Integrity Diagrams of the Ship/U-Tank System Undergoing Parametric Rolling

Marcelo A. S. Neves*, Claudio A. Rodríguez*, Jerver E. M. Vivanco**, José C. Villagómez Rosales**, Radhesh Agarwal***

* LabOceano – COPPE/UFRJ; ** COPPE/UFRJ; *** IIT-Karagpur, India

ABSTRACT

Unstable rolling motions in regular head seas are investigated in the case of a ship stabilized with an U-type anti-rolling tank (ART). A transom stern small vessel, well-known for her tendency to develop strong parametric excitation is investigated. Nonlinear equations are employed to describe the liquid motion inside the tank, the forces and moments generated by the tank on the ship and the coupled ship motions (heave, roll and pitch). These are numerically solved for different initial conditions. Nonlinear dynamics techniques are applied to the coupled ship/tank system. In particular, it is verified that the nonlinear system, despite the fact that the ship is stabilized with a tuned ART, as wave amplitude is increased there exists a process of erosion of the safe basin which is not smooth. Similarly to the case of the unstabilized ship, the integrity diagrams show a clear *cliff* tendency at some critical wave amplitudes. The implication of this aspect is relevant to the design of ART's against parametric rolling, in the sense that it allows for the definition and quantification of a critical wave amplitude for distinct tank designs in the context of an analysis that takes into account all the relevant nonlinearities of the coupled ship and tank motions under the influence of a whole set of initial conditions.

KEYWORDS

Anti-rolling tanks; Parametric rolling; Nonlinear dynamics; Safe basins; Integrity diagram.

INTRODUCTION

The phenomenon of parametric rolling of ships is a nonlinear dynamical instabilization process that has attracted much attention recently. Recent examples can be found in Neves and Belenky (2008).

By means of systematic variations of encounter frequency and wave amplitude Neves and Rodríguez (2007) numerically obtained Parametric Amplification Domains (PADs), based on coupled nonlinear equations. These new numerical diagrams display information not only on the boundaries of stability, but additionally they provide information on the nonlinear roll amplitudes in the whole domain inside the boundaries. A rich picture is obtained, including the appearance of upper boundaries, a general tendency of the system to get stiffer due to coupling, abrupt decrease in the upper boundaries with fractal geometry.

Safe basin analysis has been performed in Neves et al. (2009a) having wave amplitude as the control parameter. It was shown that for relatively small wave amplitudes the appearance of fractal boundaries was counter-balanced by an increase in other areas of the safe basin, what resulted in an overall increase of the safe area. Yet, for higher waves, erosion of safe basin came up very rapidly. Finally, the integrity curve for the ship at the *exact* tuning $w_e / w_{n4} = 2.0$ was obtained. It was observed that as erosion of the safe basin starts, a sharp decline of the safe basin area defines a critical wave amplitude for the safety of the vessel in terms of the area of the safe basins.

In this paper the nonlinear ship/tank problem under parametric excitation as modeled in Neves et al. (2009b) is revisited. That paper presented a comprehensive analysis of PADs corresponding to the coupling of nonlinear ship motions with a nonlinear model of the motion of the internal fluid. One main conclusion was that for the range of parameters simulated, an Anti-Rolling Tank (ART) may eliminate parametric rolling at some conditions but these may persist (or appear) at some other conditions.

Nonlinear dynamics techniques previously employed by the authors are now applied to the problem of the ship with tank. In particular, it is verified that despite the fact that the ship is now stabilized with a tuned ART, there exists a process of erosion of safe basin which is not smooth. Similarly to the case of the unstabilized ship, the integrity diagrams show a clear *cliff* tendency at some critical wave amplitudes. The implications of this aspect are relevant to the design of ART's against parametric rolling, in the sense that it allows for the definition and quantification of a critical wave amplitude for distinct tank designs in the context of an analysis that takes into account all the relevant nonlinearities of the coupled ship and tank motions under the influence of a whole set of initial conditions. This methodology has clear advantages when compared with the less consistent maps of numerical domains of parametric amplification.

