Uncertainty Assessment in Experiments on a Floating Body in Forced Roll Motion in Calm Water

Jakub Cichowicz, Andrzej Jasionowski and Dracos Vassalos

The Ship Stability Research Centre, Department of Naval Architecture and Marine Engineering, University of Strathclyde, Glasgow

Accurate prediction of a ship response in roll motion is one of the fundamental problems in fluid dynamics. It has inspired numerous research activities, many of them involving physical tests. The tank experiments, although seemingly the most convenient for handling such complex problems, have proven to be very challenging, both technically and conceptually. This is due to the very convoluted nature of roll motion hydrodynamics as well as the small magnitude of dissipation forces compared to inertia and hydrostatic forces. When discussing the impact of roll damping prediction on damage ship stability and survivability, before migrating from simplified theories or semi-empirical models towards more complicated and time-demanding tools, the question whether accuracy of available alternatives is sufficient to justify such transition must first be addressed. Similarly, when experimental data is provided for validating CFD codes, it is sensible to do so only if the uncertainty limits of the former are evaluated. This makes uncertainty assessment a central problem in the pursuit for high-accuracy prediction of roll damping characteristics. It may also reveal the potential limit of applicability of the adopted approach.

This paper presents uncertainty assessment of forced roll measurements performed on a floating body in calm water and discusses the main sources of errors and impact on the final prediction.

Introduction

An accurate prediction of hydrodynamic forces in roll motion is a problem of central importance in studies on ship stability and as such it has attracted numerous research studies addressing the issue both analytically and experimentally with works of Frank (Frank, 1967) and Vugts (Vugts, 1968) being among the finest examples. From the very beginning it has been clear, however, that methodologies based on linear theory and potential flow (inviscid fluid) is practically limited to small amplitude motions and does not represent well the dynamics of a dissipative system oscillating with finite amplitude. The semi-empirical method of Himeno and Ikeda (Himeno, 1981) employed for correcting damping coefficient has proven to be very useful but its applicability is limited to “standard” shapes and it generally suffers from drawbacks of regression-based techniques. On the other hand a more sophisticated approach, based on RANSE (CFD) codes, might help understanding the problem better but in order to provide high quality data CFD findings have to be verified with experimental results – and here is where the real problem seems to start because hydrodynamic reaction in roll is small, much smaller than the dominant inertia and restoring moments. Indeed it can be readily shown that damping moment has a magnitude comparable to uncertainty in restoring and inertia moments (for the tested body the maximum ratio of damping moment to total inertia, external excitation and restoring moment does not exceed 6%, 12% and 0.5%, respectively). Furthermore, a ship in roll motion is a non-conservative system and although dissipation forces (in calm water) can be generally decomposed into wave radiation (potential damping), friction and vortex shedding (viscous effects), it is very difficult to measure (or assess) accurately individual components, with the last two being strongly dependent on motion amplitude and all three on frequency. Going down this route, one can quickly realise that a seemingly simple problem becomes a real "beast" with more and more difficulties appearing whenever a new state variable is added to the equation (to mention only wave diffraction of a ship rolling in waves, Pawlowski, 1999). Finally, there is also the question of the experimental technique adopted, with measurements in-waves being one of the least controllable experimental environments, practically leaving room for calm water measurements only. These are traditionally being performed as either forced oscillations about fixed axis of rotation or roll decay tests. In case of the former, the experimental setup involves constrained motions, which deviate from realistic conditions whereas applicability of the latter is limited to single frequency estimates, which is of little, if any, use for numerical tools.

Measurements performed on a freely floating body forced to roll by an internal device seem to be an attractive alternative, retaining the virtues of calm-water techniques. Furthermore, the fact that model motions can be unconstrained and the forcing moment controllable, allow for investigating roll dynamics of a ship in damaged condition in an experimental environment accounting for transient phenomena.

Understandably, the technique adopted has certain limitations and it is the intention in this paper to discuss some of the key issues related to uncertainty assessment associated with the method and, to some extent, with measurements of hydrodynamic reaction in general. Detailed discussion is confined here to uncertainty of the mathematical model, which allows for comparison of experimental data with analytical and other experimental predictions, and on phase lag assessment – a factor thought to have the highest impact on the quality of the measured data. Some sources of errors are discussed briefly-mainly to demonstrate expected sensitivities of the results.

