Examination of the Probability Results
for Extreme Roll of Naval Vessels

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ABSTRACT

Using dynamic stability simulation tools developed by the Cooperative Research Navies based on the current understanding of the various phenomena involved, the Naval Stability Standards Working Group investigated the relationship between a number of stability criteria and the risk of exceeding a critical roll angle. The methodology involves determining the probability of exceeding the critical roll angle from the time series of roll response in multiple simulations of a ship in a given seaway. This paper describes the investigation into the relationships within probability results themselves, looking for trends and patterns that may contribute to understanding the problem.

KEYWORDS

Dynamic Stability, Probability of Capsize, Simulation.

INTRODUCTION

Static stability is well understood, fully determined by developing GZ curves based on geometry and mass distribution, and validated by inclining trials. Dynamic stability – accounting for the effects of wind, waves and current – is less understood. The Co-operative Research Navies (CRNav) Dynamic Stability group was established in 1989 to undertake research into the characteristics and underlying physical phenomena of dynamic stability. By 1999, the tools developed had matured to the point where work could begin on defining new stability criteria based on dynamic behaviour of ships rather than on static behaviour with margins.

The Naval Stability Standards Working Group (NSSWG) was then formed from the naval members of the CRNav group:

Department of Defence (Australia);
Department of National Defence (Canada);
Ministère de la Défense (France);
Ministerie van Defensie (Netherlands);
Ministry of Defence, DPA (UK);
United States Coast Guard Naval Engineering (USA); and at one time
Naval Sea Systems Command (USA).

The naval members are supported by their associated research organisations:

Defence Science and Technology Organisation;
Defence Research and Development Canada;
Bassin d’Essais des Carènes (France);
Maritime Research Institute (Netherlands);
QinetiQ (UK); and in the past
Naval Surface Warfare Center (USA).

The objective of the NSSWG is “To develop a shared view on the future of naval stability assessment and develop a draft set of stability guidelines which can be utilised by the participating navies at their discretion.” At a practical level, this involves identification of methods of relating stability criteria to risk. In
the short-term, this means identification of level of safety extant in the current standards, focusing on the strengths and weaknesses of existing criteria, using a standard set of environmental conditions. In the long term, it means developing methodologies for assessing stability characteristics and practical limits for both design and life-cycle management.

CRNav has developed dynamic stability simulation tools based on the current understanding of the various phenomena involved. NSSWG have used these tools to investigate the relationship between a number of stability criteria and the risk of exceeding a critical roll angle. The methodology followed involves determining the probability of exceeding a critical roll angle using the time series of roll response in multiple simulations of a ship in a given seaway. Although the critical roll angle may take on a number of important connotations, in the present case it is related to capsize.

This paper describes the investigation into the relationships within probability results themselves. The assumed premise is that an accurate characterization of the risk of capsize for a ship at a given loading condition can be determined by the sum of the probabilities of exceeding the critical roll angle across all speeds, headings, wave heights and periods. The basic question explored is: Which operating points (speed – heading) and environmental conditions (wave height – period) are driving the total probability of exceeding the critical roll angle? Conversely, what is the minimum range of operating points and environmental conditions that will reliably represent the extreme roll probability?

A brief description of the work to date on intact stability will be presented in the next section. The following section provides a brief presentation of the chief results of looking at the relative orders of magnitude of the probability results. Next, the range of operating points and environmental conditions required to characterize the sum total probabilities is examined. Following this, the chief results of the correlation between various individual and joint combinations of probabilities and the sum total probabilities are presented. Finally, conclusions are drawn and recommendations for further work are given.

**Determination of Probabilities**

To capture the nonlinear effects of motions, it is necessary to perform simulations in the time domain.

**FREDYN Simulations**

Since 1999, the objectives of the NSSWG have been pursued through three phases of study for intact ships. Phase 1 used a Strip Theory approach (FREDYN version 8.2) to look at relationships between the risk of capsize and various stability-related and ship-form parameters. Phase 2 used Panel Methods (FREDYN version 9.9) and the emphasis of the study shifted to looking for the level of safety inherent in the current naval stability standards. Phase 3 was conducted after a complete rewrite of the software to modularize the code. The Phase 3 study still used Panel Methods, but included a more accurate modeling of the effects of deck-edge immersion (FREDYN version 10.2). The focus in Phase 3 was narrowed to finding criteria that would be suitable for stability standards, in particular the Naval Ship Code (ANEP 77 [2012]).

FREDYN (FREgat DYNamics), as described in Ypma and Harmsen [2012], is a non-linear, semi-empirical, time-domain software for simulating ship motions in both seakeeping and manoeuvring, for environmental conditions from calm water to severe wind and waves. FREDYN is capable of predicting a range of capsize modes in regular and irregular waves. FREDYN is appropriate for any type of a relatively slender mono-hull with a Froude number less than 0.5.

In the present work, the set of ships investigated includes slender hulls with twin propellers and one or two rudders. Several different load conditions are explored for each ship, with each load condition delineated by
draft (T) and vertical position of the center of gravity (KG). The radii of gyration were held constant for a given ship for all TKG.