MATHEMATICAL MODEL

Geometric characteristics of the U-tank are shown in Fig. 1. Two vertical reservoirs are connected by a transversal duct, with all elements having rectangular constant cross sections. A partial obstruction situated at the mid position of the horizontal duct allows the consideration of variable damping actions. Fluid motion, assumed to be unidirectional, is described by fluid displacement Z(t), defined in Fig.1.

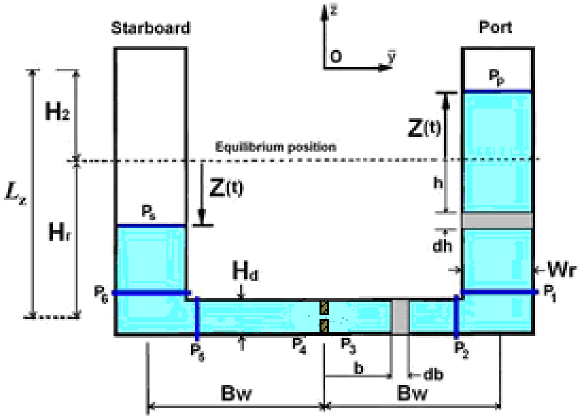

Fig. 1 Tank geometry

Nonlinear equations of ship motions may be represented as:

$$(\tilde{M}+\tilde{A})\ddot{\vec{S}}+\tilde{B}(\dot{\phi})\dot{\vec{S}}+\vec{C}_r(\vec{S},\zeta)=\vec{C}_{ext}(\zeta,\dot{\zeta},\ddot{\zeta}) \\ +\vec{C}_t(Z,\dot{Z},\ddot{Z},\vec{S},\dot{\vec{S}},\ddot{\vec{S}}) \qquad (1)$$

where vector $\vec{S}(t)= z(t),\phi(t),\theta(t)^{T}$ represents rigid body motions in the heave, roll and pitch motions, respectively. In Equation (1), $\tilde{M}$ is a 3x3 inertia matrix, $\tilde{A}$ is a 3x3 added mass matrix. Matrix $\tilde{B}$ is the damping matrix which may incorporate non-linear terms in the roll equation. $\vec{C}_r$ is a 3x1 vector which describes non-linear restoring force and moments dependent on the relative motions between ship hull and wave elevation ζ(t). On the right hand side of equation (1), the generalized vector $\vec{C}_{ext}$ represents wave external excitation, dependent on wave heading χ, encounter frequency ω_e, wave amplitude Aw and time t. Finally, the generalized vector $\vec{C}_t$ represents nonlinear forces and moments acting on the ship due to the fluid motion inside the tank. For a ship without tank, Equation (1) reproduces the system of non-linear equations introduced in Neves and Rodríguez (2004). Due to space limitations, the detailed expressions of the components of equation (1) are not repeated here, see Neves et al. (2009b) for details. From the theoretical point of view it is important to consider the internal fluid motion as nonlinear, interacting with the nonlinear ship motions. Therefore, the ship/tank problem may be investigated as a strongly coupled heave-roll-pitch-tank problem, resulting in a four degrees of freedom nonlinear problem. The 4th equation, describing the nonlinear internal fluid motion Z(t), has also been derived in Neves et al. (2009b).

SHIP AND TANK PARTICULARS

Ship TS hull forms are shown in Fig. 2; main characteristics are defined in Table 1. Dimensions and main characteristics of the ART employed in the present simulations are (see Fig. 1 for definitions): *Bw=3.00 m, Wr=1.5 m, Hr=1.5 m, Hd=0.381 m, m%*=3.00 is the percentage of mass of water relative to the mass of the ship, ω_t / ω_{n4}=1 is the tuning of the tank with the ship, where ω_t is the natural frequency of the tank and ω_{n4} is the roll natural frequency of the ship. Non-dimensional damping is $\eta_t = 0.3$.

