Onboard evaluation of the transverse stability for officers

Daisuke Terada and Akihiko Matsuda

National Research Institute of Fisheries Engineering, Fisheries Research Agency (Japan)

ABSTRACT

In this study, the convenience method for the onboard evaluation of the transverse stability is proposed based on an exponential autoregressive modeling procedure, which is a kind of time series analysis. The verification of the proposed method is implemented by using results of model experiments concerning the parametric roll resonance and onboard data under the ordinary navigation. It can be confirmed that transverse stability can evaluate by monitoring the behavior of characteristic roots on the exponential autoregressive model.

KEYWORDS

exponential autoregressive modelling, AIC (Akaike Information Criterion), navigation support system.

INTRODUCTION

For officers, the transverse stability is very important factor to keep the safe navigation of the operating ship. In general, officers confirm the static transverse stability by using a loading calculator. However, in actual navigation, the transverse stability changes due to the influence of waves. In extraordinary circumstances, the ship occurs of a serious accident such as the capsizing. Therefore, in order to protect the accident, we consider that officers should keep monitoring the roll motion, which is directly related with the transverse stability during the navigation. If we can always monitor the roll motion, then we can understand the state of the motion appropriately by using the knowledge of the time series analysis. Note that there is many studies such as have been published in the past STAB conference and the ISSW [e.g. Umeda et al (2007), Kawahara et al (2009), Francescutto & Umeda (2010), Umeda & Yamamura (2010) and so on] from the viewpoint of naval architecture concerning this issue.

In a little past, we showed the relationship between the dynamical system of the linear or the nonlinear roll motion and the time series model [Terada & Matsuda, 2011]. That is, the dynamical system of the linear roll motion can be approximated by the stationary autoregressive model, and the dynamical system of the nonlinear roll motion can be approximated by the time-varying autoregressive model. Thus, if we can get the time series data of the roll motion, we can evaluate the transverse stability directly for any data intervals by using the results of the stationary or the time-varying autoregressive modeling. However, using of these methods is difficult for the onboard evaluation of the transverse stability, because the calculation cost is too large.

To solve this problem, we attempt to apply an exponential autoregressive modeling procedure [Haggan & Ozaki, 1981]. In this procedure, the time-varying autoregressive coefficient is approximated by the exponential function, and the estimation calculation of coefficients is implemented by the least sqares method. As well as the time-varying autoregressive modeling procedure, in the exponential autoregressive modeling procedure a characteristic root calculated from a characteristic equation of the model is also very important to evaluate the transverse stability. That is, if all characteristic roots lie inside of the unit circle, then the system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as the nonlinear for the restoring force. Therefore, since officers can understand the dynamics of the roll motion under navigation in detail, it is considered that the proposed method is useful as the way to motivate the safe navigation in officers.

To confirm the effectiveness of the proposed method, we analyzed the data of the steady state and the parametric roll resonance. The obtained findings are reported.

NONLINIEAR STOCHASTIC DYNAMICAL SYSTEM

Consider the following nonlinear stochastic dynamical system concerning the roll motion:

$$\ddot{x}(t) + f(\dot{x}(t)) + g(x(t)) = u(t) \qquad (1)$$

where x(t) indicates a roll angle, the notation $(\cdot)$ and $(\cdot\cdot)$ indicate the 1st and the 2nd order deferential operator with time, $f(\ )$ indicates the nonlinear mapping function concerning the damping force, $g(\ )$ indicates the nonlinear mapping function concerning the restoring force and $u(t)$ indicates an external disturbance that is treated with the random variable, respectively. Note that $u(t)$ has the finite variance, but is not white noise sequence. And Equation 1 can be written by the following vector form:

$$\dot{\mathbf{x}}_t = \mathrm{f}(\mathbf{x}_t) + \mathbf{u}_t \qquad (2)$$

where, as the notation (T) means the transpose,

$$\mathbf{x}_t = \left[\dot{x}(t), x(t)\right]^T,$$
$$\mathrm{f}(\mathbf{x}_t) = \left(-f(x(t)) - g(x(t)), \dot{x}(t)\right)^T, \quad .$$
$$\mathbf{u}_t = \left[u(t), 0\right]^T$$

