The Inertia Contributions due to Floodwater Mass

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Abstract: The Stability in Waves committee of the 27th ITTC investigated how to deal with the ship inertia contributions due to floodwater mass from three points of view: (1) floodwater domain, (2) floodwater inertia itself, (3) floodwater entering the ship. The committee suggested three criteria indicating the concept of how to deal with floodwater and providing clues on what to consider as floodwater when examining damaged ships: (1) whether the water is moving with the ship and the mass of that volume of water, (2) whether there is a significant pressure jump across the compartment boundary, and (3) whether the dynamics of water can be solved separately. For floodwater inertia, the committee divided this into the partially flooded case and fully flooded case, and investigated the properties and showed how to deal with floodwater inertia for each case. For the case of the floodwater entering the ship, the treatment of the inertia change due to floodwater was derived using the momentum change principle. The related ITTC procedure was updated reflecting this work.

Key words: Floodwater, inertia of floodwater, domain of floodwater

1. Introduction

One of the tasks of the committee on Stability in Waves of the 27th ITTC is to investigate how to deal with the inertia due to the floodwater mass, and to update the relevant ITTC procedure for damage stability simulations. The committee investigated this task from three points of view: (1) floodwater domain, (2) floodwater inertia itself and (3) floodwater entering the ship.

The boundary of the floodwater domain is hard to determine for a large damage opening. The committee suggested three criteria indicating the concept of how to deal with floodwater and providing clues on what to consider as floodwater when examining damaged ships: (1) whether the water is moving with the ship and the mass of that volume of water, (2) whether there is a significant pressure jump across the compartment boundary and (3) whether the dynamics of water can be solved separately.

For the partially flooded compartment, the motion of floodwater is usually analysed by three techniques, namely quasi-static, quasi-dynamic and full dynamic analysis. Quasi-static and quasi-dynamic analyses consider only the centre of gravity of the floodwater, and the mass of floodwater should be included in the ship's mass. However, in full dynamic analysis, the pressure includes all static and dynamic pressure components, therefore the force derived from the pressure integration on the surface of the compartment includes all the effects of floodwater inertia and flow properties. This is subject to the condition that the body force includes the actual acceleration, that is, the gravitational acceleration and the acceleration of the

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flood water. In this case, the mass of flood water should not be included in the ship's mass.

In the case of a fully flooded compartment, the floodwater is often treated as a solid and is included in the ship's mass in many studies for the motion dynamics of ships. In order to clarify this problem, the committee reviewed the work of Lee (2014). In his study, the inertial properties of a compartment fully filled with liquid were studied based on potential flow theory. An analytic solution was obtained for the rectangular tank, and the numerical solutions using Green's 2nd identity were obtained for other shapes. The inertia of liquid behaves like a solid in rectilinear acceleration. But under rotational acceleration, the moment of inertia of liquid becomes small compared to that of a solid. The shapes of the compartments investigated in his study were ellipsoid, rectangular, hexagonal and octagonal with various aspect ratios. The numerical solutions were compared with analytic solutions, and an ad hoc semi-analytical approximate formula is proposed which gives a very good prediction for the moment of inertia of the liquid in a tank of several different geometrical shapes. The results of his study will be useful in analysing of the motion of LNG/LPG tankers, liquid cargo ships and damaged ships.

For the case of the floodwater entering ship, the treatment of inertia change due to floodwater was made clear using the momentum change principle. The related ITTC procedure was updated reflecting this work.

2. Floodwater Domain

There is the problem of which region should be treated as floodwater if the damage opening is large enough. So we first need a more reasonable and clear definition of floodwater in the analysis of a damaged ship. If we focus on the inertia properties, the floodwater can be determined by looking at whether the water is moving with the ship or not. If we focus on the hydrodynamics, floodwater may be determined by investigating whether the pressure of it is strongly related with outside water level, and whether the hydrodynamic problem of floodwater can be analysed separately, provided that the boundary condition is given for the matching of the inner and outer flow domains.

Therefore the following may be criteria that will be used to determine the floodwater.

