# Probabilistic Qualities of Stability Change in Waves

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## ABSTRACT

The paper discusses two key aspects of the evaluation of a ship's changing stability in waves: the calculation of the roll righting arm (GZ) curve in waves and the study of its probabilistic characteristics in irregular, stern quartering seas.

The calculation of the GZ curve in waves has been implemented in the LAMP (Large Amplitude Motion Program) system using two approaches. The first is a quasi-static calculation for the ship balanced on the wave and is part of LAMP's pre-processing tool PRELMP. The second is a dynamic calculation based on the ship's instantaneous motion in waves, and is implemented as part of LAMP's time-domain, approximate body-nonlinear simulation. Results of the calculations have been compared against results obtained with other methods; following the procedure defined in the ABS *Guide for the Assessment of Parametric Roll Resonance in the Design of Container Carriers* and using the EUREKA hydrostatics program. The self-consistency of the implementation has been tested by applying a heeling moment and observing heel angle for a simulation with ship speed set to match wave celerity.

Probabilistic characteristics have been studied for the principles' parameters of the GZ curve: the angle of the maximum GZ, the value of maximum GZ and the angle of vanishing stability. These GZ curve parameters were considered as stochastic processes. The consideration of practical ergodicity and development of statistical distributions of these processes were included.

## KEYWORDS

Stability in waves; Practical ergodicity; Statistical distribution

## INTRODUCTION

A renewed interest in a ship's stability change in waves has been triggered by the recent development of vessels with novel hull forms and stability characteristics that are quite different from conventional designs. Some of these new vessels have been found to be susceptible to stability failures directly related to the changing stability in waves, including parametrically excited roll of large container carriers (France, *et al.* 2003). This case and others have prompted SLF-48 to include the variation of the restoring moment in waves as one of the phenomena to be addressed in future intact stability regulations.

The phenomenon of the change of transverse stability in longitudinal waves has been known to naval architects for more than a century (Pollard and Dudebout, 1892). Methods of calculating the change of stability in waves are not considered as a new development by any means. Paulling (1961) proposed a quasi-static method for taking into account the changing pressure while a wave passes a ship. This approach was implemented in the programs EUREKA and STABW, the latter of which is a version of EUREKA adapted for the lines data representation used by the ABS SHIP-MOTIONS program.

Boroday (1967) developed a method for the theoretical prediction of the statistical characteristics of restoring moment in irregular waves, taking into account Froude-Krylov pressures. Later on, this solution was enhanced by adding radiation and diffraction (Boroday and Netsvetaev, 1982; authors are not aware of availability of this work in English). Nechaev (1978) proposed a method for the evaluation of stability in regular waves based on a series of model tests (available in English in Belenky and Sevastianov, 2007).

## CALCULATION OF STABILITY IN WAVES

### *LAMP System*

The Large Amplitude Motions Program (LAMP) is a time-domain ship motions and wave load prediction program that is built around a 3-D potential flow panel solution of the wave-body hydrodynamic interaction problem (Shin, *et al.* 2003). A key element of the LAMP code is the 3-D body-nonlinear calculation of the incident wave (Froude-Krylov) and hydrostatic restoring forces. These forces are computed by integrating the Froude-Krylov and hydrostatic pressure of the instantaneous wetted portion of the hull at its predicted position and beneath the incident wave at each time step (Figure 1).

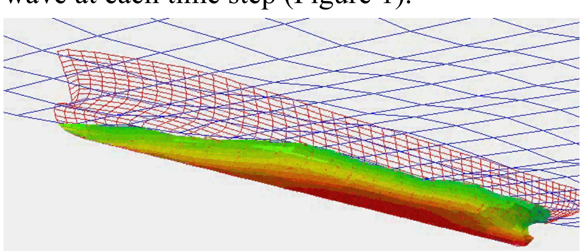

**Fig 1: LAMP Body-nonlinear Pressure Distribution for a Ship in Waves**

This body-nonlinear hydrostatics has been found to be the dominating effect in both nonlinear ship wave loads in extreme seas (Weems, *et al.* 1998) and in the parametric roll of container carriers (France, *et al.* 2003) and is the basis for the present evaluation of roll restoring (GZ) curve in waves.

