On Vulnerability Criteria for Righting Lever Variations in Waves

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ABSTRACT

This paper proposes assessment methods for use in evaluating level 1 and level 2 vulnerability, as outlined in the IMO preliminary specification for the new generation intact stability criteria under development in the Subcommittee on Stability and Load Lines and on Fishing Vessels Safety (SLF) of IMO. Particularly, these methods are developed for the identification of problems related to righting lever variations in waves– pure-loss of stability and parametric roll. Using these methods, the assessment results for a population of sample ships are presented and discussed.

KEYWORDS

dynamic stability, pure-loss of stability, parametric roll, righting lever

A REGULATORY PERSPECTIVE ¹

The international effort to not only develop, but also establish, new generation intact stability criteria in a community in which there is a general perception that adequate criteria already exists is a big challenge. History is replete with examples of efforts to replace something old with something new that have been dashed upon the rocks of prevailing contrary opinion.

In the international stability community, the example of how the 1973 IMO resolution A.265 on probabilistic damage stability for passenger ships was not incorporated into the 1974 SOLAS Convention but retained as an "equivalent" to existing criteria is a reminder that substantial effort to develop a criterion can

be met with disappointment, if the requirement for the criterion is sidelined (Robertson *et al.*, 1974). This problem was described five centuries ago in Machiavelli's famous work *The Prince* (1532):

"There is nothing more difficult to take in hand, more perilous to conduct, or more uncertain in its success, than to take the lead in the introduction of a new order of things, because the innovator has for enemies all those who have done well under the old conditions, and lukewarm defenders in those who may do well under the new. This coolness arises partly from fear of the opponents, who have the laws on their side, and partly from the incredulity of men, who do not readily believe in new things until they have had a long experience of them."

Recognizing this challenge, the case of the generation intact stability new criteria development may benefit from being considered as a companion or addition to the existing criteria rather than a replacement. In every respect, however, the need exists to demonstrate many times that the benefits of the new criteria outweigh the cost and in several forums, so that the new criteria may enjoy the best chance of acceptance. To assist this

¹ This paper expresses the personal views of the authors, which are not necessarily the official views of the U.S. Coast Guard or the Department of the Navy.

objective, the new criteria must be shown to be robust, which is a need that requires substantial verification.

The framework for new generation intact stability criteria (Annex 1, SLF 51/WP.2) covers dynamic stability failures related to righting lever variations in waves (pure-loss of stability and parametric roll), broaching, and dead ship conditions (see also Belenky, et al., 2008). A multi-tiered approach is included in the preliminary specification for the new criteria, where initial analysis is done using vulnerability criteria that progresses from a simple assessment (level 1) to a more performance-based (level assessment 2) (Annex 2, SLF 52/WP.1). If the likelihood of one or more dynamic stability failure modes is indicated by the vulnerability criteria, then direct assessment methods are applied. The identification of hull forms that may have increased risk of these stability failures early in the design process allows ship design managers to justify hull form modifications or to undertake the necessary planning and budgeting for direct assessments using advanced hydrodynamic codes, numerical simulations and/or model experiments.

These vulnerability criteria are currently under development by the Correspondence Group on Intact Stability, established by IMO's SLF Subcommittee and its latest report contains the status of this development (SLF 52/3/1; SLF 52/INF.2).

In the case of righting lever variations in waves, several methods have been proposed to assess vulnerability for pure-loss of stability and parametric roll (levels 1 and 2). In order to provide a practical tool for the designer and regulator, several considerations must be examined. These include the ability to distinguish vulnerable ships from ships that are not vulnerable to these modes of stability failure, the ease of use of the methods (including input data requirements, calculation time, and interpretation and allowable error of the results), and development of the standard (or safety level) using the criteria.