Various speed and heading (relative to the wave direction) combinations are explored. The ships were assumed to be equally likely to take on any heading relative to the waves. A generic 3-speed profile based on experience was agreed on and used for Phases 1 and 2, but reduced to the most common speed for Phase 3.

A common set of environmental conditions was used for each ship within each phase. The same set of conditions was used in Phase 2 as in Phase 1, however the set of environmental conditions used in Phase 3 was reduced. The full set of conditions is based on the Bales’ scattergram for the North Atlantic (see Bales, Lee, and Voelker [1981]) modified slightly in accordance with McTaggart and De Kat [2000]. No current was included. The wind was modeled as a function of wave height, and was set to be collinear with the wave train. To be conservative, a single sea direction was assumed and wave spreading was not used, so that all the energy associated with the sea spectrum would be concentrated in the unidirectional wave train.

Each wave condition may be defined by the significant wave height and modal wave period; however, there are typically an infinite number of ways to achieve (realize) the seaway as defined by these parameters. Each realization is capable of producing a unique time series of wave conditions. This is the key to generating probabilistic results; under the assumption that any one of the unique realizations is equally likely to occur, performing multiple simulation runs (where each run is a unique realization) generates a statistical sample.

**Post-analysis using probability methods**

For Phases 1 and 2 a block maxima method, called PCAPSIZE (see McTaggart and De Kat [2000]), was developed to derive the probability of exceeding the critical roll angle (which for Phases 1 and 2 was 90°) within an hour given each speed, heading, and wave height and period.

For Phase 3 an envelope-peaks-over-threshold (EPOT) method called LORELEI (see Ypma and Harmsen [2012]) was developed and used to obtain the probability of exceeding the critical roll angle (which for Phase 3 was 70°) within an hour given each speed, heading, and wave height and period. This method makes fuller use of the time-series data and thus theoretically provides a more accurate value.

The probability of exceeding the critical roll angle given the operating point and environmental condition is generated for each operating point (speed and heading) and environmental condition (wave height and period) as well as each load condition (TKG). Advice to the designer or operator would have to take into account the probabilities of being at each loading condition, operating point and in each environment. For the sake of the current work however, these latter probabilities are not applied, so that they will not obscure relationships between the conditional probabilities and the conditions.

**Examining Orders of Magnitude of Probability Results**

For each TKG, the simulation results can be stored as a 4-dimensional hyper-cube with each dimension representing a single variable in the set \{V, \beta, H_s, T_p\}, where V represents the ship speed and \(\beta\) the ship heading relative to the waves, while \(H_s\) is the significant wave height and \(T_p\) is the wave modal period. This makes it easy to index into the data, as well as to partition the data along any subset of variable ranges. However, for visually examining the data, it is necessary to “flatten” the data. It is clear that TKG is representative of the state of the ship itself, but the speed and heading are operational choices, while the wave height and period represent environmental conditions. Based on this intuitive distinction, the data can be examined in the form of 2-dimensional tables for each TKG.
Comparing Results across TKG

The results for different combinations of reduced/full range of ship speeds and headings were examined. No consistent patterns were observed. Similarly, results for the different combinations of reduced/full ranges of ship speed and heading were inspected, and the same lack of consistent patterns was found.

Reduced Ranges

The idea of reduced data sets suggests that fewer simulations can be run to obtain the needed results. This was in fact practiced for the Phase 3 study, based on an educated guess of the new ranges of ship speed, and wave height and period. The question naturally arises as to whether or not the guess is reasonable, and further, how far the variable ranges can be reduced in before the value of extreme roll probability is significantly affected.

Before either of these questions can be answered “significant” must be quantified. As stated above, when dealing with probabilities it is reasonable to speak in terms of orders of magnitude, and “significantly affected” can be thought of in terms of the difference between the order of magnitude of the sum of probabilities for the reduced range and that for the full range. Three levels have been examined in this study: 0.1, 0.5, and 1.0. The first level is quite demanding; one-tenth of an order of magnitude is equal to a variation of only 26% ($10^{0.1}$), while the second level allows for the full-range extreme roll probability to be up to 3x ($10^{0.5}$) that of the reduced range. The third level allows a 10x difference, and should be considered to be at or near the limit of acceptable difference, and in some cases may be too much of a difference.

To investigate the effect of reducing the range of the variables, a set of systematic reductions was conducted to find minimal ranges. Figure 1 shows the overall results of the minimum-speed-heading-range search for all TKG of all ships in all phases. The blue dashed lines indicate the range of speeds and headings in the reduced set. The blue cells show the range of conditions required to provide a probability of exceeding the critical roll angle with an order of magnitude within 0.1 of that when the whole table of conditions are taken into account. The blue cells generally underlay the green and yellow cells as well. The green cells represent a difference in the order of magnitudes of 0.5, and the yellow cells a difference of 1 order of magnitude. The figure indicates that, based on the results from all phases and ship TKG, the full range of speeds and headings are required to ensure that extreme roll probability is accurate, while all 3 speeds and headings 0° – 120° are required to get an extreme roll probability within 0.5 or even 1 order of magnitude of the full-table value. Note that a heading of 0° represents the ship in following seas.

