

Experimental Ship Dynamic Stability Assessment Using Wave Groups

Christopher C. Bassler, Martin J. Dipper, Jr., Mark Melendez
David Taylor Model Basin (NSWC/CD)

ABSTRACT

The assessment of ship performance in heavy weather, particularly dynamic stability performance, is an important but difficult assessment to make. Traditional experimental assessment methods using regular and random waves provide insight into dynamic stability performance, but may not identify, or provide a means to mitigate, specific modes of dynamic stability failure. Assessment using deterministic wave groups may provide repeatability and systematic exposure important for the assessment of ship designs, as well as aid in development and validation of numerical simulation tools. The deterministic grouped wave approach, when used to define ship behavior in heavy weather, can also be useful in the development of ship-specific operator guidance.

KEYWORDS

Wave groups, deterministic model testing, ship design, operator guidance

INTRODUCTION

Assessment of ship performance in heavy weather remains an important but difficult undertaking, due to the significant degree of nonlinearities associated with ship response to large steep waves. For heavy weather operations, it is important to assess ship performance related to crew performance and safety, mission effectiveness, and ultimately platform survivability.

Experimental assessments continue to be necessary to assess the performance of new designs, evaluate response in the most severe conditions, and enable the continued development and validation of numerical tools. An efficient, yet accurate, method is still needed for use in the early-design stage when many hull form concepts, or parametric variations in hull form, are still being evaluated. Additionally, later in the ship design process, an efficient and accurate assessment method is needed for the development of ship-specific operator guidance. In the foreseeable future, model experiments will remain a necessary component of gaining new knowledge of ship performance, evaluating ship designs, and the development ship-specific operator guidance for the most severe wave conditions.

In this paper, an experimental procedure is discussed to systematically evaluate dynamic stability performance of a ship in heavy weather conditions. The procedure relies on the ability to

experimentally generate deterministic wave sequences, based on ship-specific information. Using deterministic wave sequences, determined from ship-specific wave properties, a systematic model experiment program may be developed to reproduce and repeat conditions which will result in an undesired motions response. These properties also enable the determination of the probability of occurrence of rare wave events, which have critical characteristics, resulting in undesirable ship response. These deterministic wave sequences will enable efficient realizations in the basin, allowing for the investigation of parametric design changes, as well as assisting with the development of ship-specific operator guidance.

CURRENT DYNAMIC STABILITY ASSESSMENT PROCEDURES

A short discussion of two primary methods for dynamic stability assessment is provided. These include testing in regular waves, and testing in random waves. For the purpose of this discussion, both wave environment approaches employ remotely controlled ship models. The advantages and disadvantages of each approach are briefly summarized. For most new ship designs, experimental testing in a basin will remain the method of choice.

Regular Wave Testing

Traditional dynamic stability assessment methods, experimental as well as numerical, often use regular waves, which are fundamental and easily represented mathematically. An example of a ship model test from the Naval Surface Warfare Center, Carderock Division (NSWCDD) Maneuvering and Seakeeping (MASK) Basin with DTMB Model #5514 (Hayden, et al., 2006) is shown in Fig. 1.

For many dynamic stability related phenomena, an initial assessment made with regular waves of varying steepness ($H/\lambda \sim 1/10$) and varying length ($\lambda/L \sim 1.0$) can provide an indication of seakeeping and dynamic stability performance. These fundamental experiments enable a distinction to be made between adequate ship designs and infeasible ship designs. However, because regular waves are not directly representative of the natural wave environment in which a ship operates, subtle performance issues may not be able to be assessed or distinguished.

For hull forms which may have particular design or operational constraints, regular wave testing may not provide insight into performance in specific limiting seaway conditions. For these hull forms, regular wave testing is only the first step in the assessment process and can be used to compare new hull forms to previous designs. The operational experience from previous designs can be used to determine the general scope of potential operational restrictions for the new hull form, and eliminate infeasible or unsatisfactory hull form designs from consideration.



