Analytical Predictions of Surf-Riding Threshold and Their Experimental Validation

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ABSTRACT

As a candidate of the International Maritime Organization (IMO) vulnerability criteria for broaching, applicability of Spyrou's analytical formula for surf-riding threshold for a ship in following waves is examined. As a result, it is concluded that Spyrou's formula can be simply used and provides fairly good agreement with surf-riding threshold obtained by numerical bifurcation analysis except for large wave steepness. Then a different analytical formula is newly proposed. By applying a continuous piecewise linear approximation to the wave-induced surge force, a heteroclinic bifurcation point is analytically obtained with an uncoupled surge equation. Calculation results using this formula are presented, and show good agreement with those obtained by utilizing a numerical bifurcation analysis, even for large wave steepness. Further, extensive tests have been conducted for an unconventional ship model to experimentally obtain the surf-riding threshold. Then it is confirmed that the experimentally determined surf-riding threshold reasonably well agrees with the calculation results using the proposed analytical formula.

KEYWORDS

Broaching; Spyrou's formula; Surf-riding threshold; Heteroclinic bifurcation; Continuous piecewise linear approximation; Vulnerability criteria; Performance-based intact stability criteria.

INTRODUCTION

The IMO started to develop performance-based intact stability criteria, which should cover stability variation problems, stability under dead ship condition and broaching, with the target date of 2010. For each phenomenon, the working group of the SLF Sub-Committee at the IMO agrees that this criterion should consist of a vulnerability criterion and a direct stability assessment using first-principle tools. (Japan, the Netherlands and the United States, 2007) Since the vulnerability criterion is to be applied to all ships, it should be easily used. And a non-empirical approach is recommended to enable us to apply it to new ship-types. Therefore, it is desirable that it utilises analytical formulae rather than numerical simulation using discrete modelling of continuous variables. An analytical formula here means that an expression describing phenomena in terms of mathematical concept of limits and continuity.

Since the surf-riding is the prerequisite of broaching-to, occurrence of broaching-to can be replaced by that of surf-riding. Thus, an analytical approach for predicting surf-riding threshold in following seas is highly expected. As a candidate of this approach, Spyrou (2001) presented an analytical formula for prediction of surf-riding threshold. He approximated the thrust and resistance curve using polynomial fitting and derived an exact analytical expression of surf-riding threshold. Therefore, in this paper, we attempt to examine the applicability of his formula and to propose a different analytical approach as an alternative.

These analytical approaches are compared with numerical bifurcation analysis (Maki et al., 2007) and free-running model experiments of a new-ship types.

CO-ORDINATE SYSTEM

In this paper all the formulations are based on the co-ordinate system shown in Fig.1. An inertia co-ordinate system $o-\xi\eta\zeta$ with the origin at a wave trough has the ξ axis pointing toward the wave direction. The ship fixed co-ordinate system, $G_s - xyz$, with the origin at the centre of gravity of the ship has the x - axis pointing toward the bow and z - axis downward. ξ_G represents the longitudinal position of the centre of gravity from the wave trough.

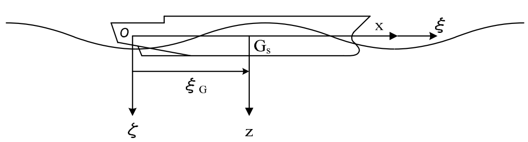

Fig. 1: Co-ordinate systems.

NUMERICAL BIFURCATION ANALYSIS

The equation of motion representing nonlinear surging is expressed as (1):

$$(m+m_x)\ddot{\xi}_G + [R(u) - T(u,n)] - X_w = 0 \qquad (1)$$

In this equation a dot denotes differentiation with respect to time *t*. Here m : the ship mass, m_x : the added mass in the *x* direction, u : the instantaneous ship velocity in the *x* direction, *R*: the ship resistance, *T*: the propeller thrust, *n* the propeller rate, X_W: wave-induced surge force. In this equation higher order terms, such as thrust variation due to wave particle velocity are ignored.

Put the state vector as follows:

$$\mathbf{x} \equiv (G,\, u)^T \qquad (2)$$

In this paper, for the sake of brevity, ξ_G/λ is denoted: G . Here equation (1) can be rewritten as follows:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x};\, n) = (f_1(\mathbf{x};\, n),\, f_2(\mathbf{x};\, n))^T \qquad (3)$$

where

$$f_1 = (u-c)/\lambda$$
$$f_2 = \{T(u;n) - R(u) + X_w\}/(m+m_x)$$

A nonlinear dynamical system described by Equation (3) could have fixed points as follows:

$$\mathbf{x}_0 \equiv (G_0,\, c)^T \text{ and } \mathbf{x}_1 \equiv (G_1,\, c)^T \qquad (4)$$

Under the definition of $\mathbf{x}_0$ and $\mathbf{x}_1$, they must satisfy the equilibrium conditions:

$$\{T(c;n) - R(c) + X_W(G_0)\}/(m+m_x) = 0 \qquad (5.a)$$
$$\{T(c;n) - R(c) + X_W(G_1)\}/(m+m_x) = 0 \qquad (5.b)$$

