Climatic Spectra and Long-Term Risk Assessment

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ABSTRACT

Modern information technologies enable a radically different approach to research on the dynamical behavior of complex marine objects. In order to utilize high-performance computational architectures effectively, it is necessary to consider alternative approaches to the mathematical formulation of these problems. In particular, new statements of a problem can be formulated which were senseless earlier, but now appear effective with these computational environments. This paper considers the problem of code development, based on potential flow formulations. It is shown that a new approach for obtaining the pressure-field in time-domain simulations could be very effective for long-term risk assessment.

KEYWORDS

ARM of wind waves; long-term ship motion simulation; high-performance computer architectures.

INTRODUCTION

Long-term risk estimation for operations in various regions of the Ocean demands the reliable consideration of the behavior of objects in specific sea areas during particular seasons, or periods, of operation. An integrated approach for the description of external excitations, on the basis of spectral approximations which account for a small amount of characteristics (more often in this aspect significant wave height and less often the average period is also considered), can result in the underestimation of risk and the loss of essential features of the estimated region. It is quite clear that the wind wave spectrum, with a particular significant wave height, will vary both for geographic location (e.g Black Sea and North Sea), resulting in different responses for the same object. Even greater variability result if we consider the wave regime – storm characteristics, superposition of different wave systems, alternation of storms and quiet weather, etc. The qualitative consequences of failing to consider these characteristics are shown in Boukhanovsky et al. (2000).

However, earlier risk estimation methods, from probability theory, and forecasting of rare events were applied exclusively. Statistical data were used only to provide estimations of one or other likelihood characteristics (moments, correlations, laws of distribution). The continued development of powerful computer resources allows one to consider alternative approaches to this problem. Such resources enable the consideration of these problems from other approaches than just the traditionally known mathematical methods. Now, the absolutely separate direction of complex problems may be considered to obtain solutions. Mapping of the problems onto particular computer architectures, especially parallel or distributed, dictates which methods are appropriate for a specific problem decision. Compared to traditional reasoning approaches (consecutive and analytical) this may seem a little bit unusual. Let us consider the general approach to the problem of computing the long-term pressure distribution under the wave surface, in both the spatial and time-domain.

STATEMENT OF THE PROBLEM

The most general description of behavior of a sea object under the action of waves may be obtained by solving the Navier-Stokes equation with traditional boundary conditions on the wave surface and the submerged portion of the body. Because the formation of waves is practically completely determined by gravitational forces, and the influence of viscosity is important to consider close to a surface of a body, in naval hydrodynamics potential flow formulations are traditionally used.

Let us follow the assumption that wave motion is irrotational and can be described by only the wave potential. In this case, the general problem is formulated by the following equation and boundary conditions:

$$\Delta\varphi = 0$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}\left(\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right) + g\zeta = p_0 \quad (1)$$

$$\frac{\partial\varsigma}{\partial t} + \frac{\partial\varsigma}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial\varsigma}{\partial y}\frac{\partial\varphi}{\partial y} = \frac{\partial\varphi}{\partial z} \quad \text{at} \quad z = \varsigma$$

where $\zeta(x,y,t)$ is the free surface, φ is the wave potential, and p_0 is the atmosphere pressure

The determination of the spatio-temporal distribution of the potential (to be exact, its derivatives) around the investigated object enables the determination of the field of hydrodynamic pressures, which when integrated on the body gives the forces and moments necessary for modeling ship behavior.

$$p(\mathbf{r}) = -\rho\left.\frac{\partial\varphi}{\partial t}\right|_{\mathbf{r}} - \rho g z_0 - \frac{\rho}{2}\left(\left(\left.\frac{\partial\varphi}{\partial x}\right|_{\mathbf{r}}\right)^2 + \left(\left.\frac{\partial\varphi}{\partial y}\right|_{\mathbf{r}}\right)^2 + \left(\left.\frac{\partial\varphi}{\partial z}\right|_{\mathbf{r}}\right)^2\right)$$

$$\mathbf{F} = -\int_{S_0} p\cdot\mathbf{n}\,dS$$
$$\mathbf{M} = -\int_{S_0} p\cdot(\mathbf{r}\times\mathbf{n})\,dS,$$

where S_0 is the wetted ship surface, **n** is the outward normal vector, and **r** = $\{x_0,y_0,z_0\}$ is a radius-vector of the wetted ship surface.

Eq. (1) is the linear problem with nonlinear boundary conditions and an unknown boundary. The last aspect makes the problem very difficult.

Therefore, the general solution for the potential is obtained only in some special cases, and first of all only for a sinusoidal wave. Accounting for the randomness of waves makes the analytical solution of the potential for a stochastic problem practically useless for applications in problems of naval hydrodynamics.

