A Method to Model Large Amplitude Ship Roll Damping

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ABSTRACT

A method is proposed to model large amplitude ship roll damping, with considerations for large amplitude roll motion effects, such as bilge keel interaction with the free-surface. The method is based on consideration of distinct ship-specific physical phenomena, such as bilge keel emergence and deck submergence. Abrupt physical changes occur with these events, resulting in significant changes in the damping of the system. Without these considerations, roll motion may be under-predicted. Additional considerations for practical implementation of the proposed method are also discussed.

KEYWORDS

roll damping, bilge keels, nonlinear oscillators, piecewise methods

INTRODUCTION

Roll damping is a complex process of energy transfer which affects the amplitude of ship motion. Bilge keels have been used on ships since the late 19th century to increase damping and reduce the severity of roll motions experienced by a ship in waves (Froude, 1865; Bryan, 1900; Martin, 1958; Kato, 1965). Because ship motions are more severe and large roll angles may occur in moderate to extreme sea conditions, it is important to understand and accurately model damping for these conditions to predict ship motions (Beck & Reed, 2001). In these conditions, the effectiveness of the dominant mechanism of roll damping, bilge keels, is reduced. This paper presents a method to model large amplitude roll damping, with consideration of the effectiveness of bilge keels at large roll angles.

BACKGROUND

Ship Roll Damping

In the classical model of ship motion as a spring-mass-damper system, damping is proportional to velocity and characterizes the energy dissipation of the system. Existing theoretical models for roll motion consider physical processes related to roll motion damping, using various mechanisms of energy dissipation. These include friction, lift, wave-making, and vortex generation from the hull, as well as the vortex generation and influence of deeply submerged bilge keels (Ikeda, 1978; Schmitke, 1978; Himeno; 1981). For ship roll motion, the effects of the bilge keels account for the largest component of energy dissipation and are most effective for small and moderate roll motion at low speeds (Ikeda, 1978; Himeno, 1981). At higher speeds, lift damping becomes more dominant (Baitis, *et al.*, 1981). Although larger size bilge keels are typically more effective, some constraints, due to hull geometry and structures, limit the practical span of bilge keels.

In most modern potential flow codes, used to predict ship motion performance, roll damping is determined using the well-known Ikeda's method (Ikeda, *et al.*, 1978), or results from roll decay experiments to obtain ship-specific damping. These methods assume small amplitude roll motion, where the bilge keels are considered to be deeply submerged and smooth changes occur between the geometry of the body and the fluid domains.

Additional work has been performed to extend the application of the component based damping model. De Kat (1988) computed roll damping coefficients at the natural roll frequency and then applied these for other roll frequencies. Blok & Aalbers (1991) decomposed the roll damping due to bilge keels into two components, the lift on the bilge keel and the eddy generation from the bilge keels. Other methods have been developed where each component is determined for zero speed and then forward speed corrections are applied. Ikeda (2004) also detailed improvements to his method to determine optimal location for placement of the bilge keels. Changes have also been made to extend Ikeda's method to high-speed planing craft, with modifications to the lift component (Ikeda & Katayama, 2000), and high-speed multi-hull vessels, with modifications to the wave-making, eddy, and lift components (Katayama, *et al.*, 2008). For these high-speed vessels, predictions were performed for speeds up to *Fn*= 0.6. Additional studies have also examined some of the limitations of Ikeda's method for application to ships with buttock flow stern geometries (Kawahara, *et al.*, 2009) and large bilge keels (Bassler & Reed, 2009).

Large ship motions result in abrupt changes in the geometry of the body relative to the fluid domains, which must be considered to accurately determine the properties of the dynamical system modeling ship roll motion (Bassler & Reed, 2009; Reed, 2009). Because existing theoretical models were developed for small to moderate roll motions, the amount of energy dissipation for large amplitude roll motion may be over-estimated, resulting in under-predicted roll motion.

For large amplitude ship roll motion, the bilge keels become less effective, due to their interaction with the free surface and, for more severe motions, due to possible emergence. An example of this occurrence is shown (Fig. 1) for the ONR Topside Series tumblehome configuration (Bishop, *et al.*, 2005). In this example, the forward section of the starboard bilge keel has emerged, the midsection is shipping water, and the aft section remains submerged.

