Split-Time Method for Surf-Riding and Broaching-To

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ABSTRACT

This paper presents the first step toward implementing the split-time method for the evaluation of the time-dependent probability of broaching-to. The main idea is to use the emergence of a surf-riding equilibrium as a condition for the separation of dynamics. The probability of attraction to surf-riding can be formulated in terms of the theory of upcrossing for the process of the difference between the instantaneous and the critical surging speed.

This paper describes the development of a simple model of nonlinear surging and surf-riding response in following irregular seas that could be used for probabilistic analysis. Some observations are reported on the influence of the spectrum bandwidth and speed setting on surf-riding in an irregular sea.

KEYWORDS

Surf-riding , Broaching-to, Split-time method

INTRODUCTION

The split-time method has been developed for the treatment of the probability of rare exceedances in nonlinear dynamical systems, and has been applied mostly for problems of capsizing in beam seas and in following seas according to the pure loss of stability mechanism (Belenky et al. 2009, 2010a). The main idea behind the split-time time method is separating the problem into two sub-problems. The first sub-problem is focused on the upcrossing of an intermediate threshold, while the second sub-problem treats the probability of initial conditions at upcrossing that lead to the behavior of interest (capsizing, large roll etc.). As a general philosophy, the split-time method shares several aspects with the "critical wave groups" approach, although there are differences in implementation, such as how the separation of dynamics is defined (Spyrou and Themelis 2005; Themelis and Spyrou 2007; a preliminary application for broaching-to has been attempted [Umeda et al. 2007]); a comparative review is available in Belenky et al. 2010b. The connection of the two methods is also interesting because the critical wave groups approach is quite promising for stability assessmment through model experiments (Bassler et al. 2009).

The application of the split-time method allows the threshold to be set in such a way that different models can be used for the sub-problems, as determined by the dominant physics. For example, when considering capsizing in beam seas, the threshold can be located around the maximum of the roll restoring (GZ) curve, which separates areas of positive and negative slope of GZ.

The dynamics of broaching-to originating from surf-riding in regular waves is now well understood from a global dynamical systems' viewpoint (Spyrou 1996). A cornerstone of this dynamics is the appearance of a pair of surf-riding equilibria of which the one that attracts in surge may be a repeller in yaw, depending on the effectiveness of rudder control. Another key issue is the possible dominance in state space of this equilibrium due to a "homoclinic connection" bifurcation that renders surf-riding inevitable.

In the application of the time-split method for the evaluation of the probability of broaching-to, it is proposed to use the emergence of a surf-riding equilibrium as a condition for the separation of dynamics.

The probability of attraction to a surf-riding equilibrium can be considered as an upcrossing problem, defined in terms of the speed. More specifically, one could consider a "critical speed" that, for a given wave environment at a given instance of time, would lead to attraction toward a surf-riding equilibrium. It should be possible to determine this value either directly or by a series of short numerical simulations with varying initial conditions. The critical instantenous speed is likely to be a stochastic process, as both the ship's surging and the wave possess certain "inertia" and are therefore dependent on the previous state.

Once the process of "critical speed" is defined, the probability of surf-riding can be formulated as an upcrossing problem on the difference between the actual surging speed and the "critical" speed.

A significant feature of this problem is that it should be defined simultaneously in space and time, as surf-riding equilibria can appear and disappear in random places and instances of time, see Fig. 1.

The random event of broaching-to after surf-riding might then be associated with the condition that the surf-riding equilibrium, while attracting in surge, repels in the yaw direction; i.e. the applied control is insufficient to keep the ship on course for the instantaneous wave conditions. This is of course a simplification, but in the first instance it provides a clear basis for the probability calculation: the probability of broaching-to can be calculated with reference to upcrossings that lead to an attraction to a yaw-repelling surf-riding equilibrium. Assuming the applicability of Poisson flow for the random event of broaching-to, one can write:

$$P(t) = 1 - \exp(-\lambda P_B t) \qquad (1)$$

Here λ is the rate of upcrossing of the difference between actual surging speed and critical surging speed, P_B is the probability of broaching-to after surf-riding and is associated with the yaw-repelling properties of the surf-riding equilibrium, and t is time duration.