Experiment Set-up

Experiments discussed in this paper have been carried out at the *Kelvin Hydrodynamics Laboratory*, a testing facility of the University of Strathclyde (UoS) in autumn 2009. These experiments are part of research activities aiming to address the hydrodynamic properties of ships in damaged condition and the results presented here are meant to validate the technique adopted.

Figure 1 *Cylindrical section of a RO-PAX vessel subjected to forced roll in calm water.*

The tested model is a 1:40 scale cylindrical section of a RO-PAX vessel of length 60 m, draught 6.287 m, beam 27.8 m and vertical position of centre of gravity (KG) equal 8.337 m (GM = 5.509 m). An internal forcing device consists of set of coupled gyros of maximum 7000 rpm spin velocities and precession rates[1] limited to 1.7 Hz. Gyros are supported by common frame, pivoted alongside of the model centre-plane and with rotation about pivoting axis constrained by the strain gauge load cell measuring force component of the generated moment.

Motions of the body are recorded with use of optical motion capture system based on high-speed infrared cameras and set of reflective (passive) markers fitted to the body. Additionally there are two devices – single axis accelerometer and solid-state gyro for reference measurements of the phase lag of response.

Uncertainty Assessment

Uncertainties associated with the mathematical model

For the purpose of this paper it is assumed that motions of a body rolling in calm water can be described by a set of linear ordinary differential equations. In such model, hydrodynamic reaction can be conveniently expressed by means of orthogonal components

[1] This is equal to maximum roll frequency of the body

given as added inertia and damping coefficients. It should be noted here that the assumption of orthogonality of the hydrodynamic moment components holds only for purely harmonic excitation - this assumption shall be discussed further in the following. Furthermore, it is assumed that the flow around the body can be considered two-dimensional and hence the problem can be reduced to vertical motions only, involving three degrees of freedom (DOF): sway (Y), heave (Z) and roll (φ). In order to describe motions of the body in space two right-handed coordinate systems are employed. The global, fixed in space, reference system is described by a set of three orthogonal axes $OXYZ$. The second coordinate system is body-fixed, with origin at the intersection of the body centre-plane and midship-section (o). The axes are denoted by x, y and z with axes ox and oz at the centre-plane, the latter positive upward. Sway and heave motions are understood as rectilinear displacements of the origin o along the global axes OY and OZ respectively, and roll as rotation about the body-fixed axis ox. If the centre of gravity (G) coincides with the origin of body-fixed system the equations of motion will take the following form

$$\begin{bmatrix} m+a_{yy} & a_{yz} & a_{y\varphi} \\ a_{zy} & m+a_{zz} & a_{z\varphi} \\ a_{\varphi y} & a_{\varphi z} & I_\varphi + a_{\varphi\varphi} \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \end{bmatrix} + \\ +\begin{bmatrix} b_{yy} & b_{yz} & b_{y\varphi} \\ b_{zy} & b_{zz} & b_{z\varphi} \\ b_{\varphi y} & b_{\varphi z} & b_{\varphi\varphi} \end{bmatrix}\begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\varphi} \end{bmatrix} + \\ +\begin{bmatrix} c_{yy} & c_{yz} & c_{y\varphi} \\ c_{zy} & c_{zz} & c_{z\varphi} \\ c_{\varphi y} & c_{\varphi z} & c_{\varphi\varphi} \end{bmatrix}\begin{bmatrix} y \\ z \\ \varphi \end{bmatrix} = \begin{bmatrix} F_y \\ F_z \\ M_\varphi \end{bmatrix} \tag{1}$$

Where

m – mass of the body

I_φ – mass moment of inertia

a_{ii} – added mass or added moment of inertia coefficient in i-mode of motion

a_{ij} – added mass coupling coefficient of j- into i-mode of motion

b_{ii} – damping coefficient in i- mode of motion

b_{ij} – damping coupling coefficient of j- into i-mode of motion

c_{ii} – hydrostatic restoring coefficient in i-mode of motion

c_{ij} – hydrostatic restoring coupling coefficient of j- into i-mode of motion

F_y, F_z, M_φ – external forces/moment in OY, OZ and (about) ox axes, respectively

