Wave celerity in a multi-chromatic sea: a comparative study

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ABSTRACT

The expansion of the theory of surf-riding form the regular to the irregular sea is continued. An alternative method of instantaneous wave celerity calculation is proposed, based on the Hilbert transform and on the concept of instantaneous frequency. The result is compared against prediction of the same by a previously presented method that was based on the propagation in time-space of a property of the wave profile. The effectiveness of the two methods for the prediction of surf-riding is discussed.

KEYWORDS

Wave, celerity, ship, stability, surf-riding, analytic signal, instantaneous frequency

INTRODUCTION

In a recent paper, various definitions of local wave celerity were introduced that could be useful for the analysis of surf – riding in multi-chromatic wave environments (Spyrou et al 2012). Specifically, an instantaneous celerity definition was proposed, as the velocity of propagation of a suitable local property of the wave profile. A number of case studies were then examined, considering several wave realizations produced by spectra of different bandwidth.

A variant (but still under the same principle) definition of local celerity was also identified, based on the propagation of the point of maximum slope in the vicinity of the ship, as an observable representative of the position of maximum surge wave force nearest to the ship. This seemed producing a smoother celerity curve, more easily integrated into the surf-riding investigation.

In the current paper is followed an alternative route for the calculation of instantaneous celerity exploiting the established concept of "instantaneous frequency". This route had been identified as a possibility in Spyrou et al. (2012) but no results had been produced. Comparative studies of wave celerity calculation according to these two principal schemes are reported here. In the first step, the comparison is based on different multi-chromatic wave realizations in time-space; while in the second, the ship motion is considered as well and the celerity value comparison refers to the instantaneous position occupied by the ship in the wave field.

BRIEF THEORETICAL BACKGROUND

There has been a lot of discussion about the interpretation and utility of instantaneous frequency in signal processing (see for example Mandel 1974 and Boashash 1992). The instantaneous frequency is usually defined as the derivative of the phase of the analytic signal, see for example Feldman (2011). The analytic signal $X(t)$ is a complex signal, whose imaginary part $\tilde{x}(t)$ is the Hilbert transform of the real signal $x(t)$ that is under consideration:

$$X(t) = x(t) + j \cdot \tilde{x}(t) \quad (1)$$

The Hilbert transform of a function $x(t)$ is the integral:

$$\tilde{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \quad (2)$$

It is common to present the analytic signal in polar coordinates:

$$X(t) = A(t) \exp\left[j \cdot \varphi(t) \right] \quad (3)$$

where $A(t)$ and $\varphi(t)$ are respectively, the instantaneous amplitude and phase:

$$A(t) = \pm \sqrt{x^2(t) + \tilde{x}^2(t)} \quad (4)$$

$$\varphi(t) = \tan^{-1} \frac{\tilde{x}(t)}{x(t)} \quad (5)$$

The derivative of the instantaneous phase is the instantaneous frequency. However, the instantaneous frequency can be derived also by direct differentiation of the signal (Feldman 2011):

$$\omega(t) = \frac{x(t) \cdot \dot{\tilde{x}}(t) - \dot{x}(t) \cdot \tilde{x}(t)}{A^2(t)} \quad (6)$$

An analytic signal appears sometimes composed of N components with amplitudes $A_i(t)$ and phases $\varphi_i(t)$:

$$X(t) = \sum_{i=1}^{N} A_i(t) \cdot \exp\left[j \cdot \varphi_i(t) \right] \quad (7)$$

Then, the instantaneous frequency can be obtained by equating with (3) and solving for the phase $\varphi(t)$ (Nho & Loughlin 1999):

$$\varphi' = [\sum_{i}^{N} A_i^2 \varphi_i' + 0.5 \sum_{i \neq k}^{N} A_i A_k (\varphi_i' + \varphi_k') \cos \Delta\varphi_{ik} + \\ +(A_i' A_k - A_k' A_i) \sin \Delta\varphi_{ik}] / [\sum_{i=1}^{N} A_i^2 + \sum_{i \neq k}^{N} A_i A_k \cos \Delta\varphi_{ik}] \quad (8)$$

where $\Delta\varphi_{ik} = \varphi_i - \varphi_k$.