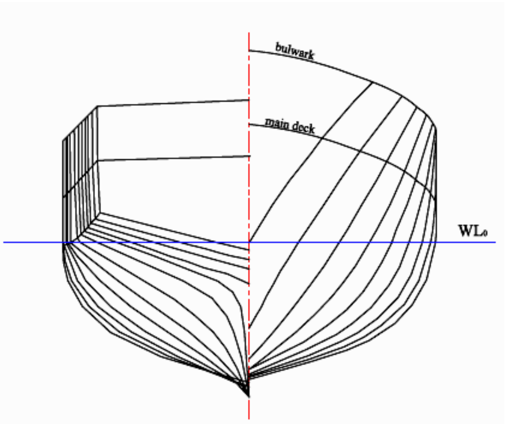

Fig. 2 Ship hull form, Transom Stern (TS).

Internal dissipative effects may be determined by means of decay tests in which an initial condition is applied to the internal fluid. The resulting oscillatory motion of the fluid inside the static tank is modeled as:

$$\ddot{Z} + p_1\dot{Z} + p_2\left|\dot{Z}\right|\dot{Z} + \omega_t^2 Z = 0 \qquad (2)$$

For reduced obstructions, the damping action is essentially linear ($p_2 \simeq 0$). The tank natural frequency is defined by geometrical characteristics of the tank (see Fig. 1 for definitions) and g, the gravitational acceleration:

$$\omega_t = \sqrt{\frac{g}{W_r\left(\dfrac{B_w}{H_d} + \dfrac{H_r}{W_r}\right)}} \qquad (3)$$

Table 1: Ship main characteristics

Denomination	TS
Length overall	25.91 m
Length between perpend.	22.09 m
Breadth	6.86 m
Depth	3.35 m
Draft	2.48 m
Displacement	170.3 ton
Longit. radius of gyration	5.52 m
Metacentric height	0.37 m

DOMAINS OF AMPLIFICATION

In the following, parametric rolling amplification is analyzed with respect to both wave amplitude and frequency. For a global analysis of these parameters, diagrams of ω_e / ω_{n4} vs *Aw* are developed in which roll amplitude is represented in a scale of colors. Fig. 3 shows the PADs for ship without and with ART considering *Fn=0.3*, χ *=180º*, $\boldsymbol{\eta_t = p_1/2\omega_t = 0.3}$, $p_2 = 0$ and $\phi_0 = 0.8^0$.

In particular, three aspects are relevant in the present context: i) existence of upper boundaries of stability are observed; ii) areas of parametric amplification corresponding to $\omega_e / \omega_{n4} \cong 2$ are bent to the right, as discussed in Neves and Rodríguez (2004, 2007) in the case of a ship without tank; iii) a tuned tank, in spite of being beneficial at most of the unstable domain, is not sufficient to avoid strong ship motions and capsize in the concave upper part of the PAD.

Therefore, contradictory to some arguments encountered in the literature stating that ARTs can in general cope well with parametric amplifications, the above results indicate that the situation may not be that simple.

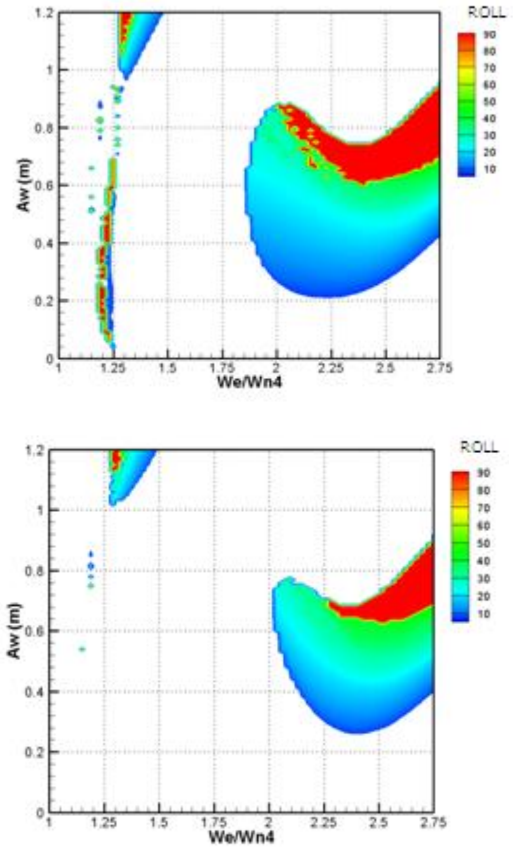

Fig. 3. PAD for ship without ART (upper); with ART (lower)

However, it is important to notice that these domains are numerically established for a single set of initial conditions (in this case, $\phi_0 = 0.8^0$). Since parametric rolling is strictly a nonlinear phenomenon, it is sensitive to initial conditions of the system, Neves and Rodríguez (2007). This aspect throws a certain level of uncertainties on the use of these domains as a practical tool for assessing the safe qualities of a ship design. Therefore, it becomes necessary to advance towards a more exhaustive and systematic analysis on the influence of initial conditions on parametric roll developments of the system.