As the heave motion is symmetrical with respect to OZ axis, it alone cannot induce any lateral or angular motions and hence $a_{iz}=b_{iz}=c_{iz}=0$. Furthermore, $c_{iy}= c_{yj} =0$. Moreover, as the internal roll motion generator produces a pure moment, force components F_y and F_z are both equal to zero and the simplified equation (1) can be expressed in scalar form as follows:

$$\left(m+a_{yy}\right)\ddot{y} + a_{y\varphi}\ddot{\varphi} + b_{yy}\dot{y} + b_{y\varphi}\dot{\varphi} = 0$$

$$a_{zy}\ddot{y} + \left(m+a_{zz}\right)\ddot{z} + a_{z\varphi}\ddot{\varphi} + b_{zy}\dot{y} + \\ + b_{zz}\dot{z} + b_{z\varphi}\dot{\varphi} + c_{zz}z + c_{z\varphi}\varphi = 0 \tag{2}$$

$$a_{\varphi y}\ddot{y} + \left(I_\varphi + a_{\varphi\varphi}\right)\ddot{\varphi} + b_{\varphi y}\dot{y} + b_{\varphi\varphi}\dot{\varphi} + \\ + c_{\varphi\varphi}\varphi = M_\varphi$$

It can be readily seen from equation (2) that with focus on roll motion and with neither sway nor roll being coupled with heave, the heave equation can be excluded. Furthermore, although external forces in horizontal and vertical directions have been dropped from the equation, hydrodynamic coefficients remain, which is the consequence of the vertical position of axis of rotation (centre of rotation) expected to lie at some point between the origin o and the centre of gravity. In fact, the centre of rotation of a freely floating body, by analogy to the coupled-mass system, will lie at the centre of coupled mass of the rigid body and the accompanying fluid (Balcer, 2004).

So far it has been assumed that the body oscillates about the axis passing through the origin o and in such case the rolling body will become a single DOF system due to vanishing of the sway component ($y=0$). In the general case however, roll about an axis not passing through o would result in sway motion given as

$$y = h \cdot \sin(\varphi) \tag{3}$$

Where, h is the vertical distance between the centre of rotation and origin o of the body fixed system (in upright position). On the other hand, in case of a floating body, the centre of rotation will not be fixed in space as due to instantaneous changes in submerged volume it will be subjected to vertical oscillations (roll-induced heave). This is a consequence of finite-amplitude angular displacement and would not be present if roll amplitude was infinitesimal or the body a circular cylinder. However, as figure (2) shows, for small and moderate roll angles roll-induced heave amplitude is expected to be small, constituting some 6% of $\overline{oG}$ at 20 deg roll angle. Bearing this is mind, for the purpose of the initial uncertainty assessment it is assumed that vertical oscillations of the instantaneous axis of rotation can be neglected and therefore roll-induced sway motion can be expressed by means of equation (3).

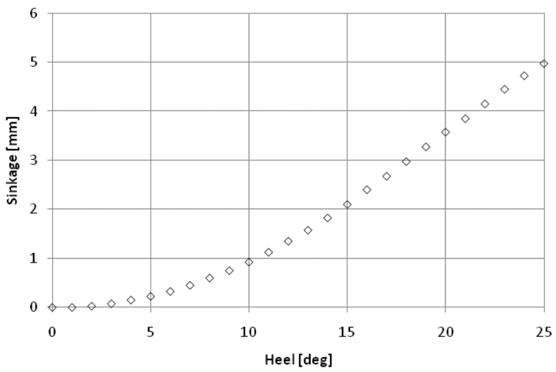

Figure 2 *Sinkage due to heel of the freely floating cylinder tested at UoS (model scale).*

As equation (3) indicates, roll-induced sway is implicit function of time thus its time derivatives can be expressed as follows

$$\begin{aligned} \dot{y} &= h\dot{\varphi}\cos\varphi \\ \ddot{y} &= h(\ddot{\varphi}\cos\varphi - \dot{\varphi}\sin\varphi) \end{aligned} \tag{4}$$

Assuming the external moment to be of the form: $M_\varphi = M_A \sin(\omega t - \varepsilon)$, where M_A, ω and ε stand for moment amplitude, circular frequency and phase lag respectively, the angular displacement and its time derivatives are given as

$$\begin{aligned} \varphi &= \varphi_A \sin(\omega t) \\ \dot{\varphi} &= \varphi_A \omega \cos(\omega t) \\ \ddot{\varphi} &= -\varphi_A \omega^2 \sin(\omega t) \end{aligned} \tag{5}$$

Where, φ_A stands for roll amplitude.