LAMP is actually a system of codes including the time-domain hydrodynamic and dynamic solver, a pre-processor (PRELMP) for setting up and checking hull geometry models and other simulation options, and a set of post-processing codes for analyzing the time-domain simulation results. The LAMP calculation of the GZ curve in waves has been implemented using two approaches. The first is a quasi-static calculation for the ship balanced on the wave. This calculation is often referred to as the "wave-pass" calculation and implemented in PRELMP. The second is a dynamic calculation based on the ship's instantaneous motion in waves, and can be invoked either as part of the time-domain simulation or as a post-processing calculation.

### *Balancing: Static and Quasi-Static Approach*

In general terms, the stability in waves is evaluated in the same way as it is done in calm seas: the vessel is rotated (heeled) about its longitudinal axis, the pressure on the hull surface is integrated to get the restoring forces and moments acting on the ship, and the uncompensated moment relative to the longitudinal axis is used to derive a restoring arm that represents the value of the GZ curve for the rotation (heel) angle. As described above, the restoring arm calculation has been implemented for both the quasi-static wave-pass problem and for the dynamic problem of the ship moving in waves. In the latter (dynamic) problem, the ship's hydrodynamic forces (including radiation, diffraction, forward speed, appendages, etc.) are assumed *not* to change with heel angle, so only the change in the Froude-Krylov and hydrostatic forces is evaluated.

In the righting arm calculation, the heel rotation disturbs the ship's static or dynamic equilibrium in pitch and heave – in other words, the heave force and pitch moment also change – so several balancing options are provided:

- No balancing; equilibria in pitch and heave are not satisfied after the hull is rotated.
- Heave balance; draft is altered to reach equilibrium in heave after the hull is rotated; equilibrium in pitch is not satisfied.
- Heave and pitch balance; both draft and trim are altered to reach equilibria in heave and pitch at each heel angle.

For the wave pass problem, the ship's dynamics are not considered, so the balancing is done with respect to the ship's static heave force (displacement) and trim moment (displacement times LCB) corresponding in calm water. Physically, this is analogous to a ship moving with wave celerity and with statically applied heeling moment.

If ship dynamics are considered, the balancing should be performed relative to instantaneous forces and moments acting on the ship. This is a direct result of application of the d'Alambert principle, as the instantaneous attitude in waves is a result of action of inertia forces and moments. As a result, two more balancing options are needed:

- Heave balance relative instantaneous heave force (displacement).
- Heave and pitch balance relative instantaneous heave force and pitch moment.

The different balancing approaches and options can lead to significant differences in the GZ curve in waves, as shown by calculations for a post-Panamax container ship reported by Shin *et al.* (2004).

***Sample Configuration***

All the sample calculations described below were computed for the tumble-home top variant of the ONR Topsides hull form study. This hull has a very fine, destroyer-like shape beneath the design waterline and an unconventional "tumble-home" shape above. This gives the hull a very non-traditional righting arm curve. Because the righting arm falls off quite quickly on this hull, standard criteria would normally require a very high static $GM_T$. However, the results below use a fairly high center of gravity in order to clearly see the geometrical effects to the stability change in waves.

***Verification: Wave Pass Comparison***

The simplest verification of the LAMP-based stability calculation is a comparison of LAMP results for a fixed ship in a wave pass to the simple wave-pass method recommended by ABS (2004) as part of its susceptibility check for parametric roll. The ABS Guide recommends evaluating GM in waves using the actual wave waterline instead of calm water plane for calculating GM in waves. As the ship is kept fixed in this procedure, the "no balancing" option was used for LAMP calculations. The comparison for a wave with length equal to ship length (154m) and a height of 4 m is shown in Figure 2.

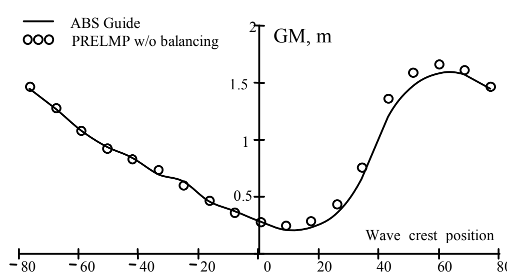


**Fig. 2: Comparison of GM value in wave: LAMP vs. ABS Guide**

The results of the two methods are very close; the small difference can be explained by the difference in hull geometry presentations: The ABS Guide method uses a station-based geometry while LAMP employs a 3-D panel-based geometry model.

A second verification was to check LAMP versus the more wave-pass calculation in EUREKA. Figure 3 shows an entire GZ curve on wave crest calculated with EUREKA and LAMP for a wave with the length equal to ship length (154m) and a height of 4 m. As EUREKA performs heave and pitch balancing to match the calm water displacement and LCB, the corresponding balancing option was used in LAMP. Again, the agreement of the results is very good and the small differences can be explained by the difference in the geometry presentation. The GZ curve in calm water is also plotted to show the magnitude of the change of stability in waves.