- Whether the water is moving with the ship and the mass of that water volume.
- Whether there is a significant pressure jump across the compartment boundary.
- Whether the dynamics of water can be solved separately.

The above three criteria indicate the concept of how to deal with floodwater and provide clues on what to consider as floodwater when examining damage ships.

3. Inertia of Floodwater

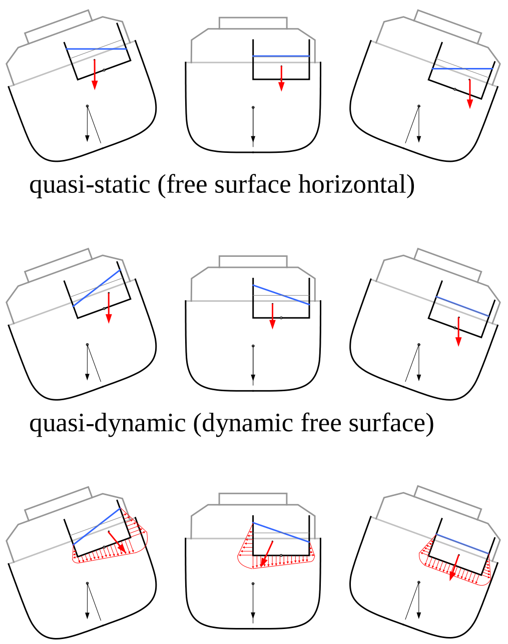
3.1 Partially Flooded Compartments

The hydrodynamics and the force on the compartment partially filled with flood water can be calculated by theory or numerical schemes, such as resonant mode analysis, potential theory, CFD with free surface, etc. In these methods, the force originated from floodwater is treated as an external force, and the motion of a ship is affected by it. However in this case, it is uncertain whether the mass of the floodwater should be included in the ship's mass or not.

The forces due to floodwater can be divided into three parts by considering their origins. The first is the one due to gravitational acceleration, the second one is due to the acceleration by the ship motion, and the third one is due to the dynamic pressure of the flow of floodwater. The interactions of floodwater and ship motion were summarised in the 26th ITTC report by the Stability in Waves committee. The interaction concept is given in Table 1, while the concepts of these three models are shown in Figure 1.

Table 1 Three models of interactions (from 26th ITTC report)

	Floodwater treatment	Interaction concept
Quasi-static	static	added weight
Quasi-dynamic	dynamic	added weight
Dynamic	dynamic	added force



(a) quasi-static (free surface horizontal)

(b) quasi-dynamic (dynamic free surface)

(c) dynamic (dynamic free surface, fluid pressure force)

Figure 1 Concept of floodwater and ship motion interaction (from 26th ITTC report)

In quasi-static or quasi-dynamic analysis, because it considers only the centre of gravity of the flood water and only the gravitational force, the mass of flood water should be included in the ship's mass in order to represent the inertia force, that is, the force due to the acceleration by the ships motion. However in fully dynamic analysis, the pressure includes all the static and dynamic pressure components, the force derived from the pressure integration on the surface of the compartment includes all the effects of floodwater inertia and flow properties. This is subject to the condition that the body force includes the actual acceleration, that is, the gravitational acceleration and the acceleration of the flood water due to the ship's motion. In this case, the mass of flood water should not be included in the ship's mass. The following conceptual equations of motion show how the floodwater inertia should be included.

Quasi-static, quasi-dynamic analysis,

$$(m + \boxed{m_F})\ddot{x} + b\dot{x} + cx = F_{ext} + F_G \quad (1)$$

Fully dynamic analysis,

$$m\ddot{x} + b\dot{x} + cx = F_{ext} + \boxed{F_{FL}} \quad (2)$$

As explained above, in quasi-static or quasi-dynamic analysis, the force due to the floodwater is a gravitational force, this is included in the right side as external force. In this case, the mass of floodwater, m_F should be included in the ship's mass, as in Eq. (1). And in fully dynamic analysis, if the floodwater force, F_{FL} includes all the forces due to gravitational acceleration, the acceleration due to the ship's motion, and dynamic pressure of the flow, the mass of floodwater should not be included into the ship's mass.