SAFE BASINS

Safe basins are used to perform analysis based on varying systematically initial conditions, Belenky and Sevastianov (2007). These are defined here as regions from which trajectories reach a determined set of maximum roll amplitudes within a time interval of 600 seconds. A resolution of 120x80 initial conditions separated regularly by steps of 1° (one degree) both in roll amplitude and velocity is employed. A comparison of safe basins for ship without (upper row) and with tank (lower row) is shown in Fig. 4, for $\omega_e/\omega_{n4} = 2.08$ for low wave amplitudes (A_w=0.2 m, 0.4 m, 0.6 m).

It is noticed that for this range of low wave amplitudes the safe basin increases continuously. This is thought to be due to the increased rigidity (dependent on wave amplitude squared) introduced by the nonlinear coupling, as discussed in Neves and Rodríguez (2007). Up to Aw=0.66 m there is no noticeable fractal erosion of the safe basin. But, as the wave amplitude is increased beyond a threshold value (around Aw=0.67 m) the safe basin area undergoes a progressive level of erosion in the proximity of the saddle points. This may be observed in Fig. 5, which shows the safe basins for the ship/tank system for a given tuning ($\omega_e / \omega_{n4} = 2.154$) and for wave amplitudes in the range Aw=0.68 m ~ 0.82 m.

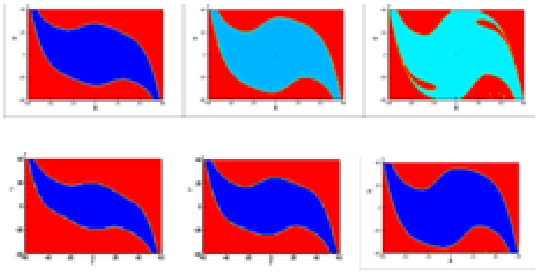

Fig. 4 Comparison of safe basins for ship without (upper row) and with tank (lower row)

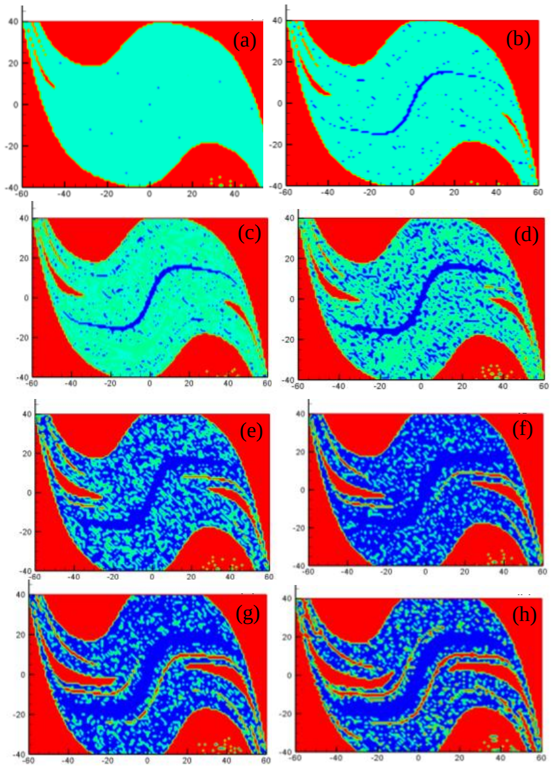

Fig. 5 Safe basins for the ship/tank system, higher range of wave amplitudes, $\omega_e/\omega_{n4} = 2{,}154$, a) Aw=0.68 m, b) Aw=0.70 m, c) Aw=0.72 m, d) Aw=0.74 m, e) Aw=0.76 m, f) Aw=0.78 m, g) Aw=0.80 m and h) Aw=0.82 m.