Fig. 1. Experimental testing of ship performance in heavy seas using regular waves

Random Wave Testing

The next level of complexity in experimental assessment uses random waves, which are more representative of the natural wave environment. These more realistic conditions can provide additional insight into the behavior of a ship in sea conditions deemed critical to its hull design and load condition. For most dynamic stability related phenomena, this next level assessment made with random waves of varying significant height, modal period, and spectral shape, can provide a more realistic indication of seakeeping and dynamic stability performance.

Random wave testing relies on long exposure times in the model basin to observe critical, but statistically rare, events which can occur in a given seaway. To achieve long exposure times, multiple test realizations in the seakeeping basin are required. Random wave testing can be used to determine response to a limited range of severe seaway conditions and provide more details regarding potential operational restrictions for the hull form.

However, these long exposure times may be practically limited by the ability of the wave-maker in the experimental basin to generate specific, more complex wave conditions, and also by concerns over reflections and self-repeating of the generated waves. Additionally, the time (and cost) associated with experimental testing limits the number of seaway conditions to which the ship may be exposed.

Limits associated with run time, reflections, and statistical uncertainty, may be alleviated by the use of large-scale model testing in open-water conditions, to enable long exposure times. However, the lack of control over weather conditions, and the techniques used to measure environmental conditions in open water testing become technical challenges which must be addressed.

Once a rare event is observed in the basin, repeatability can become an issue in random wave testing. Because of the nonlinearity of ship motion response at large angles of pitch and roll, there is sensitivity to initial wave and model attitude conditions. In random wave testing, lack of control of initial conditions prior to encountering a critical event, may result in significant difficulty

in reproducing the event, such as a large roll angle, in a particular seaway condition. Some gross approximation of the initial conditions, such as relative phase with the wave, can be made, but the specific initial conditions for the ship at the initialization of an encounter with a wave sequence resulting in a critical event are generally not provided. This not only makes probabilistic assessment of the dynamic stability performance difficult, but also is a challenge for systematically assessing parametric design changes and validating numerical simulations of ship performance in the seaway.

DIFFICULTIES IN DYNAMIC STABILITY ASSESSMENT

As discussed, critical ship motions are most often the result of either extremely high or extremely steep waves, or a particular sequence of waves. However, the rarity of occurrence of these wave events makes the assessment of ship response in these conditions difficult. The response of a ship to these critical wave events is expected to result in large amplitude motion, and to be significantly influenced by nonlinearities from wave forcing, damping, and hydrostatic restoring. As is well known, when a dynamical system has significant nonlinearities, its behavior becomes very sensitive to initial conditions (Poincaré, 1890; Lorenz, 1963). Depending on the initial conditions, the ship response to a large wave may range from small motions, to catastrophic motions including possibly capsizing. The difficulty with the assessment of dynamic stability “failures,” is as result of both their rarity of occurrence and the significant nonlinearity of the ship response in these seaway conditions, and both must be addressed simultaneously.

Problem of Rarity and the Principle of Separation

Assessment of the dynamic stability behavior in random seaway conditions constitutes the general problem of rarity – when the time between “failure” events is long, compared to a specific time-scale of interest, such as roll period (Belenky, et al., 2008; 2010). As discussed in Belenky, et al., the principle of separation considers distinguishing the nonlinear phenomena which results in an undesired ship response from

the conditions which lead to its occurrence. This enables the possibility of modeling of the ship response as a combination of two sub-problems: non-rare and rare. The non-rare problem is used to determine the probability of occurrence of conditions which may lead to severe response, and determining the distribution of the appropriate initial conditions. The rare problem is used to determine whether large responses occur for a particular set of initial conditions.

As further discussed in Belenky, et al. (2008; 2010), the main assumption behind the principle of separation is that a mechanical system can be “restarted” at a particular moment of time, *if* the state variables at that instant are fully determined. This is an assumption for the ship response, because the hydrodynamic memory effect cannot be fully considered in this form.