In both cases the unknown border, ζ (x, y, t), is defined in the process of the problem solution. For example, in the linear definition of the problem

$$\zeta(\mathrm{x},\mathrm{y},\mathrm{t}) = -\frac{1}{g}\frac{\partial\varphi}{\partial t},$$

where the derivative of the potential with respect to time is considered on an unperturbed wave surface. Therefore, in practice other approaches are applied.

COMPUTATIONAL APPROACH: MAPPING OF THE PROBLEM

On the other hand, the problem (1) could be seriously simplified if the spatio-temporal realization of random wave field is known *a priori*. From the analytical point of view, it does not give any special advantages, but for direct modeling it permits the development of effective computing procedures. The question of reconstructing random spatio-temporal wave-fields depends on its hydrodynamic adequacy, i.e. the waves simulated by any others means should fit the physical laws presented in problem (1). At the same time, such a wave model should be effective from computational point-of-view and enable one to reproduce not only a stationary wave process, but also the evolution of waves in time.

We can consider as one of criteria for assessing the hydrodynamic adequacy of the generated random wave-field, corresponding to natural observations, the statistical wave characteristics which are not used as input data for the wave generation procedure. For example, if we use the correlation surface only, its frequency directed spectrum, for free-surface generation, after statistical processing of the model realization we should obtain both the higher moments and laws of distribution for the other wave elements.

It has been shown that it is possible to obtain such a result in the specification of a model using the classical scheme of autoregression – the moving mean (Davidan 1988; Rozhkov and Trapeznikov 1990). Such a wave model is in the form of a class of linear differential systems with distributed parameters and a random input signal of type of a field of white noise:

$$\left[\prod_{k=1}^{N} L_k\right]\varsigma(\mathbf{v}) = \left[\prod_{k=1}^{N} Q_k\right]\varepsilon(\mathbf{v}), \qquad (2)$$

where L, Q are differential operators.

Stationary solutions of the differential equations of type (2) define a class of random fields with the generalized rational spectral density:

$$S_\varsigma(\vec{\omega}) = \frac{1}{2\pi}\frac{\left|\sum_{j_1=0}^{P_1}\ldots(N)\ldots\sum_{j_n=0}^{P_n} C_{[j_1\ldots j_n]} i^{\sum_m j_m}\prod_{k=1}^{N}\omega_k^{j_k}\right|^2}{\left|\sum_{j_1=0}^{N_1}\ldots(N)\ldots\sum_{j_n=0}^{N_n} B_{[j_1\ldots j_n]} i^{\sum_m j_m}\prod_{k=1}^{N}\omega_k^{j_k}\right|^2} \quad (3)$$

Thus it is possible to show that the model of three-dimensional waves, traditionally put into practice offered by Longuet-Higgins, represents model of a moving mean. Therefore, in the limiting case, both of the considered approaches could be considered as equivalent. However, the field model of moving mean has weak convergence. Because of computing difficulties for the application of Longuet-Higgins model for sea waves generation (especially three-dimensional), the combined model of autoregression can be used to establish a nonlinear procedure for parameter assessment.

Therefore, the field autoregressive model is more attractive, and can better characterize the processes. It is known, that the procedure of a moving mean is the best way which is applicable for processes with uniform spectral density, whereas autoregression model is more suitable for processes with strongly pronounced peaks (Box and Jenkins 1970).

For the proper development of a computing process for the model, it is necessary to transition from a continuous model to a model with discrete arguments. So, for example, a finite-difference equation of wave can be defined as

$$\varsigma_{(x,y,t)} = \sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{k=0}^{N_t}\Theta_{(i,j,k)}\varsigma_{(x-i,y-j,t-k)} + \varepsilon_{(x,y,t)} \qquad (4)$$

where Θ(i,j,k) are generalized coefficients of autoregression and ε(x,y,t) is a field of white noise.

Procedures for autoregression parameters and the variance of white noise field assessment are developed based on a generalized Yule-Walker equations system:

$$K_\varsigma(x,y,\tau) = \sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{k=0}^{N_t}\Theta_{i,j,k}K_\varsigma(x-i\Delta_x, y-j\Delta_y, \tau-k\Delta_t) \quad (5)$$

The variance of the white noise field can be determined from equation (5), when i,j,k=0:

$$\sigma_\varepsilon^2 = D[\varsigma] - \sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{k=0}^{N_t}\Theta_{i,j,k}K_\varsigma(i\Delta_x, j\Delta_y, k\Delta_t) \quad (6)$$

It is possible to see from (4) that the autoregressive model is capable of modeling ergodically, at minimal computing expense, a periodic realization of a random process, which its stochasticity is limited only by the period of the pseudo-random number generator. Additionally, the model does not use the property of the likelihood of convergence, Gaussian assumption, as, for example, Longuet-Higgins model or other known models. It allows effective application to the research of extreme events, both in oceanography, and in naval hydrodynamics. It is also important that, on the basis of linear inertial transformation, the model can be used to easily construct nonlinear inertia-less transformations to any law of distribution.

