A Critical Assessment of Ship Parametric Roll Analysis

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ABSTRACT

Analysis of ship parametric rolling has generally been restricted to simple analytical models and sophisticated time domain simulations. However, simple analytical models do not capture all the critical dynamics while time-domain simulations are time consuming to implement. Our model captures the essential dynamics of the system without over simplification. This work incorporates important aspects of the system and assesses the significance of including or ignoring these aspects. Many of the previous works on parametric rolling make the assumption of linearized and harmonic behaviour of the time-varying restoring arm or metacentric height. This assumption enables modelling the roll as a Mathieu equation.

KEYWORDS

Parametric Rolling; Mathieu Equation, Hills Equation

INTRODUCTION

Analysis of ship parametric roll has generally been restricted to simple analytical models and sophisticated time domain simulations. Simple analytical models do not capture the all the critical dynamics while time-domain simulations are often time consuming to implement. Our model captures the essential dynamics of the system without over simplification. This work incorporates various important aspects of the system and assesses the significance of including or ignoring these aspects. Many of the previous works on parametric roll make the assumption of linearized and harmonic behavior of the time-varying restoring arm or metacentric height. This assumption enables modeling the roll motion as a Mathieu equation.

It is well known that most hull forms especially container ships, Ro-Ro ships and fishing trawlers are found to be prone to parametric roll instability are asymmetric about the design water line. Hence the variation in the metacentric height will be asymmetric as well. This asymmetry invalidates the harmonic approximation. Studies by other researchers (ABS, 2004; Spyrou, 2000) have shown that the harmonic assumption is very crude.

Many of the past research on ship parametric roll have been to predict the occurrence of parametric roll. Fewer analytical methods have been developed to predict the resulting roll amplitude. Some studies were done by (Bulian, 2003). In his study a harmonic form was assumed for the response with a slowly varying amplitude and phase. However this required a complicated calculation and statistical linearization. Due to the large amplitude of motion resulting from the parametric instability the effects of non-linear damping also become important. Non-linear damping controls the bounded roll motion amplitude. So far there have been very few attempts to incorporate the effects of non-linear damping into analytical model to predict roll motion amplitude. Many researchers have attempted to evaluate the effects of non-linear damping using time simulations which is very time consuming and does not help in understanding the behavior of the non-linearity throughout the entire domain.

Ships typically have varying forward speeds and hence varying encounter or exciting frequency. This property of ships makes them susceptible to both sub and super harmonic parametric resonance and possible instability as compared to offshore structures. Perturbation methods and harmonic assumption greatly affect the domain under which boundaries between the stable and unstable regions are valid. Extending the model to higher harmonics will enable accurate prediction over the entire range of operation. Such simple yet more accurate models can be used as benchmarks to predict parametric instability as well as bounded roll motion amplitude which in-turn can be utilized in the preliminary design stage so as to avoid hull forms prone to parametric rolling.

BACKGROUND

Ship rolling motions is perhaps the most studied of the ship motions considering the disastrous consequences of failure. Large amplitude ship rolling motions can lead to progressive flooding and may eventually lead to the capsizing or foundering of a ship. Roll motion for ships is more complicated as compared to the other ship motions due to the presence of a non-linear restoring moment and small linear radiation damping. The presence of light damping leads to large amplitude motion when forced at the resonant frequency. As a consequence of the large amplitude roll motion the non-linear viscous damping becomes important and this further adds to the complexity of the analysis. Hence many studies have been carried to out to predict ship roll motion in regular seas. The beam sea condition is believed to produce maximum rolling and hence has been extensively analyzed, see e.g (Nayfeh, 1986). Falzarano, (1990) analyzed the complicated dynamics involved in roll motion leading to capsize using the Melnikov method. The beam seas rolling can be controlled with additional dampening such as that provided by bilge keels, roll tanks, stabilizing fins, etc. Apart from the beam sea capsizing condition, capsizing in the astern or following seas has also been analyzed (Hamamoto et al., 1996; Paulling, 1961; Umeda et al., 1995).