Assessment of Parametric Design Changes

In addition to the difficulty of assessing dynamic stability, due to the nonlinearities and rarity, systematic assessment of parametric design changes are also difficult. Just as the ship performance is highly sensitive to initial conditions, subtle changes in hull form geometry or appendages (such as bilge keels, rudders, etc) may have significant impact on the dynamic stability performance. In order to assess this impact in realistic seaway conditions, repeatability becomes very important.

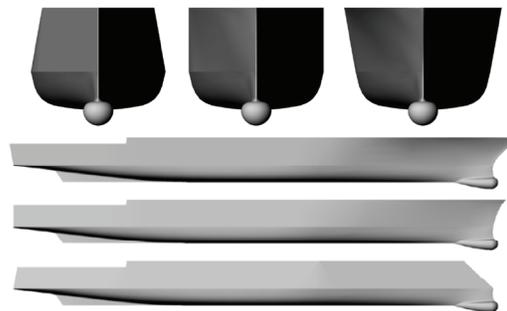


Fig. 2. ONR Topside Series Hull Forms, including tumblehome (top left, bottom), wall-sided (top middle, middle), and flared (top right, top) topside variations.

An example of this was the study of the effect of topside geometry variation on dynamic stability performance, using the ONR Topside Series (Fig. 2), which were designed at NSWCCD by Dipper, Campbell, and Belknap (Bishop, et al., 2005) and include flared, tumblehome, and wall-sided

variants (DTMB Model #5613, 5613-1, and 5613-2). Similar to DTMB Model #5514, they have become an international standard for ship performance testing of naval combatant type hull forms.

Validation of Numerical Simulations

Repeatability is important for assessing the effect of parametric design changes, and is also important for validation of numerical simulations. The use of numerical simulations can become a practical method for assessing a large number of ship designs, or for developing ship-specific operator guidance. However, numerical tools with the required fidelity to assess design changes or provide the level of accuracy necessary for detailed operator guidance are still in a developmental stage. Therefore, it is currently necessary to use experimental assessments for development and validation of these numerical simulation tools. A detailed discussion regarding precisely what constitutes validation is outside the scope of this paper, but is a subject of much recent development.

In order to provide useful data for this, initial conditions, wave conditions, and ship response must all be measured and recorded at a significant level of detail. As discussed, this is currently difficult in the random wave testing methods typically used for more physically realistic ship dynamic stability assessments.

WAVE GROUP APPROACH FOR DYNAMIC STABILITY ASSESSMENT

Because of the practical limitations of providing long exposure times and a broad range of seaway conditions for ship models in basin experiments, several alternative methods have previously been proposed. A more detailed review is given in Bassler, et al. (2008; 2009). At this time, the approach that appears most likely to address the aforementioned issues for experimental dynamic stability assessment is the wave group approach.

The concept for this method is to extract a sequence of waves which can result in large amplitude excitation of the ship model and evaluate the dynamic response to these particular sequences of waves, or “wave groups,” with random initial conditions. A definition for this type of wave sequence, or wave group, from the

ship response perspective is proposed in Bassler, et al. (2010; 2010a). The first complete implementation of this type of approach with quantitative results was proposed during the SAFEDOR project (Spyrou & Themelis, 2005; Themelis & Spyrou, 2007; 2008). A similar approach has also been followed by Umeda, et al. (2007).

Overview

The goal of using the wave group approach for experimental evaluation of ship dynamic stability performance is to:

- 1) be able to evaluate and compare specific hull form design variants and loading conditions
- 2) determine specific potential operational restrictions for a given hull form design,
- 3) verify simulation tools and aid with further development
- 4) provide an assessment process that can be used to eliminate infeasible or unsatisfactory hull form designs.

The wave group method relies on two primary components: precise generation of a deterministic wave-field, and control of initial conditions.