Making use of orthogonality of roll motion and its derivatives, equations (4) and (5) can be substituted into sway formulae (2), which after simple manipulation yields

$$\begin{aligned} a_{y\varphi} &= -(m + a_{yy})h\cos\varphi_A \\ b_{y\varphi} &= -h \cdot b_{yy} \end{aligned} \tag{6}$$

These equations express the relationship between coupling coefficients of roll into sway, sway coefficient and distance from centre of rotation to the origin o of body-fixed coordinate system. They can be combined with roll equation and after some rearrangement, roll added inertia and damping coefficients for oscillations about the natural axis of rotation can be expressed as follows[2]

$$\begin{aligned} a_{\varphi\varphi} &= \frac{c_{\varphi\varphi}}{\omega^2} - \frac{M_A \cos\varepsilon}{\varphi_A \omega^2} - I_{\varphi_o} + \\ &+ h^2\left(m\left(1 - \cos^2\varphi_A\right) + a_{yy}\cos^2\varphi_A\right) \\ b_{\varphi\varphi} &= -\frac{M_A \sin\varepsilon}{\varphi_A \omega} + h^2 \cdot b_{yy} \end{aligned} \tag{7}$$

Where I_{φ_o} is mass moment of inertia of the body about the axis passing through the origin o.

The above equations form the basis for uncertainty assessment and sensitivity analysis. In the case of the experimental technique being discussed, sway coefficients are not assessed experimentally, so they can either be ignored or assessed by means of theoretical prediction.

[2] These equations have been derived following an assumption that coupling coefficients are symmetrical, i.e. relations $a_{\varphi y} = a_{y\varphi}$ and $b_{\varphi y} = b_{y\varphi}$ hold.

It is understood that if the body’s centre of gravity lies close to the waterplane the last terms of equation (7) can be neglected (with h^2 being second-order) but in the general case their contribution to the results is expected to be significant. For the purpose of the analysis presented here theoretical predictions of the sway added inertia and damping coefficients were taken into account. On the upside it can be said that, as previous experimental works indicate, predictions of added mass and damping coefficients in sway demonstrate good conformity with physical model tests.

Given that variables present in equation (7) are not correlated (i.e. there are no underlying functional relationships between measured variables), the systematic (bias) part of uncertainty can be expressed in a form based on second-order total differential (Coleman and Steele, 1999):

$$u_S(f)^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} u_S(x_i) \right)^2 \qquad (8)$$

Where, f is a functional relation between measured variables x_i , $u_S(.)$ denote systematic errors in derived quantities and measured variables; *n* stands for number of variables. Partial derivatives in the above formula are referred to as sensitivity coefficients. In principle, function *f* should be decomposed to the level of directly measured variables, i.e. mass, distance, force, motions and time as the remaining quantities are derived from them, e.g.:

1. Hull mass, moment of inertia in air and vertical coordinate of centre of gravity are measured prior to testing and the two quantities - force due to generated moment and motions of the body – are directly measured during the experiments.

2. Circular frequency and phase lag of the response are derived from the time history of force and motion recordings and, as such, they are subjected to uncertainties associated with the former. Similarly, moment of inertia in air will be affected by uncertainties in measured mass and period of oscillations in air.

For the purpose of illustrating the process of uncertainty assessment, only errors in moment, force and response lag will be discussed in detail in the following paragraphs whereas for other terms, only sensitivity of the results will be briefly presented, based on estimates.

The following figures show comparison of experimental data with potential flow predictions obtained for the actual body shape and a rectangular cylinder of B/T ratio equal 4. Error bars correspond to the systematic part of uncertainties estimated for all variables present in the LHS of equation (7).