In Degtyarev and Boukhanovsky (1996) it is shown that this model is hydrodynamically adequate, as compared to natural conditions. For the verification of the field autoregressive model, a series of tests for complete analysis of wind and complex sea has been carried out. In addition, the analysis of wind-wave evolution in storm and with spatially non-uniform current was carried out. The latter showed that the autoregression model, together with nonlinear inertia-less transformation (Degtyarev and Boukhanovsky 1996; Boukhanovsky et al. 2000) can successively reproduce nonlinear wind-waves when the distribution law of ordinates is distinct from normal.

As a test, simulated aerial images were used (Degtyarev and Boukhanovsky 1996). Statistical characteristic of the visible waves were used for verification. The criterion of verification was the agreement of the cdf and the joint distributions and conditional moment curves between the measured waves and simulated waves.

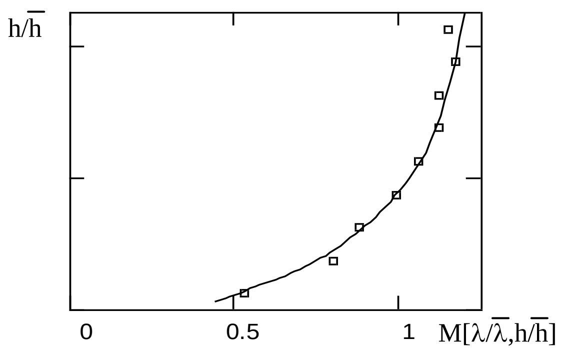

Fig. 1: Regression of lengths and heights of waves. □ – model, line – experiment

All experimental results concerning the distributions of visible wave's elements have been confirmed. In particular, the characteristic form of a curve of the conditional variance of wavelengths from their heights (Fig. 2) has been presented. Such agreement cannot be achieved by any of known ways of wave modeling, including the Longuet-Higgins model.

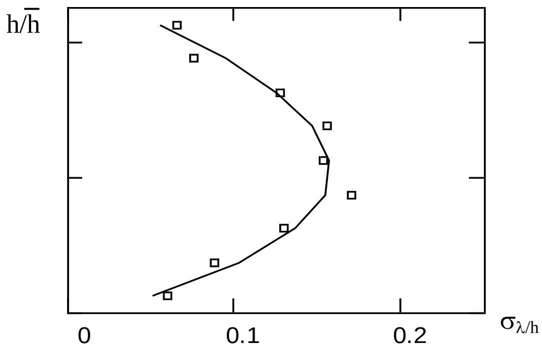

Fig. 2: Scedastic curve of lengths and heights of waves

Modeling of a complex sea has also been carried out. Some variants of wind-waves and systems of swell have been investigated. It is shown that the distribution law of the wave periods of a complex sea, represents a combination of Weibull laws with various parameters (Rozshkov 1990). The number of elements of a combination is equal to the number of wave systems. For the usual wind-waves distribution law of periods, the solution is well-smoothed on a grid, using a Weibull law with k=3, however at narrowing, a spectrum parameter of distribution law increases, approaching 4.

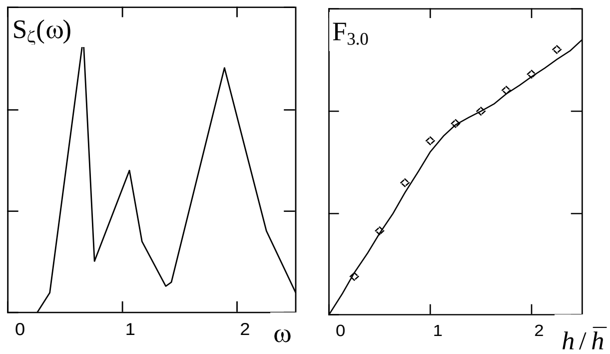

Fig. 3: Spectrum with two swells and distribution of wave periods

On the basis of the obtained results, it is possible to discuss the high physical adequacy of the presented model of sea waves on quasi-stationary time-domain. In Fig. 3, one of the interesting examples of three investigated wave systems is shown.