Let us then have a wave profile typically represented by a Fourier series:

$$\zeta(x,t) = \sum_{i=1}^{N} A_i \cos \varphi_i \quad (9)$$

The phase of each harmonic component is $\varphi_i = k_i \cdot x - \omega_i \cdot t + r_i$ (k_i, ω_i and r_i stand for the wave number, frequency and random phase of each component). The amplitude of each component is constant and it is derived from the spectrum S in the standard way.

The "instantaneous" celerity at a specific instant of time t and location x can be obtained by means of the partial derivatives of the phase in time and in space:

$$c(x,t) = \frac{-\partial\varphi(x,t)/\partial t}{\partial\varphi(x,t)/\partial x} \quad (10)$$

From now on, we will refer to this calculation of the instantaneous celerity as the 1st scheme. The method presented in Spyrou et al (2012) will be referred as the 2nd scheme.

CELERITIES FOR DIFFERENT SPECTRAL BAND-WIDTHS

A JONSWAP spectrum having H_S=5 m and T_P=12 s was selected, in order to produce 4 different wave realisations corresponding to different ranges around the peak. The simulation time was fixed. Then the number of discrete frequencies used for each scenario is determined by the band-width (Table 1). Also, the frequency increment was different for each case. The considered points (x, t) where the calculation takes place are shown in Figure 1, for one of the band-width scenarios. In general, they are based on the time-space evolution of a crest.

Table 1: Discretization of spectrum and range of frequencies

scenario	%peak (one side)	ω_{start} (rad/s)	ω_{end} (rad/s)	N of freq.	ε
1	2.5	0.511	0.537	2	0.014
2	5	0.497	0.550	4	0.029
3	10	0.471	0.576	8	0.057
4	20	0.419	0.628	16	0.108

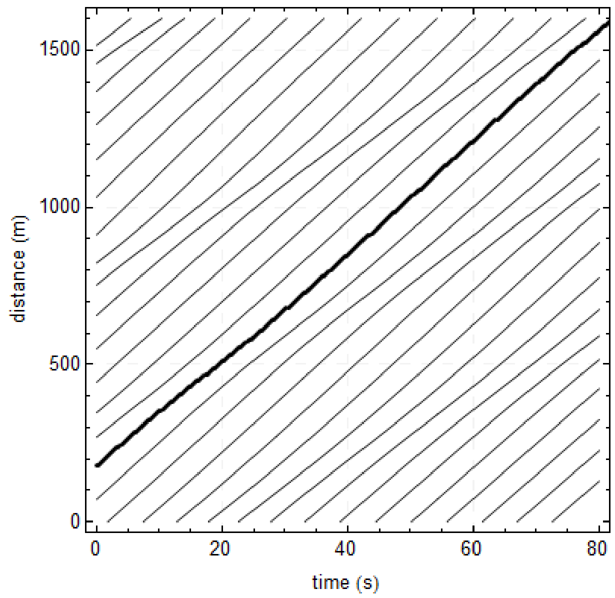

Fig. 1: Contours of points of zero wave slope in the wave field (thin lines) and the locus of points where celerity is calculated corresponding to a crest (thick line).

In Figure 2 can be seen how the instantaneous frequency, calculated through the analytic signal scheme, varies in time. It is remarked that, it is not always the case that the instantaneous frequency remains within the limits of the spectral range. In the graphs we have included also the frequency curve calculated as weighted mean of the participating frequencies, according to the following formula:

$$\bar{\omega}(t) = \left(\sum_{i=1}^{N} A_i^2(t)\varphi_i'(t) \right) \Big/ \left(\sum_{i=1}^{N} A_i^2(t) \right) \qquad (11)$$

As observed, the instantaneous frequency fluctuates around the weighted mean.