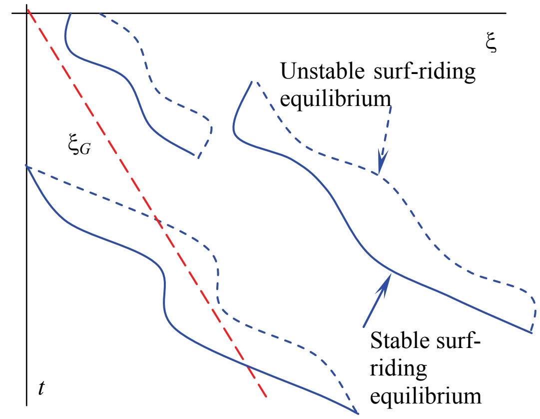

Fig. 1 Random surf-riding equilibria

NONLINEAR SURGING IN IRREGULAR WAVES

Consider a simple model for one-degree-of-freedom nonlinear surging:

$$\begin{aligned}(M + A_{11})\ddot{\xi}_G + R(\dot{\xi}_G) \\ - T(\dot{\xi}_G, n) + F_X(t, \xi_G) = 0\end{aligned} \qquad (2)$$

Here M is mass of the ship, A_{11} longitudinal added mass, R is resistance in calm water, T is the thrust in calm water, n is the number of propeller revolutions, F_X is the Froude-Krylov wave force, and ξ_G is longitudinal position of the center of gravity in the Earth-fixed coordinate system. The dot above the symbol stands for temporal derivative.

Polynomial presentations are used for the resistance and thrust in calm water (for compatibility with Spyrou 2006):

$$\begin{aligned}R(U) &= r_1 U + r_2 U^2 + r_3 U^3 \\ T(U, n) &= \tau_1 n^2 + \tau_2 n U + \tau_3 U^2\end{aligned} \qquad (3)$$

Since the Earth-fixed coordinate system is used, irregular waves are presented as a spatial-temporal stochastic process using the standard Longuet-Higgins model, based on the dispersion relation:

$$\zeta_W(t, \xi) = \sum_{i=1}^{N} a_i \cos(k_i \xi - \omega_i t + \varphi_i) \qquad (4)$$

As the model is meant at this stage to be qualitative, a linear wave-body formulation seems to be appropriate for the case. Therefore:

$$F_X(t,\xi_G)=\sum_{i=1}^{N} A_{Xi}\cos(k_i\xi-\omega_i t+\varphi_i+\gamma_i) \tag{5}$$

As a body-linear formulation is adopted, the amplitude A_{Xi} and phase shift γ_i are available from response amplitude and phase operators:

$$A_{Xi}=a_i RAO(k_i) \tag{6}$$

$$RAO(k_i)=\rho g k_i\left(\left(\int_{-0.5L}^{0.5L} C(x,k_i)\cos(k_i x)dx\right)^2+\left(\int_{-0.5L}^{0.5L} C(x,k_i)\sin(k_i x)dx\right)^2\right)^{1/2} \tag{7}$$

$$C(x,k_i)=2\int_{-d}^{0}\exp(k_i z)b(x,z+d)dz \tag{8}$$

Here x and z are measured in the ship fixed coordinate system (positive forwards of amidships and upward from the base line), $b(x, z)$ is the molded local half-breadth and d is the amidships section draft. Fig. 2 shows the RAO of the surging wave force for the tumble-home ship from the ONR topside series (Bishop et al. 2005). The phase shift γ_i is presented as Fig. 3:

$$\gamma_i=\arctan\left(\frac{\int_{-0.5L}^{0.5L} C(x,k_i)\sin(k_i x)dx}{\int_{-0.5L}^{0.5L} C(x,k_i)\cos(k_i x)dx}\right) \tag{9}$$

SPECTRUM BANDWIDTH

Up to this moment, significant experience has been accumulated on the numerical simulation of surf-riding in regular waves. To ensure correct interpretation of results from the numerical simulation of surf-riding in irregular waves, it can be useful to start from a spectrum with extremely narrow bandwidth; then the observed picture should not be much different from known regular wave surf-riding.

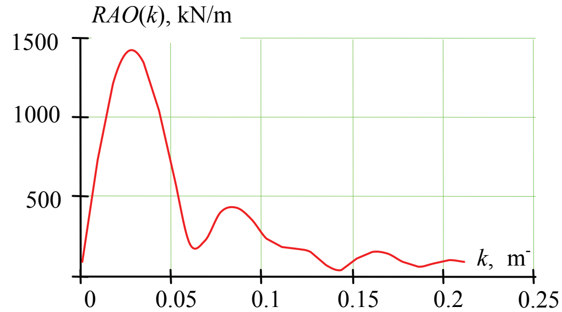

Fig. 2: RAO of Surging Force

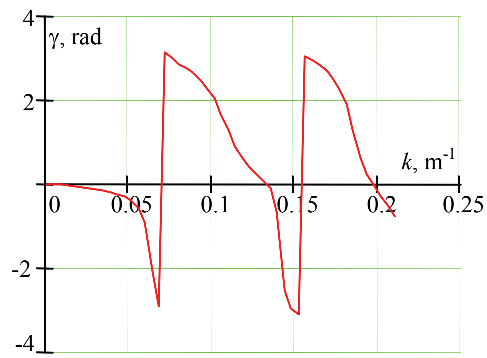

Fig. 3: Phase Shift of Surging Force

This idea has been implemented in the form of a "bilinear filter":