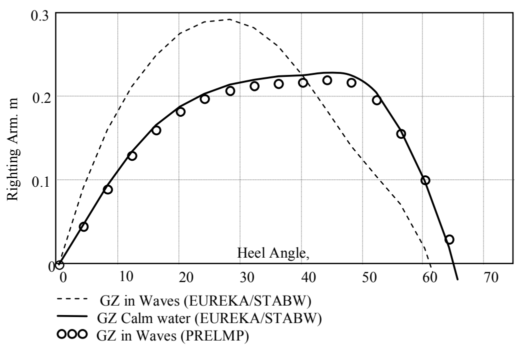


**Fig. 3: Comparison of GZ curves with Wave Crest Amidships: LAMP vs. EUREKA/STABW**

### *Verification of Self-consistency*

A self-consistency check of LAMP's righting arm curve evaluation in waves can be preformed by running a set of simulations of the ship sailing with a forward speed equal to wave celerity. The ship's position relative to wave crest is defined using initial condition, see Figure 4.

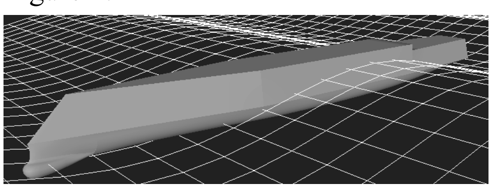

**Fig. 4: Simulation of heading with wave celerity**

Once steady state is achieved, an external heeling moment is applied to make the vessel roll until it stabilizes about an equilibrium heel angle, see Figure 5.

The value of the external heeling moment plotted against the equilibrium heel angle represents a point on a stability curve expressed in terms of moments. Figure 6 shows the curve of righting moments calculated from the GZ curve in waves plotted along with points obtained by direct simulation as described above. As the agreement between the results of simulation (points) and the GZ curve is obvious, the method passes the self-consistency test.

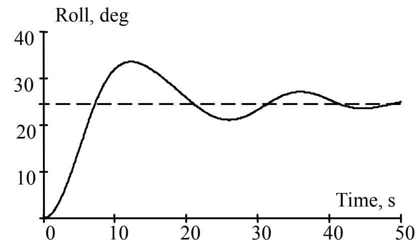


**Fig. 5: Time History of Roll After Application of Heeling Moment**

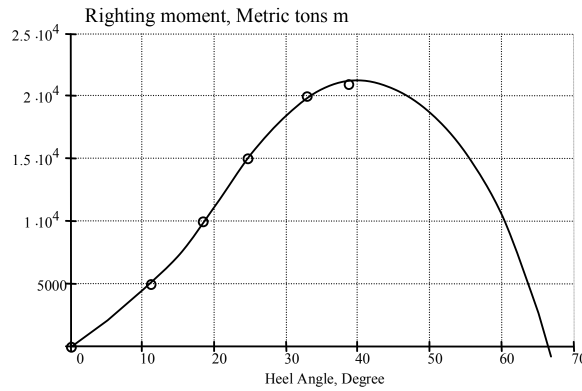


**Fig. 6: Results of Self-consistency Verification: Curve of the Righting Moments and Points Representing Direct Simulation Results**

## PROBABILISTIC CHARACTERISTICS

### *Choice of Parameters*

Strictly speaking, a GZ curve changing in time is a random field, so some parameters of the GZ curve can be chosen and considered as stochastic processes (obviously these process will depend on each other).

The most obvious candidate parameter is a value of GM. However, working with GM in waves may present certain difficulties. Usually, when considering a GM value, one implicitly assumes that this is the initial GM value calculated at initial equilibrium. So it is necessary that initial equilibrium (either stable or unstable) exists.

The existence of the initial equilibrium, however, may not be certain. Figure 7 shows a rare but possible case of an instantaneous GZ curve in irregular stern quartering seas.