To analyze the abilities of the model to generate a non-stationary wave-field, we considered a number of storm types. As the first, Hurricane "Belief" (Davidan 1988), which took place in the central part of the North Atlantic on September 2-6, 1966 was chosen.

Besides the model of non-stationary, the wave-field on a longer time interval: July 5-17, 1986 (Rozhkov 1990) has also been verified. The interval of wave evolution has been broken into thirty-six 8-hour sites, where each had waves that were assumed as quasi-stationary. The evolution of the average wave height during a storm is shown in Fig. 4. The strong agreement between experimental measurements and the model results is encouraging for the ability of the model to produce high quality results in a range of synoptic variability.

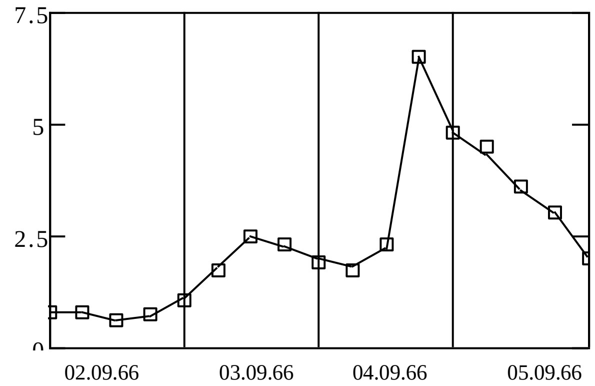

Fig. 4: Diagram of average wave height variation during hurricane "Belief"

All these results inspire confidence that this effective computing procedure allows us to generate a hydrodynamically adequate wave surface, and also possesses the ability to evolve the solution in time.

For the description of such transformation, it is necessary to address questions of wave-weather scenario modeling. Some details related to this question were presented at the Stability Workshop in 2005 (Degtyarev 2005). During the evolution of sea waves, the spectral density randomly varies in time, i.e. for description of such an evolution the spectral density should be represented by a stochastic function. One of the ideas formulated in Degtyarev (2005) consists of the parameterization of $S_\zeta(\omega)$. In this case, we consider it as a deterministic function with a set of random variables:

$$S = S(\omega, \theta, \Xi) \tag{7}$$

The feasibility of an approach like (7) obviously depends on the level of accuracy used to specify the spectrum $S_p(\omega,\theta)$. This may be specified by the parameters Ξ_p taken from their multidimensional distribution $F_\Xi(\xi)$.

In the present study, parameters of the spectrum related to wave height, spectral shape, the frequency of the spectral peak, ω_{max}, and the main wave direction, θ_{max}, are selected as parameters in Ξ. The single field model spectrum may be formulated

$$S_p\left(\omega/\omega_{\max}, \theta - \theta_{\max}, \Xi_r\right),$$

where Ξ_r signifies the rest of the parameters. More general spectra, $S(\omega,\theta)$, are obtained as

$$S(\omega,\theta) = m_{00} \sum_{p=1}^{N} \gamma_p S_p \left(\frac{\omega}{\omega_p}, \theta - \theta_{\max}, \Xi_{r_p} \right) \tag{8}$$

where m_{00}, the 0th moment of the spectrum, is equal to the total variance of wave field, N is the number of wave fields (peaks in the spectrum), and γ_p are weight factors for each system so that, $\sum_{p=1}^{N} \gamma_p = 1$.

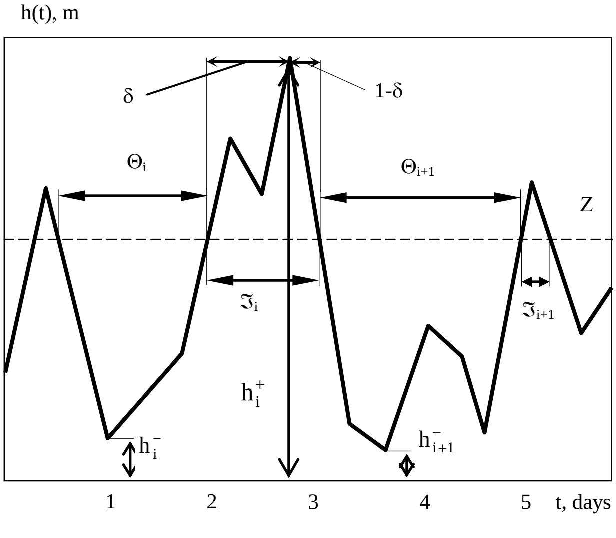

Fig.5. Parameters describing storms and weather windows

Using this approach, a procedure for a standard wave weather classification was developed (Boukhanovsky et al. 2000, Degtyarev 2005).