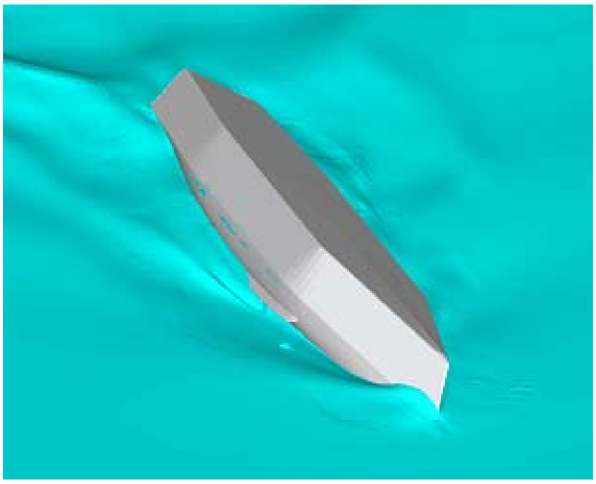

Fig. 1: DTMB Model #5613-1 at Fn = 0.30, ϕ = 30 deg. For these conditions, the bilge keel is observed to be partially emerged from the water (Miller, *et al.*, 2008).

Physical Phenomena in Large Amplitude Roll Motion

When a ship experiences large amplitude roll motion, additional physical phenomena occur which are not considered in traditional roll damping models. These include asymmetric bilge keel interaction with the free surface, where water shipping occurs for bilge keel emergence, impact loading occurs upon re-entry, and air bubble entrainment occurs under the bilge keel after re-entry.

Observations of these physical phenomena were made during a series of forced roll motion experiments, at zero forward speed, performed at DTMB (Bassler, *et al.*, 2010). Force and moment measurements on both the hull and bilge keels were obtained for a model of the midship section of the ONR Topside Series, Flared and Tumblehome configurations (DTMB Models #5699 and #5699-1). Particle Image Velocimetry (PIV) was used to measure the generated vortex-field (Fig. 2).

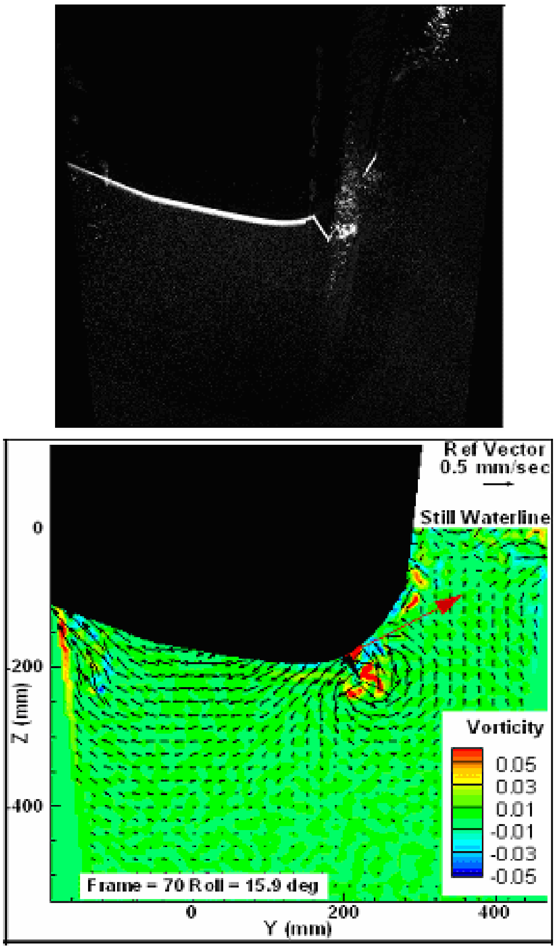

Fig. 2: Experimental measurements with DTMB Model #5699-1, ϕ_a = 45 deg, ω=2.5 rad/s. Air bubble entrainment is observed after bilge keel re-entry (top). Velocity-field measurements and bilge keel normal force measurements (red vector) are shown after bilge keel re-entry (bottom).