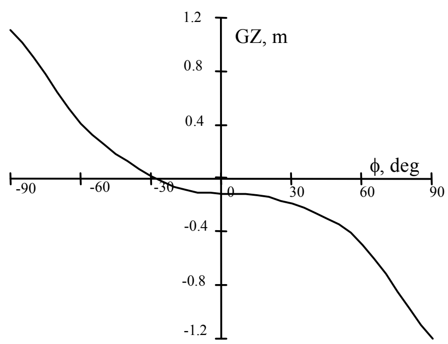


**Fig. 7: Degenerate Case of Instantaneous GZ Curve in Irregular Stern Quartering Seas.**

In the case shown in Figure 7, the restoring moment is negative for all positive roll angles. However there is a range of positive stability for negative roll angles from 0 to about -27°. The GZ curve crosses the axes of heel angles only once, at an angle of about -27°, making it the only equilibrium in this figure and it is the angle of vanishing stability for negative roll angles. As a result, there is no initial equilibrium and a conventional GM value cannot be evaluated in this case. This means that the stochastic process of GM values will not be continuous and getting useful information from such a process may not be trivial. Therefore, further study has been focused on other parameters of the GZ curve such as angle of maximum, value of maximum and angle of vanishing stability.

The evaluation of the angle of the maximum of GZ curve for such degenerate cases also deserves a discussion. First, it is suggested that if there no positive stability (like for positive roll angles in Figure 7) a value of zero should be assigned. Second, if there is positive stability, but no maximum (like for negative roll angles in Figure 7) assign a zero value as well. As the zero value for the angle of GZ maximum does not make sense itself, it can be used to identify instances where the maximum of GZ curve does not exist. This agreement cannot be used for GM, as a zero value makes sense for GM, but it does work as well for the value of GZ curve maximum as it does for the angle.

Figures 8 and 9 show time histories for the angles and values of GZ curve maximum from a typical LAMP simulation of the tumblehome ship in large, irregular stern quartering seas. To distinguish between the parameters evaluated for the two sides of the GZ curve, the words "positive" or "negative" are added to the parameter name. For these stern quartering sea cases, the positive values of roll correspond to the ship heeling into the waves while the negative values correspond to the ship rolling away from the incident wave direction. As can be seen from these figures, there are some instances when the time histories touch zero. While these points are, without a doubt, instances of decreased stability, there still may be some range of positive stability as the zero values are necessary but not sufficient indictors of completely negative stability.

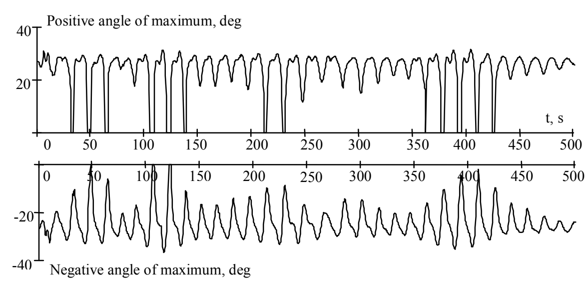


**Fig. 8 Time History of Angle of Maximum of the GZ Curve in Stern Quartering Waves**

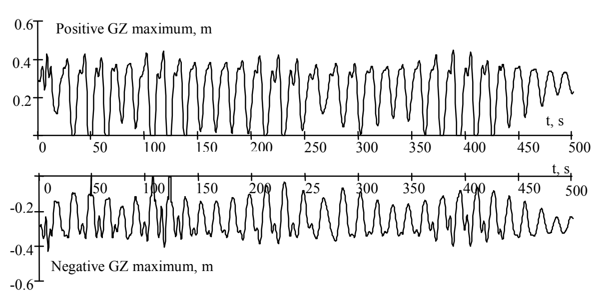


**Fig. 9 Time History of the Maximum value of the GZ Curve in Stern Quartering Waves**

This handling agreement is not applicable for angle of vanishing stability as it is an unstable equilibrium that can be located in any place. In the case of completely negative stability (on both sides), the initial equilibrium is unstable; in such a case the initial equilibrium located at zero can be identified as the angle of vanishing stability. Therefore the zero value of angle of vanishing stability is a physically plausible case of completely negative stability. For the asymmetric case shown in Figure 7, the angle of vanishing stability should be assigned to the single equilibrium point (about -27°) for both positive and negative roll angles. Therefore, negative angle of vanishing stability for positive roll angles is a necessary and sufficient indicator of completely negative stability.

Time histories for the angle of vanishing stability are shown in Figure 10. As it can be clearly seen from this figure, only negative stability could be observed for positive roll angels, while there is always some positive stability range for the other side.