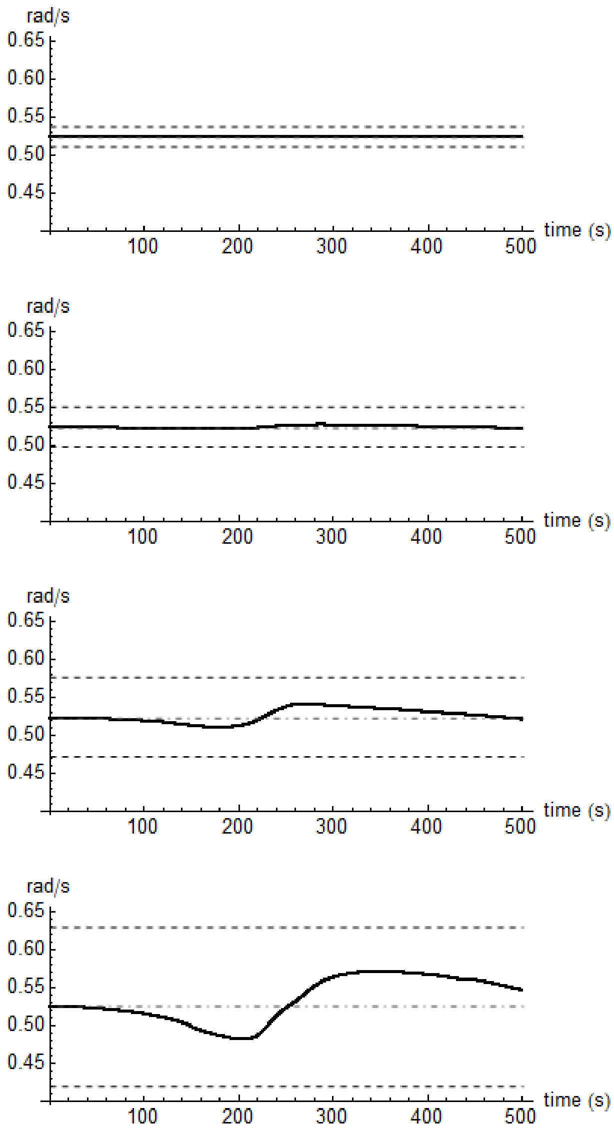

Fig. 2: Variation of Instantaneous frequency in time, for the four considered scenarios (2.5, 5, 10 and 20% of peak period). Dashed lines are the limits of the considered frequency range. The dot-dashed line corresponds to the above weighted mean frequency.

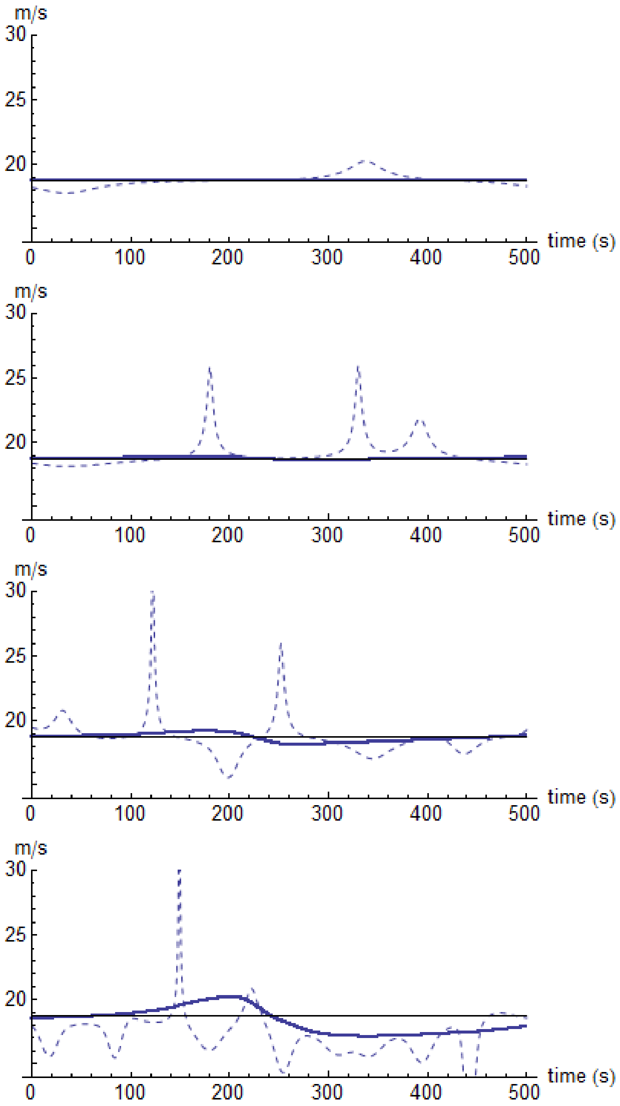

Fig. 3: Comparison of instantaneous celerities for the four considered scenarios (2.5, 5, 10 and 20% of peak period). Dashed lines correspond to the calculation scheme reported in Spyrou et al. (2012). Straight-lines correspond to the celerity of the weighted mean wave frequency for each scenario.