3.2 Fully Flooded Compartments

The flood water in a fully filled compartment is often treated as a part of the ship and treated as a solid. In rectilinear acceleration, the flood water acts like a solid. In rotational acceleration, the moment of inertia is smaller than that of a solid, because there is a part of water that does not rotate with the ship. Lee (2014) shows the ratio of the moment of inertia of flood water and that of solids for various shapes of compartments.

$$C_R = I_{Liquid} / I_{Solid} \quad (3)$$

where I_{Liquid} and I_{Solid} are the moment of inertias of the flood water when treated as liquid and solid respectively.

Figure 2 shows the shapes of compartments investigated in his study.

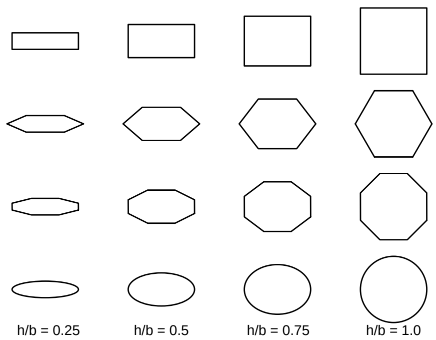


Figure 2 Various shapes of tanks useful for application from Lee (2014)

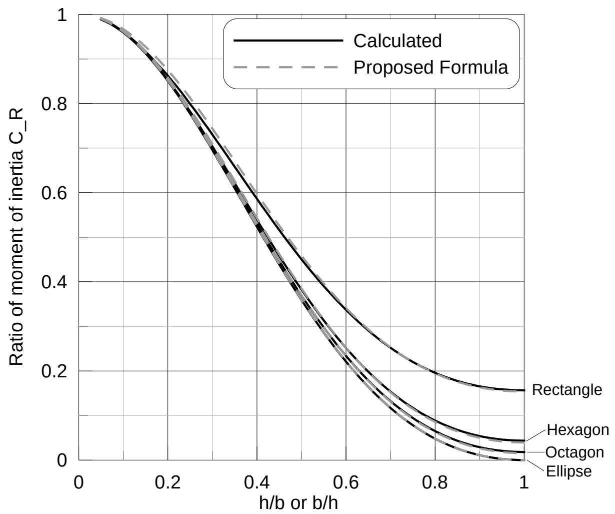


Figure 3 Moment of Inertia prediction of fully filled liquid for various shaped tanks; calculated and estimated from Lee (2014)

The inertias of the fluid in tanks of different aspect ratios and shapes become small as the aspect ratio goes to unity, see Figure 3. The solid lines in Figure 3 are analytical or numerical results while the dashed lines show an estimation formula that provides accurate results. The estimation formula is as follows:

$$I_{Liquid} = I_{Solid} - I_e = I_{Solid} - \rho k_e \frac{A^2}{\pi}\left(\frac{hb}{h^2+b^2}\right) \quad (4)$$

where the shape correlation factor k_e is

$$k_e = \left(\frac{A_{ellipse}}{A}\right)^{2/n} = \left(\frac{\pi hb}{4A}\right)^{2/n} \quad (5)$$

If we put the area A from Table 2, the factor k_e turns out as a coefficient dependent only on the type of the shape as follows,

$$k_e = \begin{cases} (\pi/4)^{1/2} & \text{for rectangle} \\ \left(\pi/2\sqrt{3}\right)^{1/3} & \text{for hexagon} \\ \left(\dfrac{\pi}{8(\sqrt{2}-1)}\right)^{1/4} & \text{for octagon} \\ 1 & \text{for ellipse} \end{cases} \quad (6)$$