The individual physical phenomena that occur for large amplitude roll motion are highly nonlinear. However, the primary consideration of these events for ship roll motion prediction is their effect on the significant changes in the dynamical properties of the system. For example, it may not be necessary to explicitly model the localized nonlinear occurrence of bubble dynamics generated by the bilge keel upon re-entry after a large roll event. To enable modeling of these events in fast numerical simulation codes, simplifying assumptions must be made, attributing the effects of these nonlinearities to the non-smooth transition at the boundary of the fluid domains.

Motivation

By including the non-smooth transition, and subsequent changes in damping, which occur at large roll angles, more accurate ship roll motion predictions can be obtained. Without these considerations, the total roll damping may be over-estimated and the resulting ship roll motion may be under-predicted.

THEORETICAL APPROACH

Overview

The proposed procedure for predicting large amplitude roll damping is based on the modeling the abrupt physical changes in the dynamical system, which correspond to events such as bilge keel emergence or deck submergence (Fig 3). For large amplitude roll motion, an explicit dependence exists between roll damping and roll angle. From these events, distinct physical regions may be identified, which are dependent on the ship-specific geometry, where a significant change in damping of the system occurs.

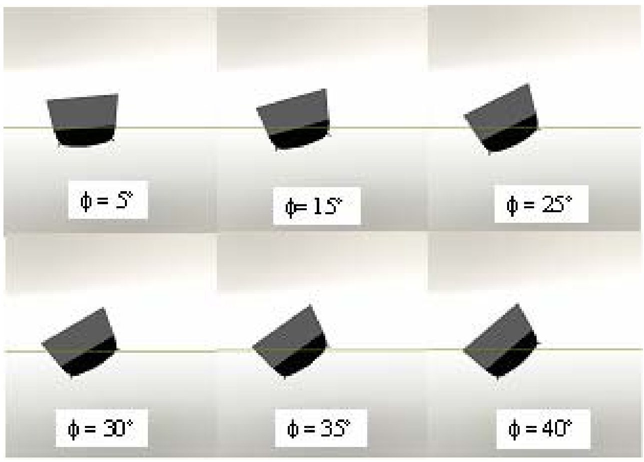

Fig. 3: Ship-specific abrupt physical changes due to variation in heel angle. For the midship section of the ONR Topside Series, flared configuration, bilge keel emergence is observed at 30 deg and deck submergence at 40 deg.

The roll angle can be used as a boundary to create a division of physical regions corresponding to abrupt physical changes associated with transition between the fluid domain boundaries. An example modeling the effect of bilge keel emergence and re-submergence on damping is considered in this paper.

Piecewise Methods

Piecewise methods are a mathematical tool that can be used to model abrupt changes in system properties. Some well known dynamical systems in mechanics with these abrupt changes are dry friction, or Coulomb damping, and clock theory (Andronov, *et al.*, 1966). Because of the abrupt physical changes, oscillator systems with this behavior must be modeled explicitly. A piecewise linear approach has also been used to model the ship motion behavior associated with changes in the GZ curve for large amplitude roll motion (Belenky, 2000; Belenky & Sevastianov, 2007).

Application to Nonlinear Damping

A piecewise method can be used to model mechanical oscillator systems with distinct physical regions, such as the interaction of the bilge keels or deck edge with the free surface. For initial consideration, single degree-of-freedom ship roll with a sinusoidal forcing function representing regular waves, is assumed. For this system a roll angle, ϕ_t, can be specified which represents a physical threshold given by the ship-specific geometry. The transition across the physical boundary for each region, from small to large amplitude damping, can be considered to occur at a discrete instant in time.

Therefore, the change in damping during this process may be modeled as a “jump” for the non-smooth transition of a component of the body, such as the bilge keel or deck edge, out of the water (or into the water). The time-scale of this transition is small compared to the time-scale of the motion of the body, such as the roll period.

Based on this formulation of the problem, a system of algebraic equations may be determined and then solved simultaneously to obtain the damping for the large amplitude region.