According to the locally linearization method [Ozaki, 1986], Equation 2 can be discretized as follows:

$$\mathbf{x}_n = \mathrm{EXP}\left[\mathbf{K}_{n-1}\Delta t\right]\cdot\mathbf{x}_{n-1} + \mathbf{B}_{n-1}\mathbf{u}_n \qquad (3)$$

where,

$$\mathbf{x}_n = \left[\dot{x}_n, x_n\right]^T, \quad \mathbf{K}_n = \frac{1}{\Delta t}\mathrm{LOG}(\mathbf{A}_n),$$
$$\mathbf{A}_n = \mathbf{I} + \mathbf{J}_n^{-1}\left\{\mathrm{EXP}(\mathbf{J}_n\Delta t) - \mathbf{I}\right\}\mathbf{F}_n,$$
$$\mathrm{LOG}(\mathbf{A}_n) = \sum_{k=1}^{\infty}\frac{(-1)^k}{k}(\mathbf{A}_n - \mathbf{I})^k,$$
$$\mathbf{J}_n = \frac{\partial \mathrm{f}(\mathbf{x}_n)}{\partial \mathbf{x}_n}, \quad \mathbf{F}_n\mathbf{x}_n = \begin{pmatrix} -f(\dot{x}_n) & -g(x_n) \\ \dot{x}_n & 0 \end{pmatrix},$$

Δt indicates a discrete interval and $\mathbf{B}_{n-1}\mathbf{u}_n$ is a two-dimensional colored noise sequence, which is obtained by the stochastic integral.

TIME SERIES MODEL

In Equation 3, since the term of the noise is not the white noise sequence, it is necessary to transform the colored noise sequence into the white noise sequence in order to deal with the problem stochastically. As the way to do the whitening, Yamanouchi (1956) showed the way to use the discrete autoregressive process. That is, in Equation 3, let

$$\varepsilon_n \equiv \mathbf{B}_{n-1}\mathbf{u}_n . \qquad (4)$$

Then this can be approximated by the following m-th order discrete autoregressive process.

$$\varepsilon_n = \sum_{i=1}^{m}\mathbf{D}_i\varepsilon_{n-i} + \mathbf{w}_n, \quad \left(\varepsilon_n = \mathbf{w}_n \quad for\ i = 0\right), (5)$$

where $\mathbf{w}_n$ is a 2×2 Gaussian white noise sequence with $N(0,\mathrm{diag}(\sigma_1^2,\ \sigma_2^2))$ and $\mathbf{D}_n$ indicates a 2×2 autoregressive coefficient matrix. On the other hands, the following relation is evident.

$$\begin{aligned}
\varepsilon_n &= \mathbf{x}_n - \mathbf{A}_{n-1}\mathbf{x}_{n-1} \\
\varepsilon_{n-1} &= \mathbf{x}_{n-1} - \mathbf{A}_{n-2}\mathbf{x}_{n-2} \\
&\vdots \\
\varepsilon_{n-m} &= \mathbf{x}_{n-m} - \mathbf{A}_{n-m-1}\mathbf{x}_{n-m-1}
\end{aligned} \qquad (6)$$

Therefore, by substituting Equations 6 into Equation 5, we can obtain the following two dimensional (m+1)–th order time-varying autoregressive model.

$$\mathbf{x}_n = \sum_{i=1}^{m+1} \mathbf{C}_i \mathbf{x}_{n-i} + \mathbf{w}_n . \qquad (7)$$

Here $\mathbf{C}_i$ (i=1,…,m+1) is the time-varying autoregressive coefficient matrix, which is expressed as follows:

$$\mathbf{C}_1 = \mathbf{D}_1 + \mathbf{A}_{n-1}, \mathbf{C}_2 = \mathbf{D}_2 - \mathbf{D}_1\mathbf{A}_{n-2}, \cdots,$$
$$\mathbf{C}_m = \mathbf{D}_m - \mathbf{D}_{m-1}\mathbf{A}_{n-m}, \mathbf{C}_{m+1} = -\mathbf{D}_m\mathbf{A}_{n-m-1}.$$