A useful standard for level 1 vulnerability assessment must be conservative, so that all

ships which may be vulnerable to this mode of stability failure fail to meet the standard and therefore, must be assessed using a higherfidelity approach (level 2 and possible direct assessment). If the standard is set at a threshold where some ships are able to pass, despite the possibility of vulnerability to the failure mode, then it fails to meet its objectives of usefulness to the designer and regulator. However, at the same time, the standard should not be overly conservative, such that nearly all ships fail, and require further assessment, which would negate the usefulness of the method.

TESTED SAMPLE SHIP TYPES

Twelve diverse ship types were examined to test the applicability of the proposed vulnerability criteria for righting lever variation modes of stability failure (pure-loss and parametric roll). The ship types considered included: a bulk carrier, a tanker (VLCC), five containerships, two general cargo ships, a and a pair of notional naval RoPax. combatants, specifically designed for research purposes (Table 1). The critical loading condition, limiting GM, is given based on the 2008 Intact Stability (IS) Code. This was used for the assessment of pure-loss of stability. For the assessment of parametric roll, a typical operational loading condition was used. The range of characteristics for the sample ship population is shown in Fig. 1.

Table 1: Ship types and general characteristics

Туре	L/B	B/T	C _B	Critical GM (m)
Bulk Carrier	5.85	2.24	0.85	4.192
Containership 1	7.07	3.05	0.62	0.1506
Containership 2	6.53	3.62	0.61	0.1507
Containership 3	7.24	3.14	0.64	0.1507
Containership 4	8.80	2.51	0.65	0.1509
Containership 5	6.55	3.12	0.55	0.1505
General Cargo 1	7.01	2.50	0.70	0.1504
General Cargo 2	7.05	2.73	0.57	0.1507
RoPax	6.76	3.64	0.60	0.3625
Tanker	5.52	2.76	0.80	1.723
Naval Combatant 1	8.19	3.42	0.54	0.20
Naval Combatant 2	8.19	3.42	0.54	1.161



Fig. 1: Sample ship population characteristics: L/B, B/T, and C_B .

Containership 5 is the C11-class containership. General cargo ship 1 is Series 60 hull form, $C_B=0.7$ variant (Todd, 1953). General cargo ship 2 is the C4 type, similar to the one used in Paulling, *et al.* (1972). Naval combatants 1 and 2 are the ONR Topsides Series, flared and tumblehome configurations, respectively (Bishop, *et al.*, 2005). The RoPax is a notional vessel similar to the one from a reported stability accident (MNZ, 2007).

LEVEL 1 VULNERABILITY CRITERIA FOR PURE-LOSS OF STABILITY AND PARAMETRIC ROLL

Because both of the modes of intact stability failure considered here, pure-loss and parametric roll, are fundamentally a result of the relation between changes in the area of the waterplane and the location of the wave crest along the hull, a common criterion to assess level 1 vulnerability is proposed. Four prospective criteria are discussed, along with the results for the sample ships.

Method

A method to assess level 1 vulnerability to pure-loss of stability and parametric roll, based on static characteristics of the hull form, is proposed and four criteria were examined. The first criterion considered the value of the total coefficient for vertical "wall-sidedness," C_{VWS} , or the variability of hull shape from the maximum dimensions over the range of draft, $\max(A_{WP}(z))$, $z \in [d - \Delta d; d + \Delta d]$, which is similar to the more traditional vertical prismatic coefficient, C_{VP} , taken from the calm waterplane. This provides an indication of the change of the shape of the hull from the volume projected using the maximum waterplane dimensions over the vertical height of the ship.

$$C_{VWS} = \frac{\int_{d-\Delta d}^{d+\Delta d} A_{WP}(z) dz}{\max(A_{WP}(z)) \cdot 2\Delta d}$$
(1)
$$\Delta d = \max\left(\frac{d}{2}, \frac{L}{20}, D - d\right)$$

The second criterion considered the average of the vertical wall-sidedness coefficients for the fore and aft quarter portions of the hull, both above and below the waterline (Fig. 2). For each of the four sections (fore, aft, above, and below), the C_{VWS} was computed as the fraction of the volume from the maximum waterplane projection for the given section. Then the average value for the four sections was used to provide an indication of the total relative changes for the bow and stern shapes, both above and below the waterline.



