CONCLUSIONS

The paper described the current state of implementation of the split-time estimation of method for probability of capsizing. The metric of likelihood of capsizing is the difference between observed and critical roll rate at the instant of crossing of an intermediate threshold. The critical roll rate (minimal perturbed roll rate leading to capsizing) is calculated with a motion perturbation method (MPM).

The split-time/MPM method has been implemented in the Large Amplitude Motion Program (LAMP). For the initial implementation and testing, the hydrodynamic forces are modeled with empirical coefficients, while hydrostatic and Froude-Krylov forces were computed with full 3D body-nonlinear formulation (LAMP-0). Motions were simulated with three degrees of freedom: heave, roll and pitch

Ongoing implementation and testing work includes the introduction of hydrodynamic memory in the perturbed motion calculations and free surge, sway and yaw motion in the non-rare and rare simulations.

9. ACKNOWLEDGEMENTS

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