Parametric rolling is a form of parametric vibration due to time varying stiffness. Studies have shown that for some ships this phenomenon can lead to larger amplitude rolling motion in comparison to the beam seas condition. The change in the underwater hull form and hence the variation of the righting lever in waves leads to a time varying stiffness. If the variation in stiffness is large enough, it can result in large amplitude motion and eventual capsize. Numerical modeling of parametric rolling of ships in regular waves has been studied (Bulian et al., 2004; Munif and Umeda, 2006; Umeda et al., 2004). The Mathieu instability criterion is the most common method used to determine the onset of parametric roll. Most of the studies have been done with stability charts that do not indicate the effects of damping. Damping dramatically affects the boundaries between the stable and unstable region. Among container ships the post-Panamax container ship (C11 class) is the most studied vessel as a result of the cargo damage it suffered in 1998. The effect of parametric roll on the failure of container lashing system was studied by the SNAME ad-hoc panel #13 on Parametric Rolling (France et al., 2001). Spyrou (Spyrou et al., 2008) also studied the prediction potential of the parametric rolling for the post Pana-max container ships. This current paper discusses the methods commonly used to study parametric roll. One of the most common methods is to use simple Ince-Strutt stability diagram for Mathieu's equation in predicting the onset of parametric roll. A major drawback of the method is that the Ince-Strutt diagram for Mathieu's equation is generic and does not depend on the ship characteristics. A stability chart which depends on the ship parameters would be a more accurate approach.

Since parametric excitation can lead to large amplitude roll motion, the effects of non-linear damping cannot be neglected. Nonlinear roll damping may lead to bounded motion. Hence incorporating the effects of non-linear damping into stability charts would give a more realistic prospect of predicting roll behavior Hence without getting into complicated analyses, we can analyze the occurrence of parametric roll and also predict the roll motion amplitude using these charts at an early design stage.

PARAMETRIC ROLL EQUATION

The roll equation of motion in general for linear uncoupled motion is given by

$$\left(I + A\left(\omega_D\right)\right)\ddot{\phi} + B\left(\omega_D\right)\dot{\phi} + C\phi = M\cos\left(\omega t\right) \quad (1)$$

Where,

Φ – Roll Amplitude

I – Moment of Inertia about Roll Axis

A(ω) – Added Interia

B(ω) – Roll radiation wave damping

C – Restoring moment in roll = $\Delta \bullet GM$

M – External roll moment

ω – Forcing Frequency

For the case of head/astern sea there would be no direct roll excitation. One would expect no motion considering (1). But as discussed, (1) only represents linear damping and stiffness. This is one of the assumptions in linear strip theory where the wave profile is approximated by a flat surface at the design draft. If one considers the actual wave profile then the underwater hull form of the vessel changes as the wave passes by the vessel. This leads to a time varying restoring moment and hence a time varying stiffness.

The parametric roll equation of motion in roll considering time varying hydrostatics is given by

$$(I + A(\omega_D))\ddot{\phi} + B(\omega_D)\dot{\phi} + C(t)\phi = 0 \quad (2)$$

Where,

C(t) = $\Delta g \bullet GZ(t)$

GZ(t) – Time varying roll restoring arm

$\omega_D = \omega_n\sqrt{1-\zeta^2}$ - Damped natural frequency

ω_n -natural frequency

ζ - Damping ratio

Note that in (2) the nonlinear viscous damping is not yet explicitly considered in the roll equation of motion.