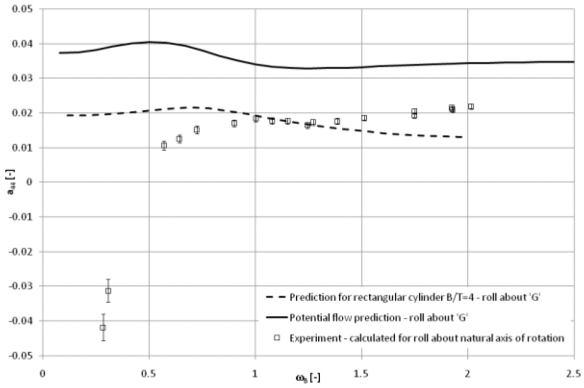

Figure 3 *Roll added moment of inertia coefficient – comparison with potential-flow prediction for the actual body shape (solid) and rectangular cylinder of B/T=4 (dashed).*

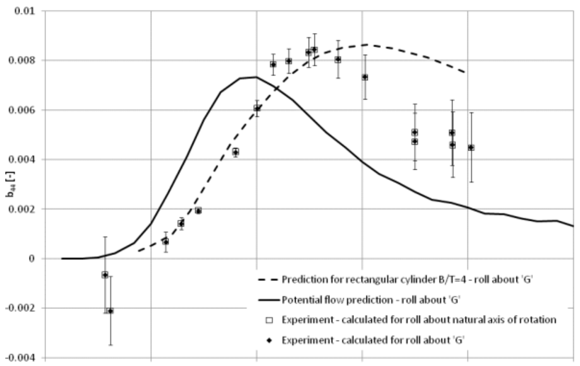

Figure 4 *Roll damping coefficient – comparison with potential-flow prediction for the actual body shape (solid) and rectangular cylinder of B/T=4 (dashed).*

As can be seen from figures (3) and (4), experimental data follow an obvious trend but special attention should be paid to the two points corresponding to the lowest frequencies of oscillation, at which both added inertia and damping coefficients take negative values. Undoubtedly, these results are wrong but serve as good indication of difficulties in measurements at low frequencies where damping is small and phase angle approaches zero asymptotically. This behaviour is also confirmed by large systematic uncertainties. It is also worth noting that damping prediction suffers from large biases not only at low- but also at high-frequency oscillations, caused by phase lag approaching -180 degrees[3].

In order to present contributions of the individual components to the total bias, the following ratio is used

$$\frac{\left(\dfrac{\partial f}{\partial x_i} u_S(x_i)\right)^2}{u_S(f)^2} = \frac{\left(\dfrac{\partial f}{\partial x_i} u_S(x_i)\right)^2}{\sum_{i=1}^{n}\left(\dfrac{\partial f}{\partial x_i} u_S(x_i)\right)^2} \qquad (9)$$

Individual contributions, expressed in terms of percentage, are presented in the following figures.

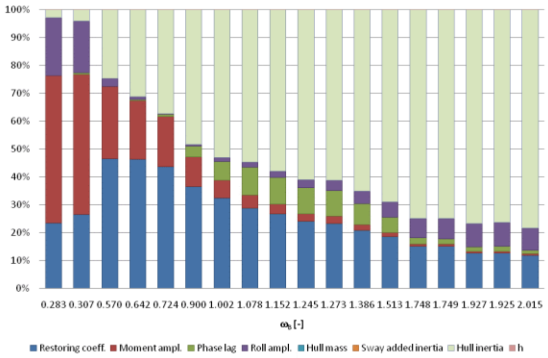

Figure 5 *Contributions of individual components in total bias error estimated for roll added inertia coefficient.*

In case of added inertia coefficient it can be seen that there are five significant contributors: errors in amplitudes of external moment and response, restoring coefficient, hull inertia and response phase lag (in the middle- and high-frequency ranges). Comparison of this characteristic with that given by figure (3) may suggest that bias in moment amplitude causes the lowest frequency values to suffer larger uncertainties than the remaining values.

[3] According to the sign convention adopted phase angles are negative.