Fig. 2: Notional ship profile with the four portions of the Cvws considered for the level 1 vulnerability assessment.

The third criterion considered the ratio of the transverse metacentric radius to the height of the transverse metacenter above the keel.

$$C1_3 = \frac{BM}{KM} \tag{2}$$

The fourth criterion considered the ratio of the transverse metacentric radius to the beam.

$$C1_4 = \frac{BM}{B} \tag{3}$$

Results

The first criterion does not show any clear separation between the ships which are known to be vulnerable and the ships which are not (Fig. 3). However, the second criterion, the average of the vertical wall-sidedness coefficient for the fore and aft quarters of the ship, seems to provide useful separation between the ships (Fig. 4) for this sample population.



Fig. 3: Total Cws, both above and below the waterline, for the sample ship population.



Fig. 4: Total average Cvws for the fore and aft quarters of the ship, both above and below the waterline, for the sample ship population. The contributions from each component of the average Cvws are identified.

Based on this sample population of ships, an initial estimate of the threshold for the standard could be proposed around 0.75-0.80. Ships above this value, the bulk carrier, tanker, and Series 60 are considered to be conventional vessels, not at risk for failures related to righting lever variations in waves. However, all of the other nine ships fall below this value, the highest being the general cargo ship 2, or C4, with a value of 0.75. The ships with the lowest values are containership 5 (the C-11 containership) and the RoPax, which have values of 0.69 and 0.67, respectively.

Of the four vertical wall-sidedness coefficients, fore and aft quarter, above and below the waterline, the aft coefficient above the waterline has the least variation for the ship population examined. However, in order to account for ships outside this population, with unconventional topside stern shapes, this effect should still be included.

The third and the fourth criteria, using ratios with the transverse metacentric radius, did not show any clear separation between the ships which are known to be vulnerable and the ships which are not.

The proposed method for level 1 vulnerability assessment does not consider the relative size of the ship and the waves. Typically it is assumed that higher sea states are more likely to result in stability failure. However, waves of large height are more likely to have larger length and waves of large length may not greatly affect stability, depending on their comparison with ship length. This important consideration is included in the proposed level 2 assessment methods.

LEVEL 2 VULNERABILITY CRITERIA FOR PURE-LOSS OF STABILITY

The procedure described for vulnerability level 2 criteria for pure-loss of stability is based on SLF 52/INF.2, Annex 6. Further refinements and improvement to the method are discussed in Belenky & Bassler (2010), including application of the method to navaltype vessels.

Methods

Pure loss of stability may be considered as a single wave event because of instantaneous changes in waterplane area. Typically, the worst-case wavelength is close to the length of the ship, $\lambda/L \approx 1.0$. However, in order to account for the effect of ship size relative to the wave conditions, righting lever variations should be evaluated in irregular waves. To characterize an event of pure-loss of stability, the distribution of random wave numbers and wave amplitudes, f(A,k), is used to evaluate statistical weight of a wave encounter:

$$W_{ij} = \int_{A_i - \Delta A}^{A_i + \Delta A} \int_{k_j - \Delta k}^{k_j + \Delta k} f(A, k) dk dA$$
(4)

The GM value is calculated for each sinusoidal wave, with characteristics as defined above. These calculations are repeated for different positions of the wave crest along the ship length, so a complete wave pass is presented.

Calculation of the time while the stability is decreased can be easily performed when the GM is considered as a function of the wave crest. The critical GM was calculated with the 2008 IS Code (Fig. 5).