The righting arm of a vessel is generally approximated by a polynomial function of the roll angle.

$$GZ = C_1\phi + C_3\phi^3 + C_5\phi^5 ... \quad (3)$$

Here GM (metacentric height) of the vessel is given by slope of the GZ curve at origin, If we linearize and neglect higher order terms (since they are important only for large amplitudes of roll), then (2) becomes,

$$\left(I + A\left(\omega_D\right)\right)\ddot{\phi} + B\left(\omega_D\right)\dot{\phi} + \Delta GM(t)\phi = 0 \quad (4)$$

If the time varying GM is modelled as

$$GM\left(t\right) = GM_0 + \delta GM\cos\left(\omega t\right) \quad (5)$$

Where, GM_0-still water GM

Using the following transformation, Eq. (3) is converted into a non-dimensional form,

$$\tau = \omega t, \quad \omega_D = \sqrt{\frac{g\Delta GM_0}{(I + A(\omega_D))}}, \quad (\ )' = \frac{d}{d\tau}$$
$$\alpha = \left(\frac{\omega_D}{\omega}\right)^2, \quad \gamma = \frac{\delta GM}{GM_0}\alpha, \ \mu = \frac{B(\omega_D)}{(I + A(\omega_D))\omega} \quad (6)$$

$$\frac{d^2}{d\tau^2}\phi+\mu\frac{d}{d\tau}\phi+\left(\alpha+\gamma\cos\left(\tau\right)\right)\phi=0 \qquad (7)$$

With μ=0, Eq. (6) represents a typical Mathieu Type equation (undamped). The Ince-Strutt diagram/Mathieu stability charts help determine the occurrence of parametric vibration.

Hence by determining the GM variation in waves one can predict the occurrence of parametric roll using the Ince-Strutt diagram. The method for developing Mathieu Charts and effects of Damping are discussed in the next section.

MATHIEU EQUATION AND STABILITY CHARTS

Mathieu equation is extensively studied in the field of parametric vibration. Several approaches are used to develop the stability charts. Hayashi, (1960) used the perturbation method to obtain his charts. The range of validity of these charts as expected is limited. Another method is called the Hill's infinite determinant method can also be used to develop stability charts. These charts are very accurate in the region where they are defined.

The standard Mathieu Equation with damping is given by

$$x''+\mu x'+\left(\alpha+\gamma\cos\left(\tau\right)\right)x=0 \qquad (8)$$

In order to develop the Mathieu charts the solution (2π & 4π) of the equation is expressed as Fourier series,

$$x_{2\pi}(t)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos(n\tau)+b_n\sin(n\tau)\right) \qquad (9)$$

$$x_{4\pi}(t)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\tau}{2}\right)+b_n\sin\left(\frac{n\tau}{2}\right)\right) \qquad (10)$$

Substituting Eq. (8) & Eq. (9) into Eq. (7) and setting the secular terms to zero we get two matrices for each solution as given below.

Neglecting the trivial case of $a_0=a_1=b_1\ldots=0$, the determinant of the parametric matrix (matrix containing α & γ) should be zero. This gives the relationship between the parameters α and γ. The instability boundaries for various damping ratios are shown in fig 1.

$$\begin{bmatrix} \alpha & \frac{\gamma}{2} & 0 & 0 & 0 & .. & 0 \\ \gamma & \alpha-1 & \mu & \frac{\gamma}{2} & 0 & .. & 0 \\ 0 & -\mu & \alpha-1 & 0 & \frac{\gamma}{2} & .. & 0 \\ 0 & \frac{\gamma}{2} & 0 & \alpha-4 & 2\mu & \frac{\gamma}{2} & 0 \\ .. & & & & & & \end{bmatrix} \bullet \begin{bmatrix} a_0 \\ b_0 \\ a_1 \\ b_1 \\ a_2 \\ b_2 \\ . \\ . \\ . \end{bmatrix} = 0 \qquad (11)$$

$$\begin{bmatrix} \alpha-\frac{1}{4}+\frac{\gamma}{2} & \frac{\mu}{2} & \frac{\gamma}{2} & 0 & .. & 0 \\ -\frac{\mu}{2} & \alpha-\frac{1}{4}-\frac{\gamma}{2} & 0 & \frac{\gamma}{2} & .. & 0 \\ \frac{\gamma}{2} & 0 & \alpha-\frac{9}{4} & \frac{3\mu}{2} & \frac{\gamma}{2} & 0 \\ 0 & \frac{\gamma}{2} & -\frac{3\mu}{2} & \alpha-\frac{9}{4} & 0 & \frac{\gamma}{2} \\ .. & & & & & \end{bmatrix} \bullet \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \\ . \\ . \\ . \end{bmatrix} = 0 \qquad (12)$$