Figure 2 shows the overall results of the minimum-height-period-range search for all ships in all phases. The table shows all the possible wave conditions (non-greyed-out cells). The blue dashed lines indicate the range of wave heights and periods in the reduced set. Again the blue cells show the range of conditions required to provide a probability of exceeding the critical roll angle with an order of magnitude within 0.1, while the green cells represent a difference in the order of magnitudes of 0.5, and the yellow cells a difference of 1 order of magnitude. The figure indicates that, based on the results from all phases and ship TKG, the range of wave heights go from 2 to 19 m and the range of wave periods is from 8.5 to 25.7 s to ensure that extreme roll probability is within 0.1, 0.5, or even 1 order of magnitude of the full-table value.

Tables similar to Figure 1 and Figure 2 were developed for each ship in each phase. The ranges were “summed” across all phases for each ship, and all ships for each phase, and finally all ships and all phases as shown in Figure 1 and Figure 2.
Figure 1. Minimum ranges of Ship Speeds and Headings

Figure 2. Minimum Ranges Wave Heights and Periods

- Yellow: Necessary to represent total exceedance by $\leq 1$ order of magnitude.
- Green: Necessary to represent total exceedance by $\leq 0.5$ order of magnitude.
- Blue: Necessary to represent total exceedance by $\leq 0.1$ order of magnitude.
Table 1. Adequacy of Phase 2 data when ranges reduced to those of Phase 3.

<table>
<thead>
<tr>
<th></th>
<th>Reduced Scattergram</th>
<th>Wave Scattergram</th>
<th>Reduced Operational Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Ship A</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ship B</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ship C</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Ship D</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
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<td>x</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
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<td>x</td>
</tr>
<tr>
<td>Ship G</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Ship H</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

The results in Figure 1 and Figure 2 tend to be an over-simplification, and the required ranges are less onerous when the ships are examined separately. Table 1 summarizes the check on the validity of the intuitive choice for reducing the ranges of ship speeds and wave heights and periods for Phase 3. The table shows that the reduction in wave conditions will still give results within half an order of magnitude of the full table, for most ships. However, reducing the range of speeds will lead to a difference in extreme roll probability of up to an order of magnitude for most ships, and greater for some ships.

Correlation of Specific Conditions and Marginal Sums with Extreme Roll Probability

The loading conditions were used as observations to examine the linear correlation of the total probability of exceeding the critical roll angle with the condition. Low correlation coefficients indicate there is weak linear correlation.

Results show more correlation between extreme roll probability and the stern-quartering and beam headings; however, this is not consistent at all wave heights and periods. Results also show more correlation between extreme roll probability and the steeper wave conditions along the diagonal of the scattergram; however, this is not consistent at all speeds and headings.

Conclusions

In this study, it is shown that the massive amount of data can be arranged in a relatively simple way to provide a visual means of finding patterns in the data; i.e., how the orders of magnitude of the probabilities might indicate which conditions tend to drive the total probability of exceeding the critical roll angle. The visual arrangement indicated that there are no consistent patterns in the speed-heading tables or the wave height-period tables across all loading conditions, all ships, and all phases. Although this is a negative result, it is at least based on examination of data rather than speculation. It is not readily apparent whether the negative result is a consequence of insufficient detail in the computer modeling method, or whether factors other than those included in the study are more influential to the risk of capsize.

An effort was made to reduce the number of simulations required by looking for the minimum ranges of the variables that would give a reasonable approximation of the order of magnitude of the extreme roll probability. Reduced ranges were found for each loading condition for each ship in each of the phases, but there was no consistent set of ranges across all phases, ships, and loading conditions. This is again a negative result, though based on the data rather than assumptions. The lack of
consistency may stem from the differences in the computer model implementations and the differences in modeling choices between phases, or other as yet unknown issues.

The loading conditions were used as separate observations to examine the correlation of extreme roll probability with the condition variables in a further attempt to find the conditions that drive the risk. Again, no discernible, consistent pattern was found.

Although no consistent patterns were found, these negative results are based on examination of data rather than intuition alone. It is not immediately apparent whether the results are a consequence of insufficient detail in the computer modeling method, or whether factors other than those included in the study are more influential to the risk of capsize. This paper is also valuable for the sake of recording the scope of the investigation and the details of the processes used, allowing others to build on the results.

**Future Work**

Future efforts will concentrate on:

1. rearranging the data into different groupings to see if patterns exist outside of the most intuitive arrangements;

2. using other variables (e.g., wave height) as the observation indices for correlation analysis, again to see if patterns exist “outside the box”;

3. re-analysis including the user-defined probability distributions for speed, heading, and wave height and period; and

4. examination of other factors such as wave steepness, steepness to ship-length ratio, and wave encounter frequency.

**REFERENCES**

ANEP-77, Naval Ship Code, NATO, Ed. 4, December 2012