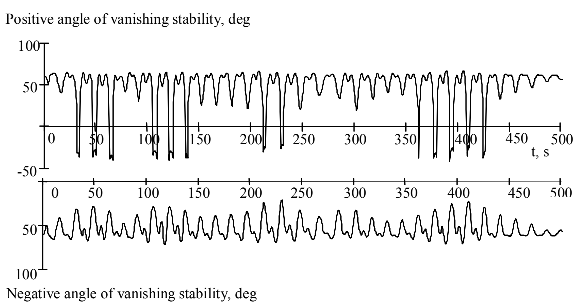


**Fig. 10 Time History of Angle of Vanishing Stability of the GZ Curve in Stern Quartering Waves**

### *Practical Non-ergodicity*

Once the parameters of the GZ curve have been chosen and degenerate cases have been dealt with, a standard procedure for the evaluation of statistical characteristic of stochastic processes related with large amplitude ship motions (Belenky & Weems 2008) can be applied. As the handling of degenerate cases (similar to shown in Figure 7) requires assignments of zeros, which will introduce significant nonlinearity into the process, special attention must be paid to practical non-ergodicity. Figures 11 through 13 show a measure of ergodicity (Belenky & Sevastianov 2007) *vs.* the number of records (realizations) for the six considered GZ curve characteristic processes.

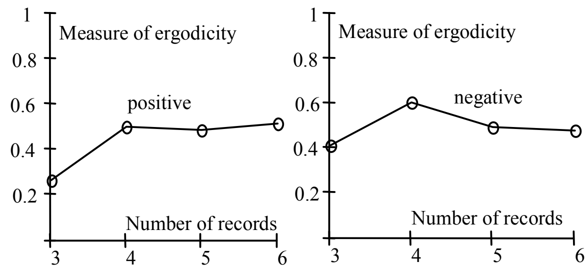


**Fig. 11 Measure of Ergodicity for Angles of GZ maximum**

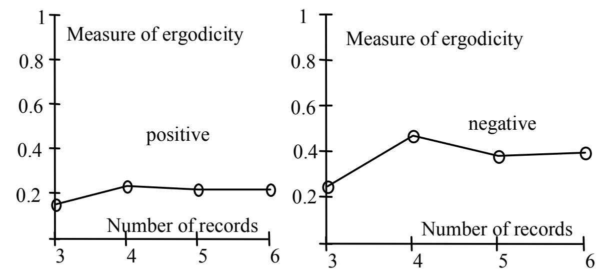


**Fig. 12 Measure of Ergodicity for values of GZ Maximum**

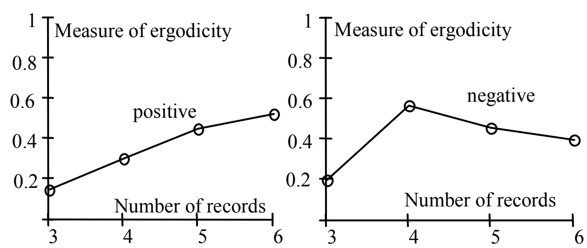


**Fig. 13 Measure of Ergodicity for Angles of Vanishing Stability**

The measure of ergodicity is a relative measure and requires comparison with the same measure calculated for a process that is known to be ergodic, usually the incident waves. For these simulations, the procedure estimates the measure of ergodicity for the wave elevation to be about 0.1 (Belenky & Weems 2008). The substantially higher values all of the considered processes indicate that they are practically not ergodic and multiple records are needed. These figures also show that the 6 records presented here are probably not sufficient, as the measure of ergodicity has not stabilized for the angle of vanishing stability.

### *Distribution*

Although the number of records may not be enough for an accurate quantitative evaluation of the statistical characteristics, they may still be enough to reveal the main features of shape of the distributions shown in Figures 14 through 16.