GM change due to wave pass



Position of wave crest, m

Fig. 5: Calculation of "time-below-critical-GM"

Points x1 and x2 (Fig. 5) show the distance when the GM remains below the critical level (based on 2008 IS Code), while the wave passes the ship. The "time-duration-belowcritical GM", tbc, can be calculated as:

$$tbc = \frac{x^2 - x^1}{c - V_s} \tag{5}$$

where c is wave celerity and V_s is ship speed. The time-below-critical GM is a random number in irregular waves. Its mean value is estimated as:

$$m(tbc) = \sum_{i} \sum_{j} tbc_{ij} W_{ij}$$
(6)

The criterion value, Cr1, is proposed as the following ratio:

$$Cr1 = \frac{m(tbc)}{T_{\phi}}; \qquad (7)$$

where T_{ϕ} is natural period of roll corresponding to critical GM

This criterion assesses the significance of stability change in waves. If stability is degraded only for a short duration, this degradation may not be significant. However, for longer durations of decreased stability below the critical level, the restoring moment may be degraded enough to result in a dangerously large roll angle.

The second criterion is set to detect if there were significant durations of negative GM. Appearance of an angle of loll may lead to the development of partial stability failure faster, as the upright equilibrium is no longer stable. It is quite possible that some ships may be more vulnerable for these types of failure than others (see the example for a notional RoPax vessel in Fig. 6).



Fig. 6: Deterioration of GZ curve near wave crest

The second criterion, Cr2, is based on characteristics of time when the angle of loll is above a certain limit angle, ϕ_{lim} (30 degrees

was used in this example). For each position of the wave crest along the hull, the indicator value, z, is calculated:

$$z = \begin{cases} 0 & if \quad \phi_{loll} < \phi_{lim} \\ 1 & if \quad \phi_{loll} \ge \phi_{lim} \end{cases}$$
(8)

The angle of loll, ϕ_{loll} , can be obtained from the "true" instantaneous *GZ* curve in waves, or from its approximation using a calm water *GZ* curve and the instantaneous *GM* in waves:

$$GZ_{W}(\phi,t) = \frac{GM_{W}(t)}{GM_{0}}GZ_{0}(\phi)$$
(9)

Here the index "0" refers to calm water conditions. The time while the angle of loll is too large during the wave pass is expressed as:

$$tbz = \sum_{k} z_k \Delta t \tag{10}$$

where Δt is the time-step and index k corresponds to a particular time instant during the wave pass.

Formulation of the second criteria is similar to the first one:

$$Cr2 = \frac{m(tbz)}{T_{\phi}} \tag{11}$$

where m(tbz) is the weighted average over the wave encounters:

$$m(tbz) = \sum_{i} \sum_{j} tbz_{ij} W_{ij}$$
(12)

Results

Results are shown for calculations using the two criteria (Cr1 and Cr2) for the sample ships. The results (Fig. 7) are given for Sea State 7 and an operational speed of 15 knots, with the

critical *KG* based on the conditions from compliance with the 2008 IS Code.



Fig. 7: Calculation results for the two level 2 vulnerability criteria for pure-loss of stability for the sample ships, ship speed of 15 kts, in Sea State 7.

Comparing the sample calculations for the level 2 probabilistic criterion, Cr1, it can be observed that there is a great distinction between the Naval Combatant 2 (ONR tumblehome topside hull), which is known to be vulnerable to pure-loss of stability (Bishop, *et al.*, 2005; Bassler, *et al.*, 2007; Hashimoto, 2009), compared to other ships, which are not known to be vulnerable to this type of stability failure, except for the notional RoPax. Given these results, and the sample calculations with a notional naval fleet (Belenky & Bassler, 2010), a standard using this criterion could be set around 1.0.

The second criterion indicates possible vulnerability for the notional RoPax vessel that is similar to one that attained large roll angles in stern waves (MNZ, 2007).

LEVEL 2 VULNERABILITY CRITERIA FOR PARAMETRIC ROLL

Method

Vulnerability to parametric roll is determined by the maximum angle of roll response on a "typical" wave group, related to a given sea state, see SLF 52/INF.2, Annex 7.