A COMMENT ON THE COMPARISON OF SURGE VELOCITY WITH CELERITY

It should be recalled here that the importance of local celerity for surf-riding prediction is phenomenological. However, ship dynamics in a multi-chromatic sea can give rise to a variety of nonlinear surge responses for which simple extrapolation from the regular sea concepts may not be workable. It is essential that one is able to classify the exhibited pattern of response and extract whether the local celerity plays some critical role. The mean and the envelope of surge response in particular, allow to effectively compare against the local celerity value as the ship proceeds in the seaway. For our further tasks, we calculated two different "mean" surge velocities:

Crude mean: Mean calculated from the consecutive local maxima and minima.

True mean: Mean of values within an apparent period of oscillation, measured between successive maxima.

Table 2: Wave input data

scenario	ω_2/ω_1	s_2/s_1
1	0.95	0.6
2	0.8	1.1
3	1.1	1.3

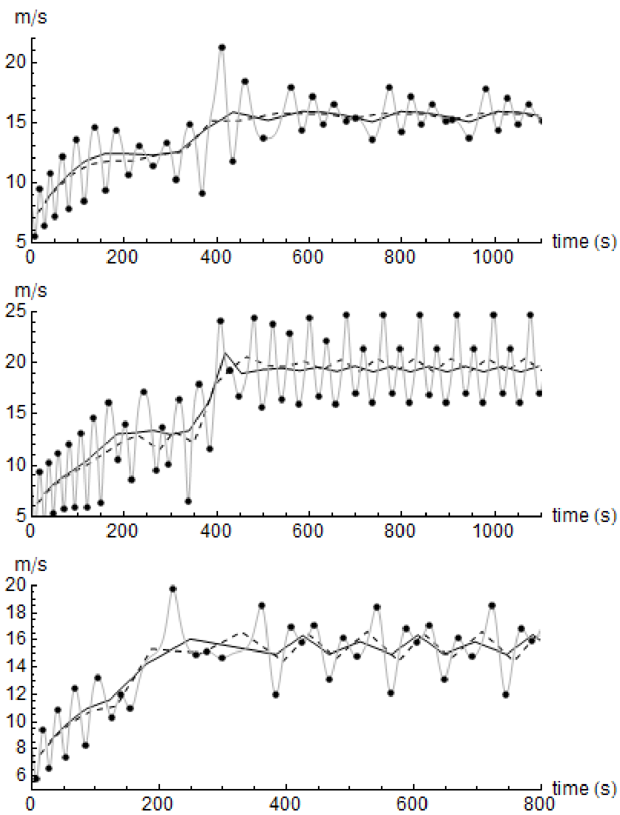

Fig. 4: Surge velocity (light grey curve), points of maxima and minima (black dots), crude mean (dashed curve) and true mean (black curve). Scenarios 1 to 3 from upper to lower graph.

In Figure 4 is shown an example of this calculation, for the well-known ONR "tumblehome topside" ship with 154 m length, exposed to a bichromatic sea. Three wave scenarios are shown. In all three, the first wave harmonic component is common. Its length is equal to ship's length and its steepness is $s_1 = 1/30$. The frequency and the steepness of the second wave harmonic are derived from the data of Table 2. Ship's nominal speed was set to Fn = 0.309 (12 m/s). There is some difference between the two "means" and, due to the nonlinearity of the response, we consider the true mean as necessary in the subsequent calculations.