Wave-Field Generation

Within one seaway spectrum, an infinite number of seaway time series realizations can be generated. However, only a small subset may have wave sequences which result in critical ship motions. These wave sequences are then the conditions of interest for ship dynamic stability performance and extreme event assessment.

In order to address the previously discussed issues for systematic exposure and repeatability, the wave-field generation process must be deterministic, such that the time and location that wave groups will occur in the model basin is known. It has been shown that groups of large waves, as well as single large waves, can be reproduced deterministically in an experimental basin (e.g. Davis & Zarnick, 1964; Takezawa & Takekawa, 1976; Clauss, 2000; Clauss, et al., 2008; Bassler, et al., 2008; 2009). However, modern paddle-type wave-makers can be used to generate more complex seaways in a deterministic manner (Fig.). A new wave-maker is currently under construction for NSWCCD (Hayden, et al., 2010), as shown in Fig. 4, and will provide

improved capability to deterministically generate complex wave-fields.



Fig. 3. A sample visualization of a complex seaway generated in the NSWCCD MASK basin, using the new wave-maker

It is also important to characterize the local wave-field surrounding the ship up to and including the dynamic stability failure event. Typically an array of ultrasonic wave probes is used in the MASK basin. However, to provide a more dense set of high-resolution local wave-field measurements, several other techniques may be considered, such as LIDAR or the Global Laser Rangefinder Profilometry (GLRP) technique, which was developed at NSWCCD (Atsavapranee, et al. 2005; Carneal, et al, 2005; 2005a; Carneal & Atsavapranee, 2006). GLRP was used to characterize deterministic realizations of steep waves in the MASK basin by Bassler, et al. (2008; 2009) – see Fig. 6.

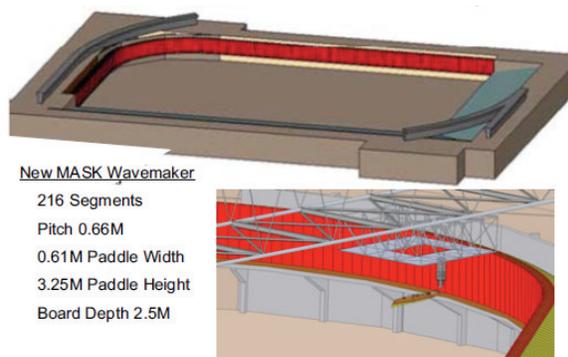


Fig. 4. NSWCCD MASK basin with new wave-maker (top), with model release mechanism shown attached to the basin carriage in the Northwest corner (bottom)

Sub-surface measurement methods, such as particle image velocimetry (PIV), have also been used to characterize wave kinematics (Minnick, et al., 2010; 2011; 2011a). In addition to the surface field measurement methods, this can provide a complete quantitative picture of the waves in the near-field to the ship model. Then, a complete data set of wave conditions (free-surface and

kinematics) will be available to compare with ship model performance, and also provide a more complete characterization for the development and validation of numerical simulation tools. Future wave-tracking methods may even be developed to provide detailed measurement of the instantaneous waterline along the hull. From this, additional insight into the primary mechanism for dynamic stability failures due to righting arm variations in waves.

Coupled with the precise local wave-field measurements, the model position can be tracked in the basin using a laser-tracker system, such as the *iGPS* system currently used at NSWCCD in the MASK basin. The *iGPS* system uses several spinning eye-safe laser transmitters with infrared LED flashes to provide stable constellation references along the perimeter of the MASK basin. In addition, an onboard detector and Position Calculation Engine (PCE) are used in the model to determine the XYZ coordinates of the detector's position in the basin, with respect to the known location of the transmitters. Using three or more detectors on a model provides six degree-of-freedom (6 DOF) motion data for any object in the MASK basin.

Initial Condition Control- A Model Release Mechanism

With the ability to repeatedly generate deterministic wave-fields, initial condition control is the next important step. For initial condition control, a model release mechanism can be used to enable repeatable, measurable initial conditions for a free-running remote controlled ship model in the basin. Previous attempts included mechanisms based on using electro-magnets or solenoids, but proved to be infeasible, due to concerns over instrumentation interference issues and reliability issues, so a new concept was developed.