$$F(\omega_i)=\begin{cases} 0 & \omega<b_{low}\Delta\omega \\ \dfrac{\omega-\omega_m-b_{low}\Delta\omega}{b_{low}\Delta\omega} & b_{low}\Delta\omega\le\omega<\omega_m \\ -\dfrac{\omega-\omega_m+b_{up}\Delta\omega}{b_{low}\Delta\omega} & \omega_m\le\omega\le b_{up}\Delta\omega \\ 0 & \omega>b_{up}\Delta\omega \end{cases} \tag{10}$$

Here ω_m is the modal frequency of the spectrum while $\Delta\omega$ is the frequency step. The filter consists of two lines: the low frequency corresponds to the index b_{low} and the high frequency index is b_{up}. These two indices are parameters for controlling the spectrum bandwidth.

To keep the variance of the wave elevation constant, a normalization coefficient is used:

$$K_N=\frac{\sum_{i=1}^{N} a_i}{\sum_{i=1}^{N} a_i F(\omega_i)} \tag{11}$$

A sample result of the bilinear filter is shown in Fig. 4. After discretizing a Bretschneider spectrum with 174 frequencies, a total filtered spectrum is created by selecting the lower boundary 10 frequencies below the modal frequency and the upper boundary 20 frequencies above the modal frequency. This corresponds to a decrease of the spectrum bandwidth parameter from 0.703 to 0.21.

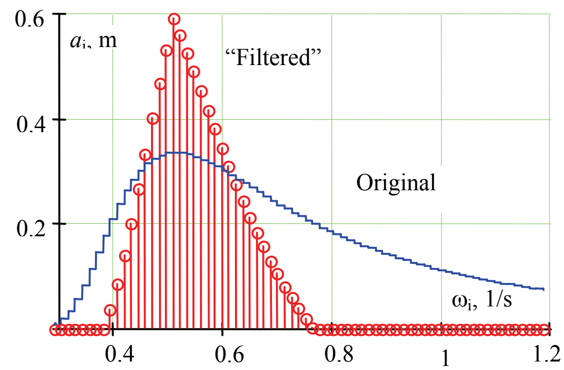

Fig. 4: Example of changing wave spectrum bandwith with bilinear filter

Using such a filter allows regular wave simulations to be carried out in an irregular wave framework; it is enough to set the indices of the high and low frequency boundaries to the modal frequency and only one component will remain in formula (4). The random phase in formula (4) allows the effect of initial conditions to be observed. It can be clearly seen in Fig. 5, which plots the time history of the surge velocity for wave spectrum and speed setting but with different wave phases. The speed setting refers to the ship's calm water speed for the specified constant propeller rate, which is also the initial speed in the simulation For this regular wave case with a speed setting of 27.5 kn (14.1 m/s), there is a co-existence of surging and surf-riding which can be realized through different initial phases.

Adding just one more frequency significantly changes the picture, as shown in Fig. 6. The surf-riding is no longer indefinite – it only exists for a finite period of time (Fig. 6a). A pattern with gradually increased surging amplitude is observed on the other record (Fig. 6b)

A further increase of the spectrum bandwith (Fig. 7) reveal a new pattern of surf-riding that is specific to the irregular sea environment: it no longer has the "constant speed" feature but the ship seems like sliding off slowly until escaping from surf-riding after several seconds (Fig 7a).