CELERITY COMPARISON INCLUDING SURF-RIDING PHENOMENA IN A MULTI-CHROMATIC SEA

We turn now to the calculation of the celerity at the position, or in the neighbourhood, of the moving ship. The earlier mentioned ONR vessel is used again. Two cases are considered:

Case 1: A bi-chromatic wave

The wave scenario 1 of Table 2 is selected. In Figure 4 is shown the obtained time history of surge velocity, its true mean and the two instantaneous celerity curves. The celerity curve obtained by the 1st method seems to behave like the mean of the respective celerity curve derived by the 2nd calculation scheme. The scatter of celerity values obtained by the 2nd method is significant, especially for the initial stage. Whilst the mean surge velocity curve does not contribute much towards establishing the beginning of attraction to surf-riding, it helps establish that, for this wave scenario, steady surf-riding is truly governed by the celerity.

Similar information, however based on the celerity of the point with maximum slope nearest to the ship ("local celerity"), is shown in Figure5.

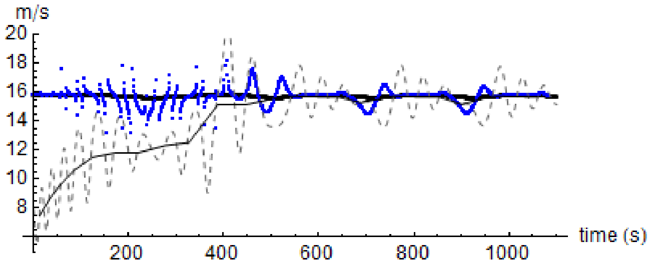

Fig. 4: Surge response (dashed curve) and corresponding mean (grey curve). Instantaneous celerities are also included. The curve corresponding to the 1st celerity calculation scheme is almost horizontal; while the one corresponding to the 2nd scheme (several dots close to each other) fluctuates.

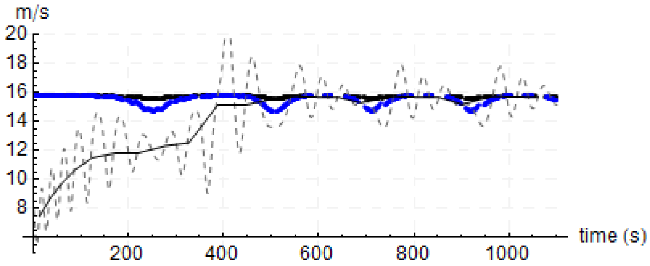

Fig. 5: Surge response (dashed curve) and its mean (grey curve). The two local celerities based on the point of maximum slope are also shown. The one that fluctuates visibly is based on the 2nd scheme.

Case 2: Narrow-band wave realizations

Three band-width scenarios were considered (see Table 3) deriving from a JONSWAP spectrum with $H_S = 3\text{ m}$, $T_P = 9.5\text{ s}$. Nominal speed was Fn = 0.36 (14 m/s).

Table 3: Data for the "narrow-band' wave realizations

scenario	%peak (one side)	ω_{start} (rad/s)	ω_{end} (rad/s)	N of freq.	ε
1	2.5	0.645	0.678	4	0.014
2	5	0.628	0.694	9	0.029
3	10	0.595	0.728	18	0.057

In Figures 6 and 7 are shown time histories of surge velocity and the celerity curves according to the two discussed calculation schemes. The curves obtained by the 1st scheme are consistently smoother and basically they seem to behave like the mean of the celerity of the 2nd scheme. The surge velocity seems to fluctuate around the celerity once surf-riding is established, thus reaffirming the relevance of the latter for the exhibited motion pattern.