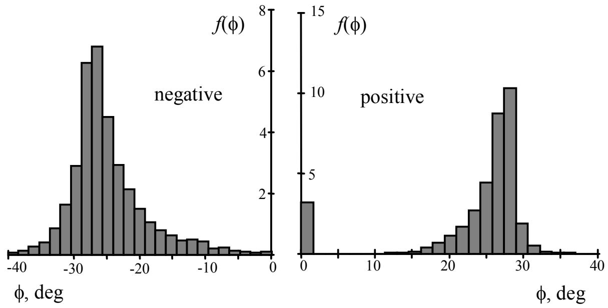


**Fig. 14 Distribution of Angle of Maximum of the GZ Curve**

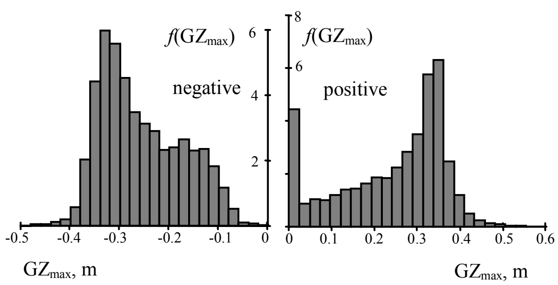


**Fig. 15 Distribution of Value of Maximum of the GZ Curve**

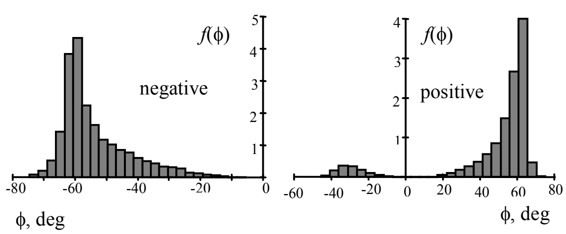


**Fig. 16 Distribution of Angle of Vanishing Stability**

The most prominent feature of the distributions of the angle and value of GZ maximum for positive roll angle is a significant occurrence of a value at zero. This can be considered as a simple statistical measure on the collective duration of degenerate cases, though it does not give a clear indication of the frequency of the degenerate cases or the duration of each degenerate case. It can also be seen that the duration of degenerate cases seems to be statistically insignificant for negative roll angles.

The distribution of the angles of vanishing stability for positive roll angle has a portion in an area of negative roll angles. This portion can be considered as a statistical measure of cumulative duration of negative stability cases altogether, although it also does not give a clear indication of the frequency of negative stability cases nor of the duration of each instance. Situations with completely negative stability were not observed for negative roll angles at all.

It is not immediately clear what theoretical distribution could be used to smooth out these statistical distributions. A very similar problem was considered by Kastner (1970, 1975) also using numerical simulations. The fitting of these distributions present the next major challenge in the current research effort.

## CONCLUDING COMMENTS

The changing stability for a ship in waves is, indeed, a very useful piece of information and is likely to be a critical element in the probabilistic evaluation of stability failures such as a parametric roll or capsizing. However, as it could be seen from the above discussion, the calculations and interpretation of stability in waves is non-trivial.

### *Methodology*

The basic calculation of stability in waves does not differ much from calm water; essentially it involves the rotation of the ship about its longitudinal axes and the evaluation of the resulting uncompensated transversal moment. The important issue here is to choose a proper balancing option: no balancing, balancing in heave or heave and pitch, balancing relative calm water equilibrium or instantaneous attitude. Different balancing options may be appropriate for different applications.

The calculations described here are based on the 3-D body-nonlinear evaluation of the Froude-Krylov and hydrostatic restoring forces and have been implemented in the LAMP system both as a quasi-static calculation based on the wave pass approach and a dynamic approach based on the simulation based ship motion in waves. The implementation has been tested against both the simple method recommended by ABS (2004) and the wave pass calculation implemented in EUREKA (Paulling 1961). Both comparisons have confirmed the LAMP implementation, as has a self-consistency verification based on the direct numerical simulation of a ship moving with wave celerity.

This implementation was used for the calculation of the stability changes for a tumble-home top ship in large, irregular stern-quartering seas. The results of these calculations showed instances in which a degenerate case was observed whereby the stability is still positive on one side of the GZ curve and completely negative on the other side. Such degenerate cases, though rare, require special handling in term of evaluating the parameters of the GZ curve. The purpose of such special handling is to avoid discontinuities in the stochastic processes of these parameters. Three GZ curve parameters have been chosen for further analysis: the angle and value of GZ curve maximum and the angle of vanishing stability.

An analysis of ergodicity as performed showed that these quantities are practically ‘not ergodic’ for the sample hull form thereby requiring multiple simulation records for the evaluation of these parameters. An initial analysis of the distribution of these quantities showed that the fitting of theoretical distributions presents the next major challenge in the present effort.

***Use for Criteria and Procedures***

As outlined by Belenky *et al.* (2008), the probabilistic characteristics of stability in waves may be used as the background for parametric probabilistic criteria related to pure loss of stability in waves. This idea is not new: a number of authors considered GM as a random quantity (Dunwoody 1989a, 1989b, Palmquist 1994, Roberts 1982, Bulian & Francescutto 2006), and the accounting of changing stability in waves was a part of probabilistic procedure proposed by Themelis & Spyrou (2007). Still, there are many questions to be asked and answered before probabilistic qualities of stability changes will become a part of intact stability regulations.

## ACKNOWLEDMENTS

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