For dynamic stability testing, a model release mechanism was designed (Figs. 5 & 6) to allow the ship model to be free to pitch, heave and roll, but fixed in surge, sway and yaw. Additionally, the mechanism could also allow for the option to fix roll in various heel conditions. For precise initial condition control, the mechanism should negate (as much as practically possible), the inertial impact of the mechanism on the motions of the model. Wave measurements near the model

would enable accurate recording of the wave conditions with respect to the model at the instant of release. The mechanism was also intended to provide the ability to release the model at a controlled point in time, with respect to at least four phase positions in waves (crest, trough, front slope, and back slope). Lastly, the mechanism was also designed with the consideration that it should allow for easy retrieval and rest of the model between realization conditions in the basin.

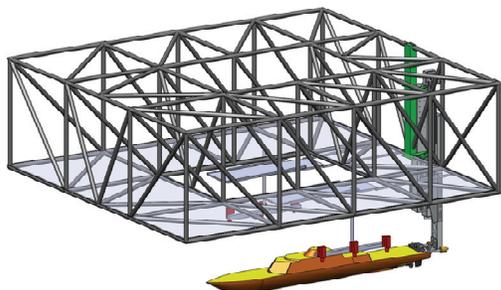


Fig. 5. Notional ship model attached the carriage with the model release mechanism and wave probes for local wave-field measurement shown (in red)

For each deterministic wave sequence condition of interest, a minimum of thirty-six variations are recommended for investigation. These include three speeds, three headings, and four wave phase realizations. The three speeds would consist of the target speed of interest and speed conditions slightly slower and slightly faster. The three heading conditions would consist of the heading of interest (e.g. stern-quartering seas) and headings slightly aft and slightly abeam. The four wave conditions would be the crest, trough, front slope, and back slope of the wave. A pre-determined acceleration profile would be programmed to enable the model to achieve the desired speed condition after release from the mechanism.

In addition to the typical instrumentation packages for ship motions, including gyroscopes and accelerometers, the remote control models can also be outfitted with instrumentation to record forces on appendages, while also recording instantaneous ship speed and propeller rpm. Pressure panels can also be installed on the hull and superstructure to measure wave-impact loads at specific regions of interest, or to measure deck wetness during green water occurrence.

Experimental Technique

After initial model tests are performed in regular waves to determine general behavior of the ship in a seaway, additional model experiments using the wave group technique can be performed to determine probabilistic risk assessment, to investigate parametric design changes, or to develop ship-specific operational guidance. The ship specific properties can also be used to identify wave sequence parameters that are more likely to result in undesirable dynamic stability performance. An example of this for ship roll motion is given in Bassler, et al. (2010; 2010a).

These parameters can also be used to search for probability of occurrence of wave sequences of interest in particular geographic regions, enabling the coupling of the risk of dynamic stability failure events to occur, with likelihood of occurrence of wave sequences that may result in dynamic stability failure, assuming a particular set of initial conditions (Themelis and Spyrou, (2007; Bassler, et al. 2010a; Belenky, et al., 2010).

The overall technique described is shown in Fig.. Synchronized computer control of the model and the wave-maker are necessary to enable the specified initial conditions to be achieved, in a given sea condition of interest. A feedback loop of the measured state conditions of the ship model and wave conditions near-field of the model can be used to determine the precise moment to release the model.

Because the wave-field in the basin is deterministic, for each set of initial conditions of the model, the wave phase can be systematically varied to examine the sensitivity of the end-state response for the wave sequence of interest.