The "typical" wave group (Fig. 8) is assumed to consist of a number of waves of the same length, and a wave period corresponding to the spectral mean period. The amplitude of the group is considered as a function of time only; its spatial change is not modeled. A more detailed method to determine the characteristics of a "typical" group for a given sea state is currently under development. Recently, a method to identify wave groups, based on ship-specific considerations for the amplitude and duration has been proposed (Bassler, et al., 2010).



Fig. 8: Model of "typical" wave group

As parametric resonance may occur both in following and head waves, the attitude of a ship is calculated based on heave and pitch response on a wave group:

$$\begin{cases} (M + A_{33})\ddot{\zeta}_{G} + B_{33}\dot{\zeta}_{G} + F_{\zeta}(\zeta_{G}, \theta, t) = 0\\ (I_{Y} + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + M_{\theta}(\zeta_{G}, \theta, t) = 0 \end{cases}$$
(13)

where *M* is mass of the ship, I_Y is mass moment of inertia relative to the transversal axes, A_{33} and A_{55} are heave added mass and pitch moment of inertia (assumed to be equal to the corresponding mass and moment of inertia), respectively; and B_{33} and B_{55} are damping coefficients for heave and pitch. Functions F_{ζ} and M_{θ} are the difference between Froude-Krylov and hydrostatic forces and moments at the instant of time, *t*. These values are expressed as follows:

$$F_{\zeta}(\zeta_G, \theta, t) = \rho g \left(V_0 - \int_{-0.5L}^{0.5L} \Omega(x, z(\zeta_G, \theta, t)) dx \right)$$
(14)

$$M_{\theta}(\zeta_{G}, \theta, t) = \rho g(V_{0} \cdot LCB_{0})$$

$$- \int_{-0.5L}^{0.5L} M_{\Omega}(x, z(\zeta_{G}, \theta, t)) dx$$
(15)

where ρ is mass density of water, V_0 volumetric displacement in calm water, LCB_0 is the longitudinal position of center of buoyancy in calm water. Functions Ω and M_{Ω} calculate an area and a static moment relative to the y-axis of a station located at abscissa *x*. The second argument of this function shows submergence of this station, as expressed by the function of instantaneous waterline $z(\zeta_G, \theta, t)$, see Fig. 9. These waterlines allow for the evaluation of the *GM* response to the wave group (Fig. 10).



Fig. 9: Sample instantaneous waterlines evaluated from heave and pitch response on a group



Fig. 10: GM response on "typical" wave group, with the GM value in calm water shown in blue.

The *GM* response to a "typical" wave group then can be approximated using a sine function with time-dependent amplitude:

$$f_{L}(\phi,t) = \frac{GM_{m} + GM_{a}(t)\cos(\omega_{e}t + \phi)}{GM} \quad (16)$$

where ω_e is the encounter frequency,

$$\omega_e = \omega_1 + k_1 V_s \tag{17}$$

and where ω_1 and k_1 are the wave frequency and wave number corresponding to the mean spectral period, V_S is forward speed, chosen to satisfy the frequency condition for principal parametric resonance, while keeping the value within the achievable range for the given vessel in the considered sea state. Roll response is evaluated by the numerical solution of the roll equation with stiffness (16) and assumed roll damping. The initial conditions for the numerical solution of roll motion can be chosen as 5-10 degrees for the initial roll angle and zero roll rate.

$$\ddot{\phi} + 2\delta_{\phi}\dot{\phi} + \omega_0^2 f_L(\phi, t) = 0$$
(18)

Equation (18) is essentially the Mathieu equation. If the amplification of roll oscillations is observed, then parametric excitation is large enough, taking into account speed limitations. The largest absolute value of the roll angle observed during the wave group pass can be used as a criterion:

$$CrL = \max(|\phi|) \quad for \quad f = f_L$$
 (19)

Due to significant nonlinearity of the GZ curve, the development of parametric resonance may be reversed as the change in instantaneous GM with roll angle may take the system out of the Mathieu instability region (Spyrou, 2004).