Moreover, by using the following relation

$$\dot{x}_n \cong \frac{1}{\Delta t}\left(x_n - x_{n-1}\right), \qquad (8)$$

Equation 7 can be approximated by the following the M-th (≧ m+1) order scalar time-varying autoregressive model

$$x_n = \sum_{i=1}^{M} a_{n,i} x_{n-i} + w_n , \qquad (9)$$

where $a_{n,i}$ indicates time-varying autoregressive coefficients, w_n is the Gaussian white noise sequence with $N(0,\sigma_2^2)$. Now, since $a_{n,i}$ is time-varying autoregressive coefficients, suppose that the following relation

$$\sum_{i=1}^{M} \left\{ \phi_i + \pi_i \exp\left[-\gamma x_{n-1}^2\right] \right\} \cong \sum_{i=1}^{M} a_{n,i} , \qquad (10)$$

where ϕ_i is a linear term of autoregressive coefficients, π_i is a time-varying term of autoregressive coefficients and γ is a scaling parameter. Thus, Equation 9 can be written as follows:

$$x_n = \sum_{i=1}^{M} \left\{ \phi_i + \pi_i \exp\left[-\gamma x_{n-1}^2\right] \right\} x_{n-i} + w_n . \qquad (11)$$

This time series model, which is called an exponential autoregressive (Exp AR) model, is firstly introduced by Ozaki & Oda (1978). And then characteristics are investigated by Haggan & Ozaki (1981). According to Haggan & Ozaki (1981), consider the following characteristic equations of Equation 11:

$$\begin{cases}
\lambda^M - \phi_1\lambda^{M-1} - \cdots - \phi_{M-1}\lambda - \phi_M = 0 \\
: \quad As\ x_{n-1} = 0 \\
\lambda^M - \left(\phi_1 + \pi_1\right)\lambda^{M-1} - \cdots - \\
\left(\phi_{M-1} + \pi_{M-1}\right)\lambda - \left(\phi_M + \pi_M\right) = 0 \\
: \quad As\ x_{n-1} = \pm\infty
\end{cases} \qquad (12)$$

Note that the Exp AR model is one class of the radial basis function (RBF) approximation model in the neural network approach. As the study of the prediction of time series for roll motion, there is Ueno & Han (2013).

If all roots of these equations lie inside of the unit circle, then the nonlinear stochastic dynamical system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as the nonlinear for the restoring force.

FITTING OF THE TIME SERIES MODEL

As to the estimation of the model order M and the coefficients $\{\gamma, (\phi_i, \pi_i;\ i = 1, \ldots, M)\}$ in the

Exp AR model, for simplicity, by fixing the parameter γ at one of a grid of values, we estimated the model order M and the corresponding ϕ_i, π_i parameters as well as Haggan & Ozaki (1981). As N is the total number of observations, after fixing $\gamma = \gamma_0$, the Exp AR model for $n = M+1, \ldots, N$; $i = 1, \ldots, M$ can be written as follows:

$$x_n = \sum_{i=1}^{M} \left\{ \phi_i + \pi_i \exp\left[-\gamma_0 x_{n-1}^2 \right] \right\} x_{n-i} + w_n . \quad (13)$$

So the matrix form of Equation 13 can be written $\mathbf{X}^{(n)} = \mathbf{H}\beta + w$, where, $n = N - M, \ldots, N$,

$$\mathbf{X}^{(n)} = \left(x_n, x_{n-1}, \cdots, x_{n-(N-M-1)} \right)^T ,$$

$$\mathbf{Y}^{(n)} = \left(\exp[-\gamma_0 x_n^2] x_n, \exp[-\gamma_0 x_n^2] x_{n-1}, \cdots \right.$$
$$\left. \cdots, \exp[-\gamma_0 x_n^2] x_{n-(N-M-1)} \right)^T ,$$

$$\mathbf{H} = \left(\mathbf{X}^{(n-1)}, \mathbf{Y}^{(n-1)}, \mathbf{X}^{(n-2)}, \mathbf{Y}^{(n-2)}, \cdots, \mathbf{X}^{(n-i)}, \mathbf{Y}^{(n-i)} \right),$$

$$\beta = \left(\phi_1, \pi_1, \phi_2, \pi_2, \cdots, \phi_i, \pi_i \right)^T ,$$

$$w = \left(w_n, w_{n-1}, \cdots, w_{M+1} \right)^T ,$$

so that the normal equations for β become $\mathbf{X}^{(n)} = \mathbf{H}\beta$. Hence β can be found from

$$\hat{\beta} = \left(\mathbf{H}^T \mathbf{H} \right)^{-1} \mathbf{H} \mathbf{X}^{(n)} . \quad (14)$$