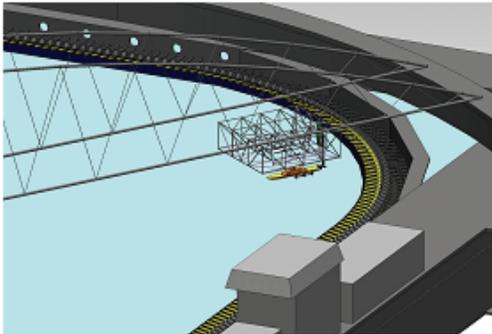
Potential Benefits of the Wave Group Approach

The wave group approach provides a practical method to experimentally assess ship dynamic stability performance. Systematically exposing the ship model to wave conditions which yield a high probability of failure can provide a more accurate and detailed determination of dynamic stability performance. This process lends itself to parametric modeling, allowing assessment of the parametric change in design by comparing performance in the same wave sequence. Because of the measurement of initial conditions and a deterministic wave-field, the data from this

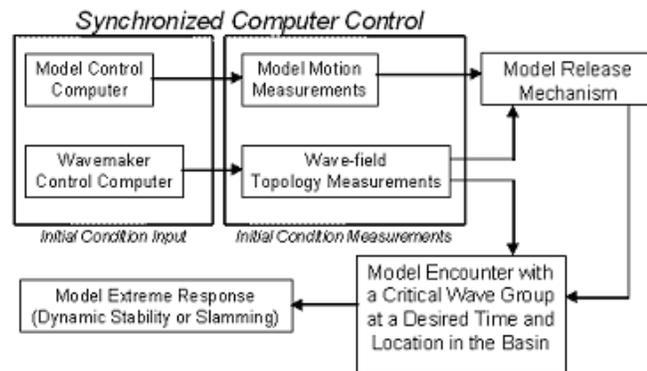
method of conducting experiments may be more helpful for conducting probabilistic risk assessment and for the validation of numerical tools used to assess dynamic stability performance.

The wave group approach may be used to provide guidance for operational environments.

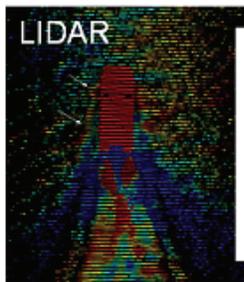
The method can provide a way to link the ship performance with the likelihood of encountering critical wave conditions for a specific ship. Then, dangerous combinations of loading conditions and operational parameters (ship speed and heading) can be identified.



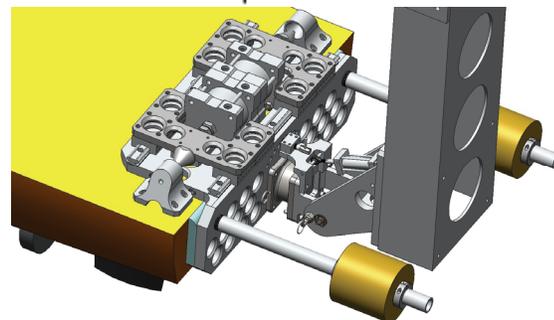
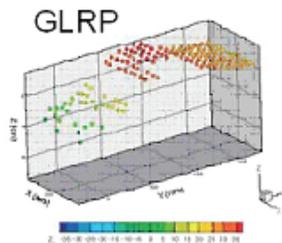
Ship Model in the MASK Basin with New Wavemaker



Model Experiment Process



Wave-field Topology Measurements



Model Release Mechanism Concept

Fig.6. Model release mechanism concept and process

The wave group approach to experimental dynamic stability assessment appears to have the potential for increased accuracy, as well as time (and cost) savings, in experimental testing over traditional methods using random waves. However, regular wave testing should remain a crucial first-step for experimental assessment of dynamic stability performance.

CONCLUSIONS

A method using wave groups for experimental dynamic stability assessment was discussed. The method can reduce testing time in the model basin when a dynamic stability problem has been identified or a design change has been implemented. This process can improve the

representation of the particular critical seaway conditions of interest, increase confidence in the dynamic stability assessment of ship performance, and increase confidence in safety in severe seaway conditions.