A time-series of wind-wave heights in the mid-latitudes and subtropical areas of the World Oceans can be used as alternating sequences of storms and weather windows. We define a storm of duration $\Im$ and intensity h^+ as a situation when random function h(t) exceeds a predefined value Z. The period Θ, during which the wave height is less than this threshold, will be called a weather window of intensity h^-. The parameter δ shows the asymmetry of the storm:

$$\delta=(t_p-t_b)/\Im$$

t_b, t_p , t_e are times of storm start, the maximum, and the end, respectively. Fig. 5 clarifies these definitions.

Such a parametrization of wave evolution permits one to simulate variations of the spectrum parameters in (7). Examples of procedures of storms classification for specific regions are shown (Boukhanovsky et al. 2000, Degtyarev 2005, Belenky and Sevastianov 2007). The uniform approach to waves, modeling (2) - (4), and its evolution, permits one to develop a set of nested autoregressive models for generation of continuous realization of spatio-temporal wave-field, in a given region of the Ocean.

At the quasi-stationary and synoptic intervals of variability, the wave process is best described by the stationary auto-regression model AR(p) of order p, namely

$$\xi_t = \sum_{k=1}^{p} \phi_k \xi_{t-k} + \varepsilon_t,, \quad \zeta_t = f(\xi_t) \qquad (9)$$

where ϕ_k are coefficients to be computed using the correlation function $K_\xi(\tau)$, and ε_t is white noise with a given distribution function, which has to be compatible with the nonlinear functional transformation $f(\bullet)$ of function ξ_t into, respectively, the Rayleigh or log-normal distribution of ζ_t. In Lopatoukhin et al. (2001) it is shown that a stationary pulse-like random process is a good model for a sequence of storms and fair weather intervals.

The actual generation of a series of random storms and weather windows is based on a Monte-Carlo approach. Thus, it becomes possible to reproduce the whole variety of values of $\{h^+, h^-, \Im, \Theta\}$:

$$\begin{gathered} \Im_k = F_{\Im}^{-1}\left(\gamma_1^{(k)}\right), \Theta_k = F_{\Theta}^{-1}\left(\gamma_2^{(k)}\right) \\ h_k^+ = F_{h^+|\Im}^{-1}\left(\gamma_3^{(k)} \mid \Im_k\right), h_k^- = F_{h^-|\Theta}^{-1}\left(\gamma_4^{(k)} \mid \Theta_k\right) \end{gathered} \qquad (10)$$

Here $\left\{\gamma_i^{(k)}\right\}$ denotes a system of four pseudo random numbers.

A stochastic model for extra-annual rhythms could be written as follows:

$$\zeta(t)=m(t)+\sigma(t)\xi_t \qquad (11)$$

Here m(t) and σ(t) are periodic functions, and ξ_t is a non-stationary process AP(p) so that

$$\xi_t = \sum_{k=1}^{p} \phi_k(t) \xi_{t-k} + \varepsilon_t \qquad (12)$$

and the coefficients $\phi_k(t)= \phi_k(t+T)$ are periodic functions of time.

A model that is capable to describe year-to-year variability of the monthly mean wave heights will therefore require twelve values of m(t) and 78 values of K(t,τ). It is possible to reduce the number of dimensions by considering the following representation of periodically correlated stochastic processes (PCSP):

$$\zeta(t) = \sum_{k=-\infty}^{\infty} \eta_k(t) \exp\left(i\omega_k t\right) \qquad (13)$$

So with the help of such nested autogression models, it is possible to reconstruct conditions of a hypothetical (artificial) weather scenario, at a specific location of interest. The idea is to look at a situation that did not yet happen, but in principle, can happen.

CONCLUSION

One of the most promising applications of the autoregression model is for advanced hydrodynamic codes. These codes are traditionally based on potential flow and external models for vortex and viscosity forces. They use Longuet-Higgins model for wave elevations and pressures, which put a limit on the length of irregular wave realizations that can be efficiently used for simulations. Another limitation is for the modeling non-stationarity. The latter one may be especially important for dynamic stability, as growing seas increase the probability of encounter for steep waves, which may represent significant danger, in terms of roll motions. The application of the autoregression model naturally solves both problems. As it was shown, the autoregression model offers a very natural way to present non-stationarity. However, for use of the autoregression model in a potential hydrodynamic code, wave pressures also need to be evaluated. Several options can be considered for pressures. The most natural way is to use formulation (1). The autoregression model of wave elevations becomes the boundary condition. Another option is use autoregression model itself for the pressures as well. In the latter case, it needs to be related with the wave elevations and a given spectrum.

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