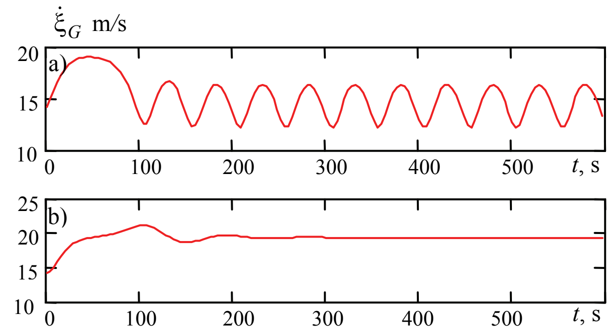

Fig.5: Simulation in regular wave – co-existence of surging and surf-riding

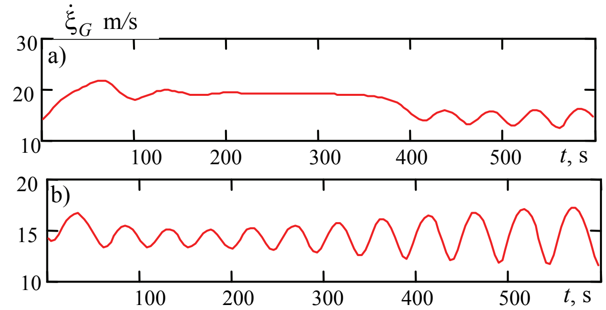

Fig.6: Simulation with 2 frequencies; spectrum bandwidth 0.02

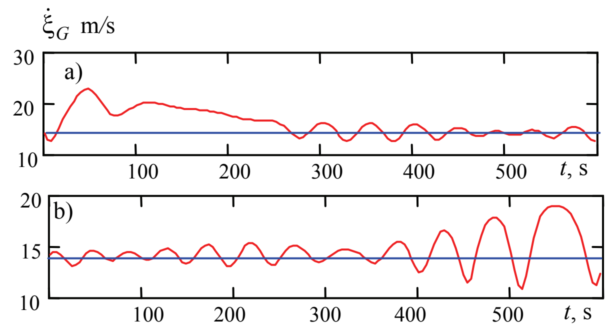

Fig.7: Simulation with 5 frequencies; spectrum bandwidth 0.046

This pattern may be related to the fact that wave celerity is not constant in irregular waves. An unsteady surging pattern with gradually increasing amplitude is observed in Fig.7b. As can be expected, the large-amplitude surging motion has a fairly flat peak—indeed, it is the influence of the coexisting surf-riding equilibrium (Spyrou 2006).

Fig. 8 shows two surging response records for the artificial spectrum with spectrum bandwidth of 0.21 from Fig. 4, while Fig. 9 shows the response for the original Bretschneider spectrum.

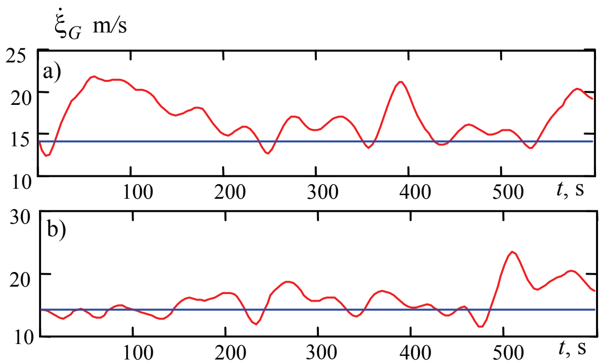

Fig.8: Simulation with 30 frequencies; spectrum bandwith 0.21

Despite a significant difference in the spectrum bandwidth, the patterns of response seem to be similar. Both records contain large amplitude asymmetric oscillation, sometimes with flatten peaks. This asymmetric behavior may be a manifestation of surf-riding in irregular seas. To prove this, however, one needs to find the positions and evolution of the surf-riding equilibria.

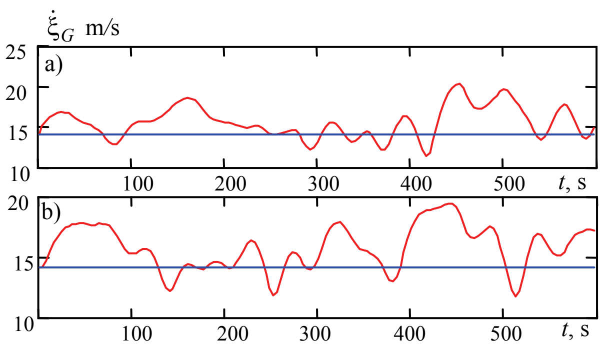

Fig.9: Simulation with original Bretschneider spectrum

SPEED SETTING IN IRREGULAR WAVES

Fig. 10 shows the influence of the calm water speed setting on the nonlinear surge response in irregular waves. The unfiltered, discretized Bretschneider spectrum from Fig. 5 was used, which has a bandwidth of 0.703. The significant wave height was 11.5 m while modal period was 16.4 s, which corresponds to Sea State 8. The wave length corresponding to the mean frequency through the linear dispersion relationship is 146 m, which makes the mean wave celerity equal to 29.3 kn (15.1 m/s).