The model order M of the fitted model is selected by using the Akaike Information Criterion (AIC) for nonlinear time series (Ozaki & Oda, 1978)

$$\mathrm{AIC}(M) = (N - M) \log \hat{\sigma}_{2,M}^2 + 2(2M + 1) \quad (15)$$

where

$$\hat{\sigma}_{2,M}^2 = \frac{\left(\hat{w}_N^2 + \hat{w}_{N-1}^2 + \cdots + \hat{w}_M^2 \right)}{N - M} \quad (16)$$

is the least sqares estimate of the residual variance of the model.

VERIFICATION

Used time series data

To verify the proposed procedure, we analysed the data of the steady state and the parametric roll resonance. As to the data of the steady state under the ordinary navigation, we used the roll angle data of the research vessel "Taka-maru", which belongs the NRIFE. Table 1 shows the principal particulars of Taka-maru. The data as shown in Fig.1 was measured at sampling interval 0.1 [s] when the ship was running 4 knots in beam waves. From this figure, it can be seen that the maximum value of the absolute value of the roll angle is about 5 degrees and the motion is steady and stable. However, since the size of the ship is small, we fell that the ship shakes intensely. Actually, some of the crew for measurement of data became seasickness.

Table 1: Principal particulars of the Taka-maru.

Items	Ship
Length between perpendiculars: L	25.0 m
Breadth: B	5.2 m
Mean draft: T	2.0 m
Block coefficient: C_b	0.442
Metacentric height: GM	0.52 m
Natural roll period: T_φ	5.95 s

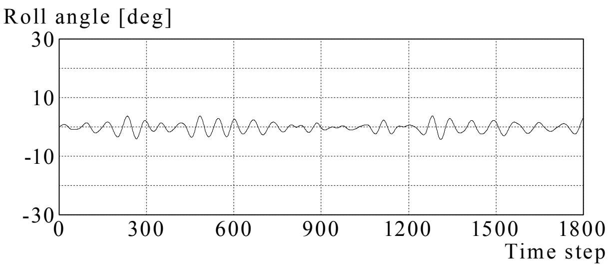

Fig. 1: Time series of the roll angle measured in onboard.

As to the data of the parametric roll resonance, we used results of model experiments concerning the Post-Panamax Container ship implemented by Hashimoto et al (2005). Table 2 shows the principal particulars of the sample ship. The data as shown in Figs.2 ~ 3 was measured at sampling interval 0.1 [s], when the ship was running under the condition of causing the parametric roll resonance in head seas. Fig. 2 shows the result in regular waves and Fig. 3 shows the result in irregular waves. From the Fig. 2 in shown the result of regular waves, it can be seen that the amplitude becomes large rapidly after about the 300th, and becomes the steady state in roll angle 25 [degrees]. From the Fig. 3 in shown the result of irregular waves, it can be seen that the absolute value of amplitude exceeds 10 degrees in about the 500th ~ 900th and about 1100th ~ 1800th.

Table 2: Principal particulars of the sample ship by using the study of Hashimoto et al (2005).

Items	Ship	Model
L	238.8m	2.838m
B	42.8m	0.428m
T	14.0m	0.14m
C_b	0.630	0.630
GM	1.08m	0.0106m
T_φ	30.3 s	3.20 s

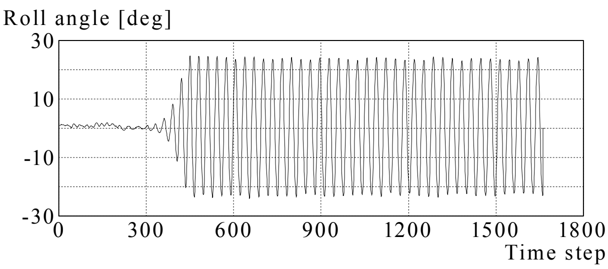

Fig. 2: Time series of the roll angle measured in model experiments: The result of regular waves.