A METHOD FOR LARGE AMPLITUDE SHIP ROLL DAMPING

System of Equations

The single degree-of-freedom ship roll equation is

$$\ddot{\phi} + 2\delta(\phi)\dot{\phi} + c(\phi) = F(t) \tag{1}$$

where $F(t)$ is the forcing function from waves, given by

$$F(t) = \alpha \sin(\omega_e t) \tag{2}$$

where α is the excitation amplitude and ω_e is the frequency of excitation. The nonlinear stiffness, $c(\phi)$, is given by

$$c(\phi) = \omega_n{}^2 \frac{GZ(\phi)}{GM} \tag{3}$$

and the roll amplitude dependent damping, $\delta(\phi)$, is given by

$$\delta(\phi) = \begin{cases} \delta_1 & if\ \phi < |\phi_t| \\ \delta_2 & otherwise \end{cases} \tag{4}$$

where δ_1 is the damping for the small amplitude mode, below a specified physical threshold, ϕ_t, and δ_2 is the damping for the large amplitude mode. The method may be implemented to obtain equivalent linear damping coefficients for each physical region. However, the damping formulation for each region is not limited to a linear formulation (4), and may include the use of more realistic models, such as a nonlinear formulation (5),

$$\delta(\phi)\dot{\phi} = \delta_a(\phi)\dot{\phi} + \delta_b(\phi)|\dot{\phi}|\dot{\phi} + \delta_c(\phi)\dot{\phi}^3 + \ldots \tag{5}$$

where δ_a, δ_b, and δ_c are linear, quadratic, and cubic damping coefficients for that particular physical region (Dalzell, 1978; Cotton &

Spyrou, 2000; Spyrou & Thompson, 2000). These multiple sets of δ_a, δ_b, and δ_c can then be combined with the piecewise method to characterize damping for large amplitude roll.

This formulation may be further extended to model additional physical thresholds which will alter the damping characteristics of the ship in roll, such as deck edge submergence. Although nonlinear damping will most likely used in any practical method for large amplitude roll motion; as a first step, in order to examine the ability of the piecewise model to reproduce the dynamic behavior of large amplitude roll motion, only linear damping coefficients are considered in this study.

An example of single degree-of-freedom large amplitude steady-state roll oscillation, with considerations for bilge keel emergence is shown (Fig. 4). The system is characterized by a natural roll frequency, ω_n, frequency of excitation, ω_e, and amplitude of excitation, α, and amplitude of response, ϕ_a. In this example, two transition points are identified, where one bilge keel re-enters the free surface after emergence, 1, and where the opposite bilge keel emerges, 2. In this example, the damping of the system is characterized by one set of coefficients, δ_1, in the small amplitude mode, and another set of coefficients, δ_2, in the large amplitude mode.

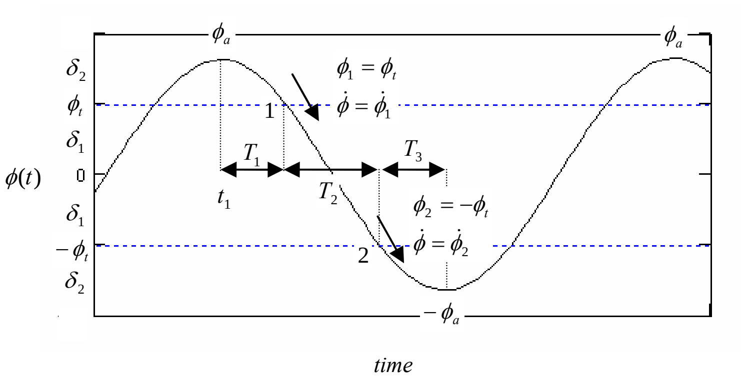

Fig. 4: 1DOF roll oscillation with amplitude, ϕ_a, for one damping physical threshold. Locations (1) and (2) indicate threshold crossings at $\pm\phi_t$ (e.g. bilge keel interaction with the free-surface).