While the speed setting V_S are low (Fig. 10a and 10b), the surging response maintains a group structure resulting from the effect of speed on the excitation bandwidth. The response is visually fairly symmetric. Increasing the speed setting to 20 kn leads to the first instance of asymmetric response (see Fig. 10c just before the 400 s mark). At 22.5 kn, two large amplitude responses can be spotted along with the asymmetric one (Fig. 10d).

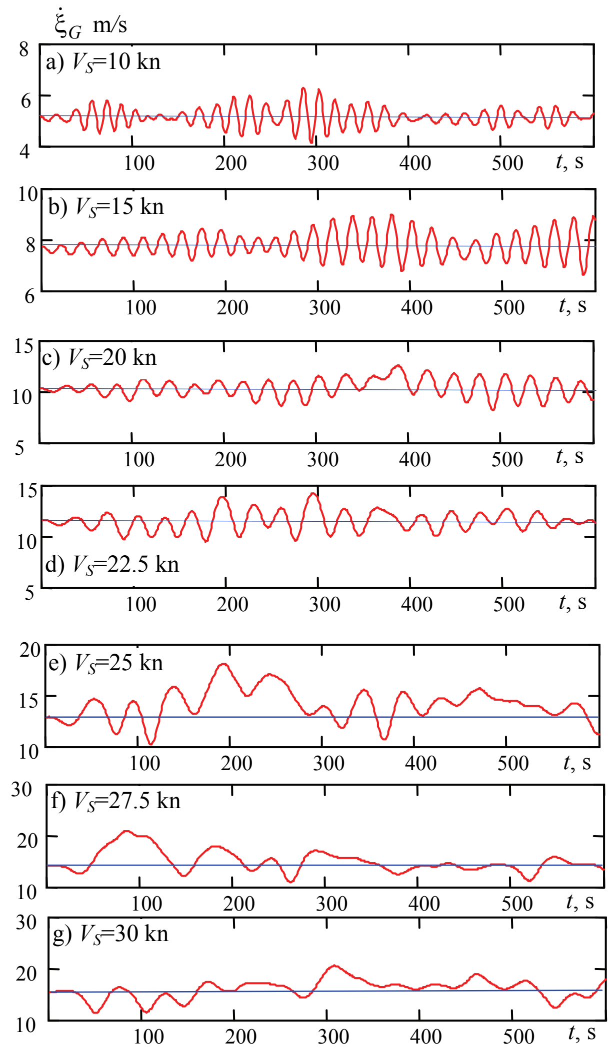

Fig.10: Influence of speed setting on nonlinear surge response in irregular waves

At the speed setting 25 kn (Fig. 10e), two large duration asymmetric responses can be observed. The large positive peaks, however, seem to be less apparent for the two higher speed settings in Figs. 10f and 10g. This may occur because the speed settings are close to the wave celerity of the mean frequency, so surf-riding may no longer require significant acceleration. A long “hanging” just above the set speed in Fig 10g is another symptom of this. However, a complete analysis will only be possible when the locations of surf-riding equilibria in time become available.

SUMMARY AND FUTURE WORK

This paper describes the first step toward applying the split-time method to the evaluation of the probability of broaching-to. The main idea is to use the surf-riding equilibrium as a "breakpoint" between two sub-problems. The first sub-problem leads to a probabilistic characterization of the attraction to a surf-riding equilibrium that can be formulated in terms of the theory of upcrossing for the process of difference between the instantaneous speed and the critical surging speed. The critical speed is defined as a speed sufficient to attract the dynamical system to the surf-riding equilibrium at a given instant of time. The second sub-problem is the probability that the surf-riding equilibrium is a repeller in the yaw direction.

To test the concept, a simple model of surging and surf-riding in irregular waves has been formulated. A bilinear filter is introduced to control the bandwidth of the waves, so the connection to surging / surf-riding response in regular waves can be made.

This filter was used for a visual analysis of the influence of the spectrum bandwidth on the surging / surf-riding response. Another visual analysis was performed on the influence of the speed settings. An asymmetric response was observed that may incorporate the surf-riding behavior. The next logical step is to find the locations of the surf-riding equilibria in time and space.

The asymmetry of the surging response can be characterized statistically through estimates of mean value and skew of its distribution. It may also be useful to c`ompare these instances of asymmetric response with the definition given by B. Campbell (Ayyub et al. 2006) and relate it to the location of surf-riding equilibria.

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