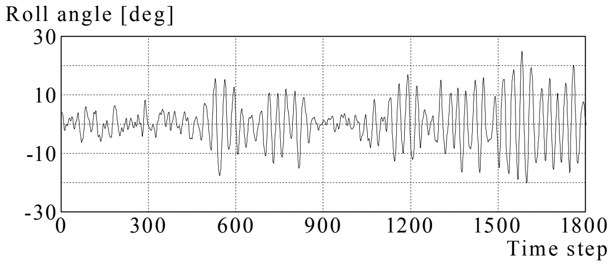

Fig. 3: Time series of the roll angle measured in model experiments: The result of irregular waves.

Results and discussions

The analysis for (N-300) samples based on the Exp AR modelling was recursively performed for 300 samples. Figs. 4 ~ 6 show the arrangement of characteristic roots calculated by using the Equation 12. Fig. 4 shows results concerning the onboard data as shown in Fig. 1, Fig. 5 shows results concerning the experimental data in regular waves as shown in Fig. 2, and Fig. 6 shows results concerning the experimental data in irregular waves as shown in Fig. 3, respectively. In these figures, the left hand side shows the result of first 300 samples and the right hand side shows all results of (N-300) samples, respectively. And the symbol "○" indicates the characteristic root in the case of $x_{n-1} = 0$ and the symbol "×" indicates the characteristic root in the case of $x_{n-1} = \pm\infty$, respectively. As mentioned before, all characteristic roots lie inside of the unit circle, then the system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as the nonlinear for the restoring force. In Fig. 4, it can be seen that all characteristic roots lie inside of the unit circle and the arrangement of symbols "○" and "×" are almost same. Therefore, we can judge that the onboard data as shown in Fig. 1 is the stationary and stable. In Fig. 5, it can be seen that all characteristic roots do not lie inside of the unit circle. As to the result of first 300 samples, one pare is outside of the unit circle. Therefore, we can judge that the experimental data as shown in Fig. 2 is the unstable from the analysis of the data of first 300 samples only, although the amplitude of the roll angle is very small. In this case, officers must devise thoroughgoing measures to prevent the large amplitude roll motion based on the applied ship operation. In the right hand side of Fig. 6, it can be seen that all characteristic roots do not lie inside of the unit circle. Thus, we can judge that the experimental data of (N-300) samples in irregular waves as shown in Fig. 3 is the non-stationary and unstable. However, as to the result of first 300 samples as shown in the left hand side of Fig. 6, all characteristic roots lie

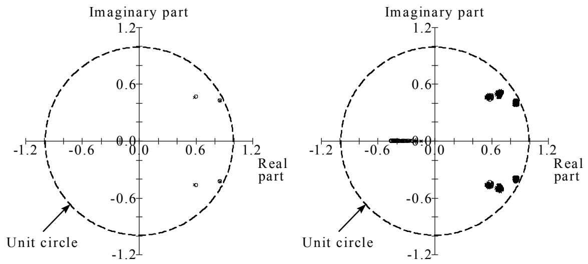

Fig. 4: Characteristic roots calculated by using Equation 12 concerning the onboard data as shown in Fig. 1: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of (N-300) samples.

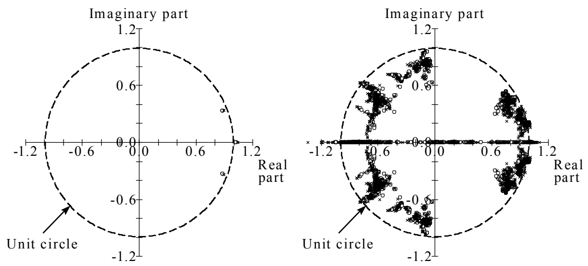

Fig. 5: Characteristic roots calculated by using Equation 12 concerning the experimental data in regular waves as shown in Fig. 2: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of (N-300) samples.