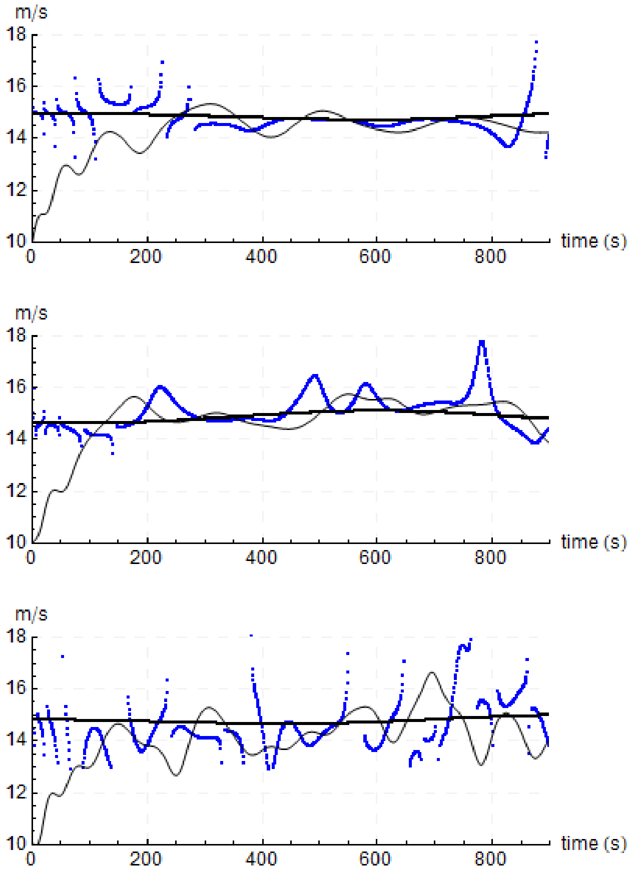

Fig. 6: Comparison of instantaneous celerities for different band-widths (2.5, 5 and 10% of peak period). The black, almost horizontal, curve is based on the 1st scheme; while the heavily fluctuating one comprised of dots) to the 2nd.

CONCLUSION

An alternative method for the calculation of celerity in irregular seas was discussed. The new calculation creates a smoother celerity curve. The reason for the differences, mainly in the intensity of fluctuations, should be considered in the future. Both methods seem to be helpful for the analysis of surf-riding.

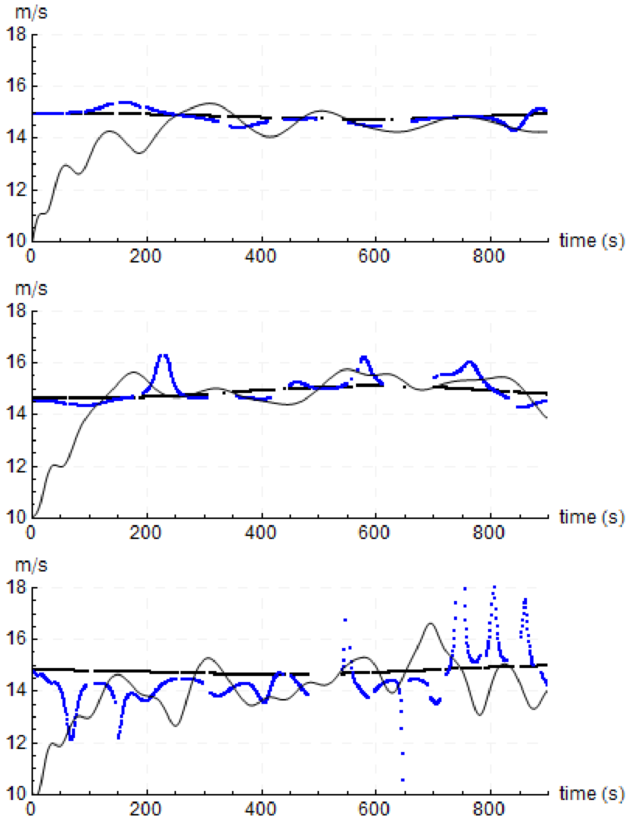

Fig. 7: Behaviour of "local" celerities based on the nearest point of maximum slope, for different band-widths.

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REFERENCES

Boashash, B. (1992) "Estimating and Interpreting The Instantaneous Frequency of a signal Part 1: Fundamentals", Proceedings of the IEEE 80, 520-538.

Feldman, M: Hilbert transform applications in mechanical vibration, John Wiley & Sons, Ltd. Published 2011.

Mandel, L. (1974) "Interpretation of instantaneous frequency", American Journal of Physics 42, 840-846.

Nho, W. and Loughlin P.J. (1999) "When is instantaneous frequency the average frequency at each time?" IEEE Signal Processing Letters, 6,4, 8-80.

Spyrou, K., Belenky, V., Themelis, N. & Weems, K. (2012) "Conditions of surf-riding in an irregular seaway", Proceedings, 11th International Conference on Stability of Ships and Ocean Vehicles, STAB 2012, September 2012, Athens, Greece.