Using this method, parametric design variations may be examined in the same deterministic wave sequences, which can be precisely repeated for each design variation. This deterministic method also provides for more precise comparisons with simulation tools, enabling validation and further tool development. Quantification of initial conditions and the position of the model relative to the wave field at an instant in time also aids in simulation tool development. Because of the short run times

inherent in the deterministic evaluation process, unsteady Reynolds-Averaged Navier Stokes (URANS) methods can also become more practical for conducting comparative assessments.

Longer-term, this deterministic method may also be considered for the identification of specific wave sequences that are critical to the ship. Then, with accurate on-board wave-field sensor measurements, and with faster than real-time nonlinear wave propagation and ship motion prediction methods, the possibility for real-time ship-specific operator guidance may be realized.

Although not yet examined, the wave group method coupled with model instrumentation techniques may also have the possibility to be used for the assessment of other rare events, such as slamming. However, this is the subject of future work.

ACKNOWLEDGEMENTS

The authors would like to express their appreciation for the support of the work in the paper from the Naval Innovative Science and Engineering Program (NISE) at NSWCCD, under the direction of Dr. John Barkyoumb. Additionally, the authors also appreciate support from Mr. James Webster (NAVSEA) and Dr. Pat Purtell (Office of Naval Research) for previous research and development, which provided the foundation and initialization of this work. The authors would also like to thank Mr. John Hoyt and Mr. Dan Hayden (NSWCCD) and Prof. Kostas Spyrou (NTUA) for their helpful discussions regarding the contents of this paper.

REFERENCES

- Atsavapranee, P., J. B. Carneal, C. W. Baumann, J. H. Hamilton, and J. W. Shan (2005), "Global Laser Rangefinder Profilometry (GLRP): A Novel Optical Surface-Wave Measurement System," Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2005/022.
- Bassler, C. C. G. E. Lang, S. S. Lee, J. B. Carneal, J. T. Park, and M. J. Dipper (2008), "Formation of Large-Amplitude Wave Groups in an Experimental Basin," Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2008/025.
- Bassler, C. C., M. J. Dipper, and G. E. Lang (2009), "Formation of Large-Amplitude Wave Groups in an Experimental Basin," *Proc. 10th Intl. Conf. on Stability of Ships and Ocean Vehicles*, St. Petersburg, Russia.
- Bassler, C. C., V. Belenky, and M. J. Dipper (2010), "Characteristics of Wave Groups for the Evaluation of Ship Response in Irregular Seas," *Proc. 29th Intl. Conf. on Ocean, Offshore, and Arctic Engineering*, Shanghai, China.
- Bassler, C.C., V. Belenky, and M.J. Dipper (2010a), "Application of Wave Groups to Assess Ship Response in Irregular Seas," *Proc. 11th Int. Ship Stability Workshop*, Wageningen.
- Belenky, V., J. O. de Kat, and N. Umeda (2008), "Toward Performance-Based Criteria for Intact Stability," *Marine Technology*, 45(2):101-123.
- Belenky, V., C. Bassler, M. Dipper, B. Campbell, K. Weems, and K. Spyrou (2010), "Direct Assessment Methods for Nonlinear Ship Response in Severe Seas," *Proc. ITTC Intl. Workshop on Seakeeping*, Seoul, Korea, 19-21 October.
- Bishop, R.C., W. Belknap, C. Turner, B. Simon and J.H. Kim (2005), "Parametric Investigation on the Influence of GM, Roll Damping, and Above-Water Form on the Roll Response of Model 5613," Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2005/027.
- Carneal, J. B., P. Atsavapranee, C. W. Baumann, J. H. Hamilton, and J. Shan (2005), "A Global Laser Rangefinder Profilometry System for the Measurement of Three Dimensional Wave Surfaces," *Proc. ASME Fluids Engineering Division Summer Meeting*, Houston, TX, 19-23 June.
- Carneal, J. B., P. Atsavapranee, and J. T. Curight (2005a), "Global Laser Rangefinder Profilometry: Initial Test and Uncertainty Analysis," Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2005/069.
- Carneal, J. B. and P. Atsavapranee (2006), "Global Laser Rangefinder Profilometry: Initial Test and Uncertainty Analysis," *Proc. ASME Joint European Fluids Summer Meeting*, Miami, FL, 17-23 July.
- Clauss, G. F. (2000), "Tailor-made Transient Wave Groups for Capsizing Tests," *Proc. 7th Intl. Conf. on Stability of Ships and Ocean Vehicles*, Launceston.
- Clauss, G. F., C.E. Schmittner, and J. Hennig (2008), "Systematically Varied Rogue Wave Sequences for the Experimental Investigation of Extreme Structure Behavior," *J. Offshore Mechanics and Arctic Engineering*, 130, May.