As shown in Fig. 4, t_1 is the time of the maximum amplitude, T_1 is the time between the maximum amplitude and the first threshold crossing (e.g. when the bilge keel re-enters the water), T_2 is the time of the second threshold crossing (e.g. when the opposite bilge keel emerges) and T_3 is the time of the maximum amplitude in the opposite direction. The roll angle and roll rate at the first threshold crossing, 1, are given as ϕ_1 and $\dot{\phi}_1$, respectively. The roll angle and roll rate at the second threshold crossing, 2, are given as ϕ_2 and $\dot{\phi}_2$. Because the process is periodic, a system of equations describing the half-period behavior of the system can be determined.

Formulae (6–8) express the roll and roll rate processes as solutions to ordinary differential equations for a system with linear damping in each physical region. The system of equations can also each be represented numerically, using a Runge-Kutta solver for each formula. Numerical evaluation for each equation enables a more robust model, and the use of varied nonlinear damping formulations for each physical region.

The transition from the maximum roll amplitude to the first threshold crossing, the re-submergence of the bilge keel, is given by

$$\begin{aligned} \phi_2\left(\phi_a, \dot{\phi}_a = 0, t_1, t_1 + T_1\right) &= \phi_1 = \phi_t \\ \dot{\phi}_2\left(\phi_a, \dot{\phi}_a = 0, t_1, t_1 + T_1\right) &= \dot{\phi}_1 \end{aligned} \quad (6)$$

where ϕ_2 is the roll process using the second region damping coefficient, δ_2.

The transition from the first threshold crossing to the second crossing, the emergence of the opposite bilge keel, is given by

$$\begin{aligned} \phi_1\left(\phi_1, \dot{\phi}_1, t_1 + T_1, t_1 + T_1 + T_2\right) &= \phi_2 = \phi_t \\ \dot{\phi}_1\left(\phi_1, \dot{\phi}_1, t_1 + T_1, t_1 + T_1 + T_2\right) &= \dot{\phi}_2 \end{aligned} \quad (7)$$

where ϕ_1 is the roll process using the first region damping coefficient, δ_1.

The roll process from the second threshold crossing to the maximum amplitude at the opposite side of the roll cycle is given by

$$\begin{aligned} \phi_2\left(\phi_2, \dot{\phi}_2, t_1 + T_1 + T_2, t_1 + T_1 + T_2 + T_3\right) &= -\phi_a \\ \dot{\phi}_1\left(\phi_2, \dot{\phi}_2, t_1 + T_1 + T_2, t_1 + T_1 + T_2 + T_3\right) &= 0 \end{aligned} \quad (8)$$

In order to demonstrate the method, a solution for the amplitude (the direct problem) was obtained. Given the solution for amplitude, the system of equations was then solved for damping (the indirect problem).

Solution for Amplitude- The Direct Problem

For the direct problem, the evaluation of amplitude, in addition to the system of equations given by (6)–(7)–(8), the times corresponding to the threshold crossings, T_1, T_2, and T_3, must also be included. Because the roll process considered in the model to obtain the damping coefficients is periodic, the times can be obtained with the inclusion of the following additional equation, where T_e is the roll excitation period.

$$T_1 + T_2 + T_3 = \frac{T_e}{2} = \frac{\pi}{\omega_e} \tag{9}$$

The values for ω_n, ω_e, α, δ_1 and δ_2 are specified and the values for ϕ_1 and ϕ_2 (=$\pm\phi_t$), and $\dot{\phi}_a$ are known. The system of equations (6)–(9) is solved to obtain ϕ_a, t_1, T_1, T_2, T_3, $\dot{\phi}_1$ and $\dot{\phi}_2$.

Solution for Damping- The Indirect Problem

In this system model, the first region, or small amplitude, damping, δ_1, can be determined using Ikeda’s method or from experimental measurements, such as roll decay tests. For the piecewise linear formulation discussed in this paper, the use of the equivalent linear damping coefficient formulation enables continuity with existing methods, which have traditionally been very appropriate for their intended use— modeling small to moderate amplitude roll motions.