Table 2 Area and moment of inertia of solid for various shapes from Lee (2014)

shape	Number of edges (n)	Area	Moment of inertia for roll
Rectangle	n=4	$A = hb$	$\frac{I_{Solid}}{\rho} = \frac{1}{12}A(h^2+b^2)$
Hexagon	n=6	$A = \frac{\sqrt{3}}{2}hb'$	$\frac{I_{Solid}}{\rho} = \frac{5}{72}A(h^2+b^2)$
Octagon	n=8	$A = 2(\sqrt{2}-1)hb$	$\frac{I_{Solid}}{\rho} = \frac{3-\sqrt{2}}{24}A(h^2+b^2)$
Ellipse	n=∞	$A = \frac{\pi}{4}hb$	$\frac{I_{Solid}}{\rho} = \frac{1}{16}A(h^2+b^2)$

4. Inertia of Floodwater Entering Ship

Newton's Second Law states that the force (moment) on a body is equal to its time rate-of-change of momentum (angular momentum). For a body of constant mass (moment of inertia) this translates to $\vec{F} = m\vec{a}$ ($\vec{M} = I\,d\vec{\omega}/dt$). However, for a body such as a rocket which is burning fuel and ejecting gas or a damaged ship in a seaway taking on and possibly discharging water, the $\vec{F} = m\vec{a}$ analogy is not correct, but in fact the time-rate-of-change of mass must be taken into account. As the force must remain independent of the coordinate system, a simple application of the rule for differentiation of the product of two functions is not correct. The contribution from the time-rate-of-change of mass term belongs on the left-hand side of the equation with the force. In the context of rocket propulsion, the time-rate-of-change of mass contribution is the equivalent of the thrust of the rocket motor, and the entire system must be looked at as a constant mass system. Similar analogies apply to the time-rate-of-change of moment of inertia.

If we represent the momentum of the vessel as $\vec{p}$ and the angular momentum as $\vec{L}$, where $\vec{p}=m\vec{v}$ and $\vec{L}=I\vec{\omega}$, with m the mass of the ship, $\vec{v}$ the velocity, I the moment of inertial tensor and $\vec{\omega}$ the angular velocity, then Newton's second law can be written as:

$$\vec{F}=m\frac{d\vec{v}}{dt}, \qquad \vec{M}=I\frac{d\vec{\omega}}{dt}. \tag{7}$$

When the mass and hence the moment of inertia are constant, then these equations reduce to the traditional $\vec{F}=m\vec{a}$ form. However, in the damaged condition, the vessel's mass and moment of inertia vary with time and the equations of motion must be written in the above form. Rewriting equation (7) to account for the intake or discharge of floodwater as for a closed system yields:

$$F-\vec{v}\,'\frac{dm}{dt}=m\frac{d\vec{v}}{dt}, \qquad M-\vec{\omega}\,'\frac{dI}{dt}=I\frac{d\vec{\omega}}{dt}, \tag{8}$$

where $\vec{v}\,'$ and $\vec{\omega}\,'$ are the velocity and angular velocity of the flooding (discharging) water relative to the vessel, respectively. All of the quantities $\vec{v}\,'$, dm/dt, and $\vec{\omega}\,'$ can be determined from analysis of the flow at the damaged opening (if there is flow between flooded compartments, then the flow between the compartments must be incorporated in a similar manner.) The evaluation of dI/dt is somewhat more complex as it involves the actual shape of the compartment.

The above material dealing with the inertia change due to floodwater was included in the ITTC procedure 7.5-02-07-04.4.

5. Conclusions

The committee investigated how to deal with the inertia due to floodwater mass from three points of view: (1) floodwater domain, (2) floodwater inertia itself and (3) floodwater entering the ship.

For the floodwater domain, the committee proposed the criteria that will be used to determine the floodwater. For floodwater inertia, the committee divided this into the partially flooded case and fully flooded case, and investigated the properties and showed how to deal with floodwater inertia for each case. For the case of the floodwater entering the ship, the treatment of inertia change due to floodwater was made clear using the momentum change principle. The related ITTC procedure was updated reflecting this work.

Acknowledgments

The aim of this paper is to introduce the work of the ITTC Stability in Waves committee on the damage simulations. The present paper contains a chapter of the SiW report to the 27th ITTC in a rearranged format.

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