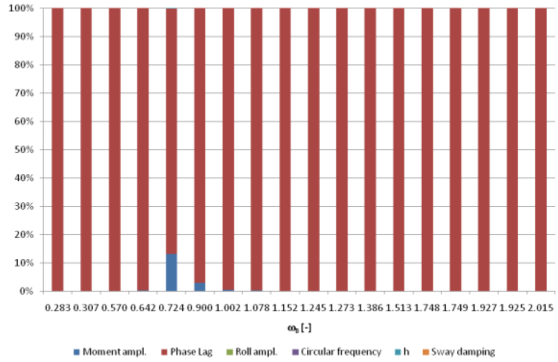

Figure 6 *Contributions of individual components in total bias error estimated for roll damping coefficient.*

In case of damping coefficient it is clear that, practically, the sole contributor to the total bias is response phase angle with the exception at mid-range frequencies where there is some bias associated with moment measurements. In any case, systematic errors in roll and moment amplitudes as well as phase lag estimation are dominant. This being the case, the last two will be discussed in some detail. Before this, it might be useful to have a closer look at equations describing the rolling moment produced by the gyroscopic generator in order to justify the aforementioned assumption of orthogonality of hydrodynamic moment components.

Uncertainties associated with moment generation and measurement

In the most general form the equation of motion of the single gyro fitted to the hull is given as

$$\begin{aligned}&\ddot{\varphi}\left(J_z\cos^2\theta_G + J_x\sin^2\theta_G + J_Z{}^o\right) + \\ &-\dot{\theta} h_G\cos(\theta_G) + \\ &+2(J_x - J_z)\dot{\varphi}\dot{\theta}_G\sin\theta_G\cos\theta_G = \\ &= M_c\end{aligned} \qquad (10)$$

Where, φ is the roll angle, θ_G gyro precession angle, J_x, J_z gyro moments of inertia with respect to local coordinate system, $J_Z{}^o$ is the system inertia with respect to roll axis and M_c is external moment about roll axis.

Without going into detail - this can be found in (Cichowicz, Vassalos, & Jasionowski, 2009) - it can be assumed that the moment M_c taken with minus sign can represent damping and restoring components (i.e.: $M_c = -b_{\varphi\varphi}\dot{\varphi} - c_{\varphi\varphi}\varphi$) and therefore the equation (10) can be rewritten as

$$
\begin{aligned}
&\ddot{\varphi}\left(J_z \cos^2\theta_G + J_x \sin^2\theta_G + J_Z{}^o\right) + \\
&+ 2\left(J_x - J_z\right)\dot{\varphi}\dot{\theta}_G \sin\theta_G \cos\theta_G + \\
&+ b_{\varphi\varphi}\dot{\varphi} + c_{\varphi\varphi}\varphi = \\
&= \dot{\theta} h_G \cos(\theta_G)
\end{aligned}
\tag{11}
$$

The above equation shows clearly that the motions produced by the gyroscopic roll generator are not, in the general case, purely harmonic as there are quadratic and double-frequency terms present. However, it is so unless moments of inertia of the gyro and its gimbal (J_x and J_z) about local axes are equal, in case of which, the quadratic and double-frequency components vanish:

$$
\begin{aligned}
&\ddot{\varphi}\left(J_G + J_Z{}^o\right) + b_{\varphi\varphi}\dot{\varphi} + c_{\varphi\varphi}\varphi = \\
&= \dot{\theta} h_G \cos(\theta_G)
\end{aligned}
\tag{12}
$$

Where, J_G is a substitute gyro moment of inertia following that $J_z \cong J_x \cong J_G$

Assessment of the gyro inertia properties has shown that indeed, differences in the moments are small and can be ignored – this can also be verified by observing the response power spectrum in figure 7, where there is no double-frequency component present[4]

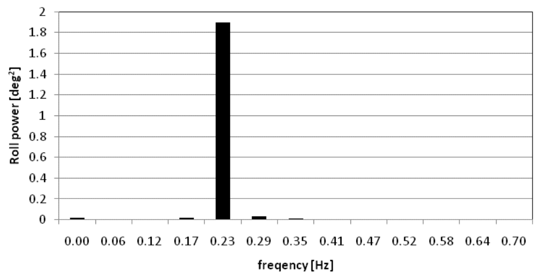

Figure 7 *Example of roll power spectrum for low frequency case (ω=1.5 rad/s).*

Moment (force) measurement is thought to be very reliable, mainly due to use of simple strain gauge and pivoting gyro frame to eliminate (or minimize) impact of lateral inertia forces that might cause bending of the transducer. The load cell itself has low inertia and therefore short response time as well as linear characteristics with very low hysteresis in a broad range of loads. All this is particularly important for accurate prediction of the body response lag as discussed in the next section.