INTEGRITY DIAGRAMS

Soliman and Thompson (1989) proposed and investigated the use of several different integrity measures. In the present study a definition will be considered which is particularly revealing of some important characteristics of coupled parametric resonance.

Integrity diagrams – curves of integrity for a determined range of wave amplitudes – for the ship with and without tank are drawn in Fig. 6 for the same tuning $\omega_e/\omega_{n4} = 2.154$. All these areas are non–dimensionalized with respect to the safe basin area of roll decay of ship without tank: $\bar{A}(A_W) = A_S(A_W)/A_0$ where *As(Aw)* defines the safe basins areas for different wave amplitudes and *Ao* defines the safe basin area for *Aw=0.0 m* (for the ship without tank). It is possible to appreciate in Fig. 6 the differences between their trends and the fact that for both curves the non-dimensionalized area starts to diminish dramatically – due to the fractal erosion - for higher wave amplitudes. The two-sided phenomenon of increased fractal erosion together with enlargement in other areas of the same basin persists in the case of ship without tank, however the area starts to be eroded for lower wave amplitudes than in the case of the ship with tank. When the U-tank is implemented, a shift of its integrity curve towards the right is observed, corresponding to the beneficial effect of the anti-rolling tank, since it represents the increase of wave amplitude after which the safe basin is dominated by fractal erosion. But it becomes clear that the integrity diagram is not smooth. Analogous to the case of the ship without tank, there is a clear change of tendency at certain *critical wave amplitude*, reflecting a sharp reduction of engineering integrity of safe motions, Thompson et al. (1990). This dynamical aspect of the ship/tank system enables the consideration of a quantitative safety measure of a given ART design.

INTEGRITY SURFACES

A more general view of the trends of the areas of safe basin is obtained when surfaces of integrity are computed, resulting from the computation of integrity diagrams for a range of tunings. Integrity surfaces for the ship are shown in the upper part of Fig. 7, whereas the surface for the ship/tank system is shown in the lower figure; the range of tunings (ω_e/ω_{n4}) is from 1.6 to 3.2.

An interesting characteristic to highlight in the surfaces of integrity is the fact that, like with the ship without tank, for the ship/tank system the non-dimensionalized areas starts to fall dramatically for all frequencies – due to fractal erosion - retaining the *cliff* tendency when the anti-rolling tank is on. The consideration of a critical wave amplitude may be associated to this characteristic tendency of the surfaces of integrity, common to the dynamics of the ship with or without an U-tank. It is possible to observe in the surface of integrity of the ship with tank that the *cliff* appears after a wave amplitude of about Aw=0.63 m.

Fig. 8 shows, for the range of tunings ω_e / ω_{n4} from 1.6 to 2.7, the curves of critical wave amplitudes for the ship with/without tank. It is observed that a positive variation of critical amplitudes occurs, with respect to the values reached for the ship without tank, for a range of tunings. It is also observed that for the higher tunings no gain in increase of critical wave amplitude is obtained. Clearly, the general increase of critical amplitude for the ship with tank is considered beneficial, since it reflects its greater resistance to capsizing, keeping the ship stable for higher wave amplitudes.