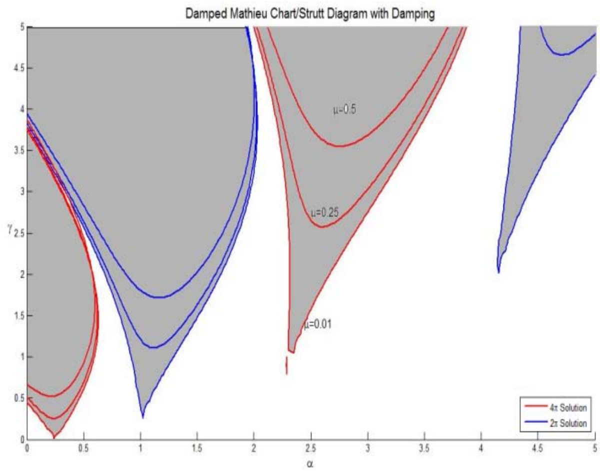

Fig1. Ince-Strutt diagram for Mathieu's Equation with constant damping. The shaded region indicates the unstable zone.

As evident from the charts, the effect of damping is to elevate the curves from the α axis, thereby reducing the unstable region. In terms of energy one can imagine damping tending to drain the energy from the excitation until the threshold energy is reached to instigate parametric vibration. Hence one method of avoiding parametric roll in ships would be to increase the damping.

The advantage of the chart above is that it can be used to study the parametric instability of any dynamical system whose equation of motion can be modeled as a Mathieu equation. This is so because the charts are not affected by the parameters of the system under study. Depending on where the (α, γ) pair falls in the chart, it becomes trivial to predict parametric instability.

If the stiffness variation is not single frequency harmonic and sinusoidal the system cannot be represented by a Mathieu equation. In such a case we can always represent the time varying coefficient (stiffness for ships) as a Fourier expansion. The resulting equation is called Hills Equation. Since the formulation of the Hills equation depends on ship parameters, these charts give a better prediction model. Our current and future work has concentrated on studying the details of Hills equation and developing the corresponding stability charts.

GM VARIATION

Ship Details

As discussed in the previous sections modern container ships seem to be more prone to parametric excitation. In order to develop realistic charts for prediction it is necessary to use a model which has parametric instability. It has been shown that post-Panamax C11 hull form exhibit parametric rolling(France et al., 2001). Here a modified C11 hull form is analyzed. The stern of the hull is modified to have fuller form, this model is named Pram aft body (MARIN Report No 17701-2-SMB, 2005). The main particulars of the vessel are shown in the Table 1.

Table 1. Main Particulars of C11 Hull Form (pram aft body)

L_{PP} (m)	262.00
B (m)	40.00
D (m)	24.45
Mean Draught (m)	11.50
Displacement (tones)	69128.00
KG (m)	18.37
GM_t (m)	1.96
Natural Roll Period ,T_Φ (sec)	25.14

The body plan of the modified C11 hull form is shown in Fig2. A 3D-wire mesh model of the vessel is shown in Fig3. The fine underwater hull form and wide flare above design draft is clearly evident from the wire mesh model. Such hull characteristics are one of the main reason for drastic variation of the submerged hull form and hence metacentric height. Hence ship stability in waves is a lot different from static stability.