Uncertainties associated with response lag estimation

It is a well known fact that accurate phase lag estimation is the most difficult task in measurements to determine hydrodynamic reaction. In the case of oscillations about the natural axis, the body motions can be recorded only by using non-contact techniques, based either on measurement of acceleration (velocity) components performed using an internal device or by an optical motion capture system. The former method can be referred to as "classical" approach and in principle such devices are widely used even as independent wireless units (La Gala, Gammaldi, 2009). In case of electromechanical devices, however, their response characteristics (i.e. internal damping and inertia) may be a serious issue and accurate dynamic calibration can be very difficult. Additionally, multi-mode (multi-axis) devices may suffer inaccuracies due to cross-coupled response. Optical motion capture systems are affected neither by mechanical factors like inertia nor by cross-coupling but, as it has been found in the course of experiments carried out at UoS, their real-time output may suffer time shift due to data processing. As a matter of fact the time shift itself would not be considered as a major issue but the real problem lies in its randomness, which makes correcting for the time lag practically impossible.

Electronic (solid state) devices, in turn, are compact, easy to use and calibrate but their accuracy might be questionable and hence readings should be approached cautiously.

During the present measurements the response phase angle has been estimated on the basis of three devices: single axis accelerometer, single axis solid-state (electronic) gyro and optical motion capture system. The last device proved to be unreliable for the aforementioned reasons whereas the first two devices have performed much better. However, as the comparison of

[4] This also implies that the following relation holds: $J_z \cos^2\theta_G + J_x \sin^2\theta_G \cong J_G$.

damping prediction shows, there are significant discrepancies at super-critical frequencies (see figure below), for which a consistent explanation has not been found yet.

Comparison of the results shows clearly that low frequency predictions match well but high-frequency characteristics are completely different. In principle, it would be expected that mechanical devices performed worse at higher frequencies but accelerometer based characteristics follow the theoretical prediction better. Bearing this in mind it should be said that the problem has to be investigated in detail as to avoid speculative and somehow counter-intuitive judgment. For the time being it is assumed that accelerometer readings should be used as a basis for further analysis until the question of solid-state gyro accuracy is resolved.

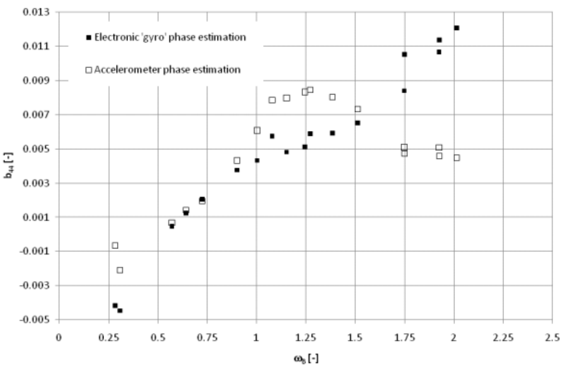

Figure 8 *Comparison of damping prediction based on solid-state gyro and single-axis accelerometer-based phase prediction.*

Independently on the measurement method, there is also a question of assessing phases of the time histories of excitation moment and roll motion. Estimates presented in this paper are based on least-squares fit to the steady-state part of the raw data with standard deviations from the analysis constituting the basis for the bias error. Such approach is relatively easy to apply and provides instant information on errors but it is semi-manual and might be considered as not particularly systematic. Spectral techniques may be better alternative but there is some concern associated with them and related to the resolution of harmonic decomposition, particularly for low frequency oscillations and large phase velocity of the radiated waves, which if not damped sufficiently may have difficult to assess impact on the results[5].