- Davis, M.C. and E.E. Zarnick (1964), "Testing Ship Models in Transient Waves," *Proc. 5th Symp. on Naval Hydro.*, Bergen, Norway, 10-12 September.
- Hayden, D. D., R. C. Bishop, J. T. Park, and S. M. Laverty (2006), "Model 5514 Capsize Experiments Representing the Pre-Contract DDG51 Hull Form at End of Service Life Conditions," Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2006/020, April
- Hayden, D. D., J. G. Hoyt III, M. Melendez, H. J. Moeller, Y. Bargman, S. Carpenter, and S. R. Turner (2010), "Naval Surface Warfare Center's Wavemaker Modernization Program," *Proc. 29th American Towing Tank Conference (ATTC)*, Annapolis, MD, August.
- Lorenz, E.N., "Deterministic Non-Periodic Flow," *J. Atmospheric Sci.* 20, 130-141, 1963.
- Minnick, L., C. Bassler, S. Percival, and L. Hanyok (2010), "Large-scale Wave Kinematics Measurements of Regular Waves and Large-Amplitude Wave Groups," *Proc. 29th Int. Conf. on Ocean, Offshore and Arctic Engineering*, Shanghai.
- Minnick, L. M., C. C. Bassler, S. Percival, and L. W. Hanyok (2011), "Characterization of Regular Wave, Irregular Waves, and Large-Amplitude Wave Group Kinematics in an Experimental Basin" Hydromechanics Dept. Technical Report, NSWCCD-50-TR-2011/012, February
- Minnick, L, C. Kent, C. Bassler, S. Percival, and L. Hanyok (2011a), "Kinematics of Experimentally Generated Severe Wave Conditions and Implications for Numerical Models," *Proc. 30th Intl. Conf. on Offshore Mechanics and Arctic Engineering*, Rotterdam, The Netherlands, 19-24 June
- Poincaré, H.J. (1890), "Sur le problème des trois corps et les équations de la dynamique," *Acta Mathematica*, 13, 1-270.
- Spyrou, K. J. and N. Themelis (2005), "Probabilistic Assessment of Intact Stability," *Proc. 8th Intl. Ship Stability Workshop*, Istanbul.
- Takezawa, S. and M. Takekawa (1976), "Advanced Experiment Technique for Testing Ship Models in Transient Water Waves, Part I: The Transient Test Technique on Ship Motions in Waves," *Proc. 11th Symp. on Naval Hydrodynamics*, University College, London.
- Themelis, N. and K. J. Spyrou (2007), "Probabilistic Assessment of Ship Stability," *Trans. Society of Naval Architects and Marine Engineers*, Vol. 115 pp. 181-206.
- Themelis, N. and K. J. Spyrou (2008), "Probabilistic Assessment of Ship Stability Based on the Concept of Critical Wave Groups," *Proc. 10th Intl. Ship Stability Workshop*, Daejeon, Korea, 23-25 March.
- Umeda, N., M. Shuto, and A. Maki (2007), "Theoretical Prediction of Broaching Probability for a Ship in Irregular Astern Seas," *Proc. 9th Intl. Ship Stability Workshop*, Hamburg.