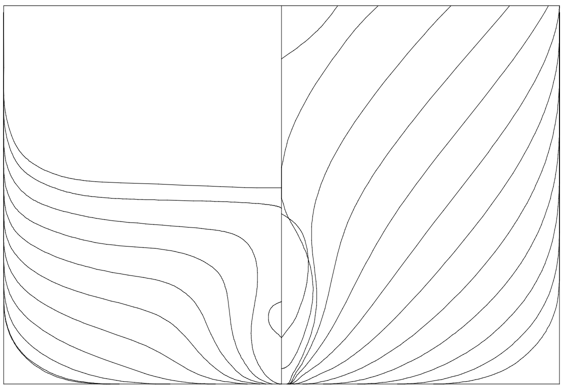

Fig 2. Body Plan of modified C11 Hull Form (not to scale)

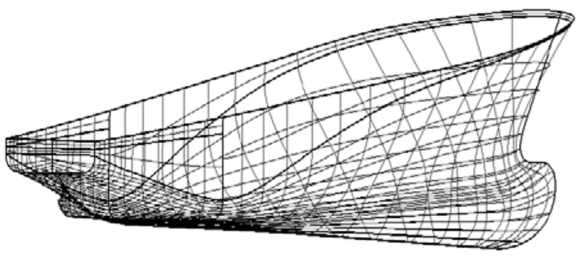

Fig3. Wire mesh model of modified C11 Hull

Fig4 shows the variation of the submerged hull with wave crest at midship and wave trough at midship.

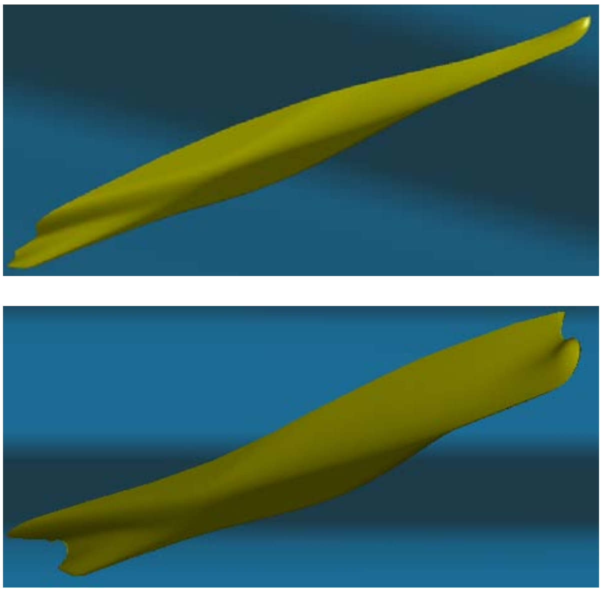

Fig4. Change in Underwater Hull form in waves of modified C11 Hull form. Top -Wave Crest Midship, Bottom -Wave Trough Midship. Wave Length=Ship LPP

GM in regular waves

In order to estimate the GM variation in regular waves, the roll restoring curve (GZ) for 10 different wave crest positions along the ship are calculated. The slope of the GZ curve at origin gives the GM. Standard hydrostatic software is used to obtain the GM for different wave crest position. Calculations are done for zero forward speed and free trim condition (hydrostatic balance). The details of the regular wave used for estimation is given below,

Wavelength $\lambda = L_{PP} = 262$m

Wave Number $= k = \dfrac{2\pi}{\lambda} = 0.024$

For deep water the wave frequency is given by

$\omega^2 = gk$, $\omega = 0.485$ rad/s

The ship's natural frequency of roll is given by

$\omega_n = \dfrac{2\pi}{T_\phi} = 0.25$ rad/sec

The damping ratio $\zeta = \dfrac{B(\omega_n)}{2(I + A(\omega_n))\omega_n}$ ~0.003

Hence $\omega_D \sim \omega_n$

Hence the parameter $\alpha = \left(\dfrac{\omega_n}{\omega}\right)^2 = 0.265$

Looking at the Mathieu Chart (Fig.1) this value is very close to principal parametric resonance zone ($\alpha = 0.25$). The value of metacentric height (GM) for the wave crest at different position along the ship length is shown in Fig. 5.