Regarding the formal uncertainty assessment, there are certain points requiring attention. Firstly, there is strong dependency of phase angle estimate on circular frequency. This dependency is not clearly exposed in the state equation but consequences of its propagation into the results are apparent. In short, although expected variations in frequency estimates across the measured variables are small, even these negligible discrepancies introduce significant variations in the phase angle estimates. For this reason, in the least-squares fit, frequency is estimated on the basis of force recordings (considered to be most reliable) and passed as a constrained parameter to the estimates of the remaining signals. Although such approach is formally correct it is thought that harmonic analysis might be more suitable as it automatically averages frequency and, which is perhaps even more important, it makes phase angle formulae easy to process for purpose of uncertainty assessment.

From the formal point of view, uncertainties associated with curve fitting (or harmonic decomposition) are considered to be systematic errors. However, unlike "ordinary" biases, curve fitting errors can be reduced by increasing the length of the sample and in this respect they behave more like precision errors. This is very important as low and high frequency errors in damping could be further reduced by means of detailed investigation of all the motion components and more systematic selection of the data sample, i.e. by making sure that within the selected time the model was not subjected to transient motions, e.g. yaw caused by imperfections in gyros' ramp velocity characteristics.

[5] In simpler terms there is certain concern that the radiated wave can be reflected from the wavemaker and add energy to the system. This, given the relatively high phase velocity of the low-frequency radiated waves, combined with the long transient period of the forced motions in consideration, constitutes a serious problem and may serve as an explanation for the large uncertainty in both added inertia and damping at very low frequencies, as well as for their negative magnitude.

Additional considerations

Although the results do not exhibit sensitivity with respect to sway there is some indication that its contribution might be somehow underestimated. The figure below shows the vertical distance from origin of body-fixed coordinate system *o* to the predicted mean position of the natural axis of rotation (parameter *h*) as derived on the basis of equation (3).

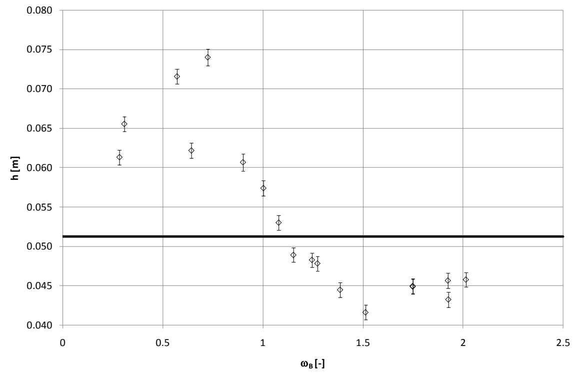

Figure 9 *Predicted mean vertical position of the instantaneous axis of rotation. The solid line corresponds to the vertical position of the centre of gravity (oG).*

As the figure indicates the predicted value of *h* varies with frequency, which can be explained by analogy to coupled-mass system with the axis of rotation passing through the centre of mass of the system. However, for lower frequencies the estimated *h* is larger than *oG*, which suggests negative added inertia. As by default added mass and inertia of mono-hulls must be non-negative the only reasonable explanation is, assuming correctly estimated centre of gravity, that the body "slides" due to asymmetric pressure distribution leading to sway amplitudes larger than the expected maximum, i.e. $(y_A)_{\max} = oG \cdot \sin(\varphi_A)$. Should this proved to be the case a mathematical model might have been revised to accommodate for such behaviour.

Conclusions

The results presented in this paper demonstrate the outcome of the preliminary stage of uncertainty assessment and they clearly do not provide answers to many important questions. Nevertheless, even such rough estimates of errors allow narrowing down the broad spectrum of problems associated with the measurements and these can be addressed in detail in a more efficient way. Discussion on precision errors has been omitted entirely but, as it was shown, some of the systematic errors associated with the measurements are "precision" in their very nature with the only difference stemming from the way they are handled.

It should also be emphasised that the conclusions, as far as sway contribution is concerned, are valid for the particular case tested but their generalisation should be approached carefully. It is possible that for a more realistic, in terms of GM, example the sway damping and added mass might have more significant impact on the roll motion hydrodynamics. This is even more important as the present error estimates do not explain the divergence of the experimentally derived coefficients from the theoretical prediction for the very low frequencies and therefore prove only that such discrepancies cannot be solely justified by inaccuracy of measurements.

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