To model this nonlinearity, formula (9) can be used in the roll equation with nonlinear stiffness. Equation (20) is a variation of Hill's equation:

$$f_N(\phi, t) = \frac{GZ_W(\phi, t)}{GM}$$
(20)

$$\ddot{\phi} + 2\delta_{\phi}\dot{\phi} + \omega_0^2 f_N(\phi, t) = 0$$
(21)

However, it may be necessary to extend (20) up to 180 degrees to avoid numerical issues while solving equation (21), see Fig.11.



Fig. 11: GZ curve modeled for response on "typical" wave group

Instead of the approximation (9), the actual GZ curve in waves can be used as well. Based on the solution of (21), the second criterion, CrN, is formulated:

$$CrN = \max(|\phi|) \quad for \quad f = f_N \tag{22}$$

Due to nonlinearity of the time-dependent stiffness, it is not known in advance what frequency region may lead to parametric resonance, so several speeds with the achievable range must be used.

Results

Results are shown for the two criteria (*CrL* and *CrN*) for the sample ships (Fig. 12). The values used for the evaluation for each ship are given in Table 2. For the ships considered, a common damping ratio was chosen, typical for these types of ships. For the two naval combatants, which typically have larger bilge keels and therefore, a larger damping ratio was specified. The *GM* condition used was a typical operational load condition for each of the sample ships, GM_{OP} . Sea States 5-8 were evaluated, but only the particular sea state where parametric roll was observed and the given speed condition to satisfy the frequency ratio conditions are presented.



Fig. 12: Calculation results for the two level 2 vulnerability criteria for parametric roll for the sample ships in Sea State 7.

Туре	Sea	GM _{OP}	Roll	Speed
	state	(m)	damp.	(kts)
Bulk Carrier	7	9.41	0.05	10
Containership 1	7	1.12	0.05	10
Containership 2	7	1.84	0.05	2
Containership 3	8	1.64	0.05	1
Containership 4	7	1.06	0.05	10
Containership 5	7	1.91	0.05	0
General Cargo 1	6	0.25	0.05	9.3
General Cargo 2	6	1.10	0.05	5.1
RoPax	6	1.77	0.05	25
Tanker	7	9.76	0.05	10
Naval	6	1.03	0.15	15
Combatant 1				
Naval	6	3.01	0.15	25
Combatant 2				

Table 2: Ship types and general characteristics

Modern containerships, particularly the C11-class containership, are known for their vulnerability to parametric roll (France, *et al* 2003). The proposed criteria shows large roll angles for all five containerships, as well as the notional RoPax vessel, encountering representative wave groups in Sea State 6, 7, and 8. As expected, Series 60, which is representative of a conventional ship type, the tanker, and bulk carrier did not show any vulnerability for the considered loading and operational conditions.

Both ONR Topside configurations (flared and tumblehome) have relatively large bilge keels. The damping ratio used was meant to model the fully appended hulls. While the ONR Tumblehome Topside did not show any parametric roll for the analyzed loading condition, parametric roll was observed for Flared Topside, using the linear ONR formulation. However, parametric roll was not from earlier numerical observed and experimental investigations for these hull forms (Bassler, 2008; Olivieri, et al., 2008; Hashimoto and Matsuda, 2009), including for the flared topside configuration with roll damping coefficients, corresponding to the fully appended hull. However, when the instantaneous GZ curve is used instead of the approximation, parametric roll was not indicated, which corresponded to previous findings.

CONCLUSIONS

Several methods were proposed to assess vulnerability to righting lever variations in waves (pure-loss of stability and parametric Calculation results for a sample roll). population of 12 ships were examined for both simple, geometry-based (level 1) and more complex (level 2) analysis methods. Of the proposed criteria, one for level 1 vulnerability to pure-loss and parametric roll, and two for level 2 pure-loss of stability and one for level 2 parametric roll show promise for possible criteria to assess these modes of stability failure in early-stage ship design. However, additional work remains to determine results for the methods with a larger population of sample ships and then determine possible standards for the criteria.

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