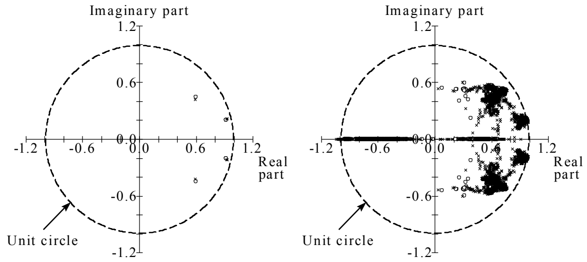

Fig. 6: Characteristic roots calculated by using Equation 12 concerning the experimental data in irregular waves as shown in Fig. 3: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of (N-300) samples.

inside of the unit circle, this case is the stationary and stable, although a few nonlinearity with respect to the restoring force because bath characteristic roots indicated with "○" and "×" are different concerning imaginary part. It means that officers must always pay attention to the roll motion in order to prevent the large amplitude roll motion in irregular waves. From these results, we can conclude that officers can keep the safe navigation in the meaning of preventing the large amplitude roll motion by monitoring it based on the Exp AR modelling.

CONCLUSIONS

In this study, we propose the onboard evaluation method of the transverse stability for officers based on the exponential autoregressive modeling procedure. To confirm the effectiveness of the proposed method, we analyzed the data of the steady state and the parametric roll resonance. As the result, we can confirm that the characteristics of the dynamical system for the roll motion can understand by monitoring the behavior of the characteristic root of the characteristic equation in the exponential autoregressive model. Therefore, we conclude that the proposed method can be used as the navigation support system to protect the serious accident.

ACKNOWLEDMENTS

This work was supported by JSPS KAKENHI Grant Number 25289328. Authors would like to thank Capt. Toshiyuki HIRAI and the crew of Taka-maru for cooperation to onboard measurement. Authors are grateful to Dr. Hirotada HASHIMOTO and Prof. Naoya UMEDA (Osaka University) for acceptance with respect to using the experimental data. And authors would like to thank President Genshiro KITAGAWA (Research Organization of Information and System) and Prof. Kohei OHTSU (Tokyo University of Marine Science and Technology) for helpful suggestions.

REFERENCES

A. Francescutto and N. Umeda (2010): Current status of new generation intact stability development, Proceedings of the 11th International Ship Stability Workshop, Wageningen, pp.1-5.

D. Terada and A. Matsuda (2011): Study on a relationship between nonlinear dynamical system and stochastic model - Application to stability judgment system of ships - (in Japanese), Journal of the Japan Society of Naval Architects and Ocean Engineers, 14, pp.75-84.

H. Hashimoto, A. Matsuda and N. Umeda (2005): Model Experiment on Parametric Roll of a Post-Panamax Container Ship in Short-Crested Irregular Seas, Conference Proceedings of the Japan society of naval architects and Ocean Engineers, vol.1, pp.71-74.

K. Ueno and C. Fan (2013): Quasi-chaotic behaviors of narrow-band response of a non-deterministic resonant System -Application to analysis of ship motion in irregular seas, Japan Journal of Industrial and Applied Mathematic, Vol.30, No.1, pp.203-225.

N. Umeda, M. Shuto and A. Maki (2007): Theoretical prediction of broaching probability for a ship in irregular astern seas, Proceedings of the 9th International Ship Stability Workshop, Hamburg, pp.151-157.

N. Umeda and S. Yamamura (2010): Designing new generation intact stability criteria on broaching associated with surf-riding, Proceedings of the 11th International Ship Stability Workshop, Hamburg, pp.17-25.

T. Ozaki and H. Oda (1978): Nonlinear time series model identification by Akaike's information criterion, in Information and Systems, Dubuisson eds, Pergamon Press, pp.83-91.

T. Ozaki (1986): Local Gaussian modelling of stochastic dynamical systems in the analysis of nonlinear random vibrations, in Essays in Time Series and Allied Processes, Festshrift in honour of Prof.E.J.Hannan, Probability Trust, pp.241-255.

V. Haggan and T. Ozaki (1981): Modelling nonlinear random vibrations using an amplitude-dependent autoregressive time series model, Biometrika, 68, 1, pp.189-196.

Y. Kawahara, K. Maekawa and Y. Ikeda (2009): A simple prediction formula of a roll damping of conventional cargo ships on the basis of Ikeda's method and its limitation, Proceedings of the 10th International Conference on Stability of Ships and Ocean Vehicles, St. Petersburg, pp.387-398.

Y. Yamanouchi (1956): On the Analysis of ships, Oscillations as a Time Series, Journal of zosen kyokai, Vol. 99, pp.47-64.