Large amplitude forced oscillation tests may be carried out using experiments (e.g. Bassler, et al., 2007; 2010) or high-fidelity simulations tools, such as RANS (e.g. Miller, *et al.*, 2008). In these tests, the maximum amplitude of the forced oscillation, ϕ_a, and frequency of oscillation, ω_e, are specified and the physical threshold, ϕ_t, is known from the ship-specific geometry. Because forced oscillation is used, the amplitude of wave excitation and phase become virtual quantities. Therefore, the excitation, α, the time of the maximum amplitude, t_1, and the large amplitude, or second region damping, δ_2, are unknowns and are determined from the solution to the system of equations using the indirect problem formulation.

For the indirect problem, ω_n, ω_e, δ_1, and ϕ_a, are specified and the values for ϕ_1 and ϕ_2, $\dot{\phi}_a$, $\dot{\phi}_1$, $\dot{\phi}_2$, and T_1, T_2, and T_3 are known. These values can be obtained from forced oscillation tests, using either experiments or high-fidelity simulations tools, such as RANS solvers. The system of equations is then solved to obtain α, t_1, and δ_2. The system of equations (6)–(8) is over defined, which enables robust solutions to be obtained with very approximate initial values.

ADDITIONAL CONSIDERATIONS FOR A PRACTICAL METHOD

The procedure presented in this paper to model the change in roll damping for bilge keel interaction with the free-surface in large amplitude roll motion may also be extended to include additional physical regions which may significantly affect damping based on the ship-specific hull geometry, such as deck-in-water effects (Grochowalski, 1990; Grochowalski, *et al.*, 1998).

Several additional considerations are needed in order to implement the method in time-domain numerical simulations and use the procedure for practical prediction of ship roll damping. These include multiple degree-of-freedom ship motions (such as heave and pitch), forward speed, irregular waves, and roll frequency dependence.

The use of a sectional approach in the time-domain, with the instantaneous relative position of the ship section and the free-surface from irregular waves near the ship (Fig. 5), may provide more accurate determination of when the physical threshold for a given ship section is crossed and which corresponding damping should be used. By integrating the sectional damping along the hull at each time-step, the ship-specific roll damping for large amplitude ship motions can be determined.

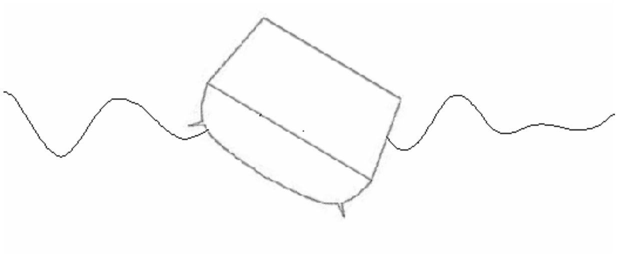

Fig. 5: Sectional view of the instantaneous relative position of the ship and irregular waves for determining roll damping at a given time-step.

SUMMARY

A method for modeling large amplitude roll damping has been presented, based on modeling the abrupt physical changes that occur with events, such as bilge keel emergence or deck submergence. When these events occur, a significant change in damping of the system occurs that can be modeled explicitly using a piecewise approach.

By considering the discrete physical events in the time-domain, which alter the damping properties of large amplitude ship roll in waves, a series of damping coefficients for these different regions can be obtained. These can then be included in a look-up table and used in sectional time-domain evaluation. An example, with considerations for bilge keel emergence, was shown using the method. Despite the formulation of the method to only consider periodic roll, similar to excitation from regular waves, the damping coefficient information can be used to predict ship motion from a stochastic excitation. However, some additional considerations must still be addressed for practical implementation.

To examine the feasibility of the proposed method for modeling large amplitude roll damping, comparisons will be made to experimental measurements (e.g. Bassler, *et al.*, 2010). The suitability of the damping formulation for each region, small and large, and the frequency dependence of large amplitude damping will also be investigated. As mentioned previously, in order to develop a practical method using this theoretical model for large amplitude ship roll damping, several additional issues must still be examined. The ability to account for additional ship motions, forward speed effects, and the local wave-field, may be possible using a sectional time-domain approach.

ACKNOWLEDGEMENTS

The authors appreciate the support of this work from Dr. Pat Purtell (ONR) and Dr. John Barkyoumb and the NSWCCD Independent Applied Research (IAR) program. The authors would also like to thank Dr. Vadim Belenky (NSWCCD) for several helpful technical discussions.

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