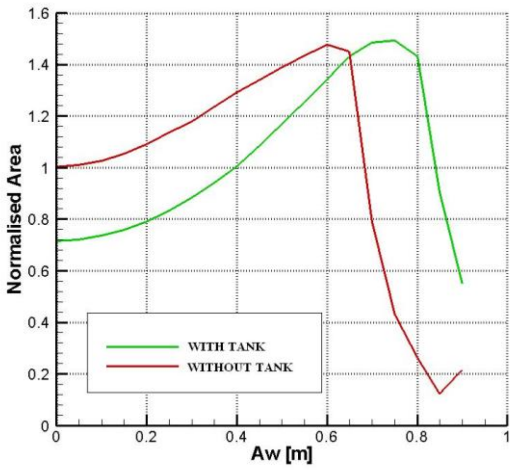

Fig. 6 Integrity diagrams for ω_e / ω_{n4} =2.154

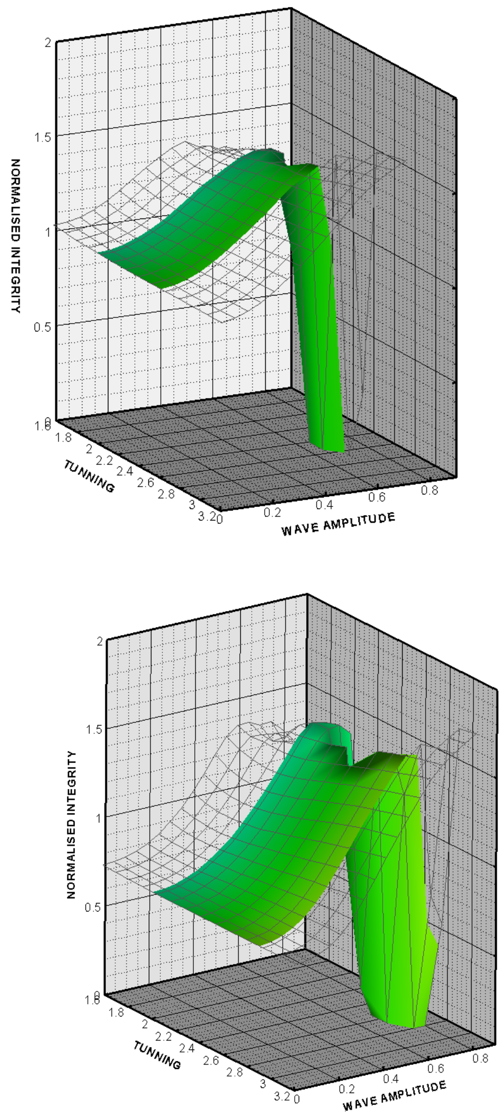

Fig. 7 Integrity surfaces for the ship and ship/tank system.

CONCLUSIONS

A methodology to quantitatively analyze nonlinear responses of the unstable movements of the ship coupled with the water inside the tank in head seas for a broad set of roll initial conditions has been introduced. For the ship with tank the PAD is smaller and there is a general tendency to less intense roll amplifications but, equally important, areas adjacent to the upper boundaries still register ship capsize.

Analysis of integrity diagram and surface has shown that for the ship with tank, despite the fact that the ship is stabilized with a tuned tank, there exists a process of erosion of the safe basin which is not smooth; as in the case of the unstabilized ship, the integrity surface shows a clear cliff tendency at critical wave amplitudes. It has also been shown that the resulting critical amplitudes are higher when the U-tank is activated, particularly at low frequencies. Based on that, it is concluded that there is an overall positive effect of the anti-rolling tank on the control of parametric roll, but safety from capsize remains as a necessary design target.

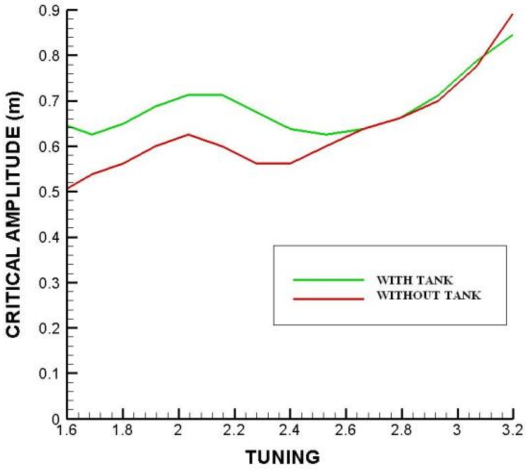

Fig. 8 Critical wave amplitude, ship with/without tank.

The present analysis incorporates sensitivity of the coupled nonlinear system ship/tank to a large set of initial conditions and allows quantification, in terms of increase of critical amplitude and reduction of safe basin, of the net effect of the anti-roll tank on the control of parametric rolling. Considering that as a result of the present analysis a measure of the safety of the ship at sea is obtained, now independent of initial conditions, the methodology could be of practical interest to the development of ship/tank designs. These aspects must be studied in more detail in the future, as well as their relation with irregular seas and introduction of active control to the systems.

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