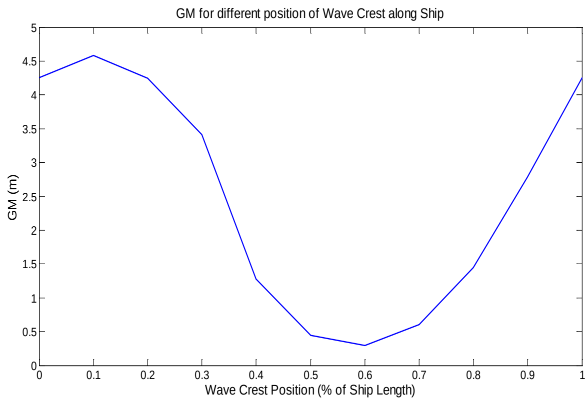

Fig5. GM for different position of wave crest

A wave height equal to 1/40 of wave length is used to estimate *GM*, $H_W = 6.55$m. The non-linear coupling effects of pitch and heave on the hydrostatics of the vessel is neglected.

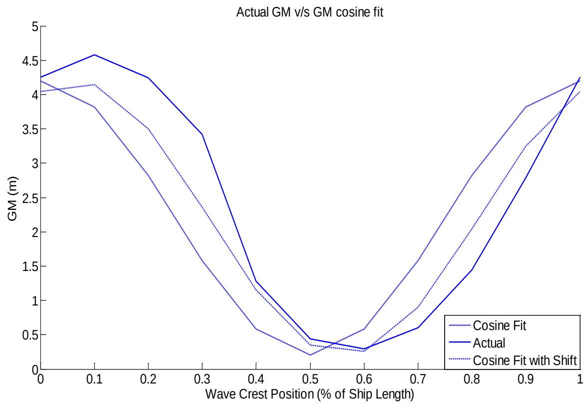

Fig6. Comparison of Cosine Fit of GM with Actual GM. (-- Cosine Fit and -.- Cosine Fit with Shift of π/8)

Fig. 5 depicts a form for the GM variation and hence can be approximated into Mathieu's equation. The comparison between the Mathieu fit and actual GM is shown in Fig. 6. As shown by (Spyrou et al., 2008) a case of cosine fit of GM with a phase shift (Fig6) has a better fit. The phase shift used here is π/8. The poor fit of the Mathieu approximation (even phase shifted) to the actual GM variation is clearly evident. Hence, there is a need to use a method with which we can approximate the GM variation more accurately. The Hill's equation and the corresponding stability charts could be a solution to this problem.

CONCLUSIONS

The abrupt changes in the underwater hull of the vessel are one of the primary reasons for the drastic change in stiffness of the vessel. The analysis carried out in the paper clearly exhibits the usefulness of simple Ince-Strutt diagrams or instability chart in predicting parametric roll of ships. The chart also demonstrates the implicit dependence of the phenomenon on damping.

The ability of the charts to predict the bounded roll motion amplitude is perhaps a feature so far not discussed. The effects of non-linear damping (which is important due to large amplitude motion due to parametric roll) which is to bound the motion can be explained using these stability charts. Being able to estimate the bounded roll motion amplitude can be very helpful in the initial design stage to study the implications of parametric roll on the stability of the vessel.

The Hill equation tend to consider the time varying stiffness better in comparison to a Mathieu and hence the use of a stability diagram for Hill's equations would give a much more accurate prediction of the occurrence of parametric roll especially in higher instability zones. The charts can also be used to calculate the critical frequency and the threshold wavelength which would initiate large amplitude rolling motion.

The parametric stability of the vessel for different forward speeds can also be predicted using these charts. The charts also enable the study of parametric stabilization. For example by merely increasing or decreasing the speed of the vessel we might be able to avoid parametric roll or worsen the situation by moving into a more unstable region. These instability charts can act as a guide for crew onboard a ship experiencing large amplitude motion in head/following sea in deciding whether to increase or decrease the vessel speed and to what extent.

Hence apart from serving the purpose of a simple and practical tool for parametric roll study during the initial design stage the Mathieu or Hill stability charts can also be helpful during the operation of the vessel in a seaway.

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