Linearizing Equation (3) at $\mathbf{x}_0$ yields the conditions about eigenvalue μ_α and eigenvector $\mathbf{x}_\alpha = (G_\alpha, u_\alpha)^T$ as follows:

$$2\mu_\alpha - (\alpha_{11} + \alpha_{22}) - \sqrt{D_\alpha} = 0 \qquad (6.a)$$
$$(\alpha_{11} - \mu_\alpha)G_\alpha + \alpha_{12}u_\alpha = 0 \qquad (6.b)$$
$$G_\alpha{}^2 + u_\alpha{}^2 - \delta^2 = 0 \qquad (6.c)$$

where $D_\alpha = (\alpha_{11}+\alpha_{22})^2 - 4(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})$. Here α_{ij} denotes the Jacobi matrix of linearized equation of Equation (3) at $\mathbf{x}_0$.

Heteroclinic bifurcation requires the unstable invariant manifold from a saddle coincides with the local stable invariant manifold of another saddle. Mathematically this idea can be written as follows:

$$\|\mathbf{\Psi}(\mathbf{x}_0 + \mathbf{x}_\alpha, \tau) - \mathbf{x}_1\|^2 - \delta_2^2 = 0 \in \mathbf{R}^1 \qquad (7)$$

Here $\mathbf{\Psi}(\mathbf{x}_I, \tau)$ means the solution of the state equations where $\mathbf{x}_I$ is the initial value of the state vector, τ is the duration of integration in time. Then, if we find the propeller rate, n_0 , which satisfies Equations (5)-(7), this is a heteroclinic bifurcation point.

Numerical calculations were carried out, and an example of the results is shown in Fig.2. The wave steepness is 0.0667 and the wavelength to the ship length ratio is 1.5. In this case the heteroclinic bifurcation point obtained is the nominal Froude number, Fn, of 0.3483.

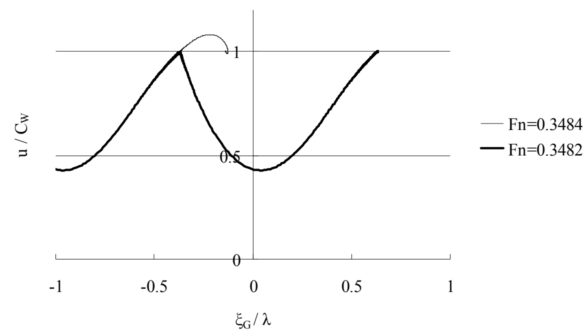

Fig.2 Comparison of phase trajectories in the vicinity of a surf-riding threshold (Maki et al. 2007).

The iterations required by numerical bifurcation analysis to converge to 0.3483 in the above sea state are shown in Fig.3.

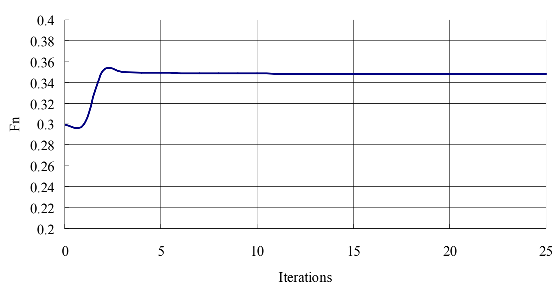

Fig. 3 Convergence process of the numerical bifurcation analysis for nominal Froude number (Maki et al. 2007).

Though numerical bifurcation analysis gives accurate solution described by Equation (1), it requires numerical integration with discrete time variable. Therefore this method is not so suitable for the vulnerability criteria but can be used to validate analytical methods.

MATHEMATICAL MODEL FOR NON-LINEAR SURGING AND SPYROU'S FORMULA

Assuming that the hull form is almost longitudinally symmetric, the wave-induced surge force is represented as the first order approximation as follows:

$$X_w \approx f \sin(k\xi_G) \tag{8}$$

where $k \equiv 2\pi/\lambda$ and λ is the wavelength. The relative ship speed to waves, $\dot{\xi}_G$ can be obtained by the formulae, $\dot{\xi}_G = u - c$. Here c is the wave celerity. Then we can simplify the Equation (1) as Equation (9).

$$\begin{aligned}&(m+m_x)\ddot{\xi}_G + A_1(c\,;n)\dot{\xi}_G + A_2(c)\dot{\xi}_G^2\\&+A_3\dot{\xi}_G^3 + f\sin(k\xi_G) = T(c,n) - R(c)\end{aligned} \tag{9}$$

If relative speed variation is limited, a method of least-square fit can be applied to the nonlinear damping so that it can be approximated with an quadratic function as follows (Spyrou, 2001):

$$\begin{aligned}&(m-m_x)\ddot{\xi}_G + \gamma(n)\,\mathrm{sgn}\,\dot{\xi}_G \cdot \dot{\xi}_G{}^2 + f\sin(k\xi_G)\\&= T(c,n) - R(c)\end{aligned} \tag{10}$$

where $\gamma(n)$ is defined as:

$$\gamma(n) = -\frac{A_1(c\,;n)\sum_{i=1}^{l}\dot{\xi}_G^3 + A_2(c)\sum_{i=1}^{l}\dot{\xi}_G^4 + A_3\sum_{i=1}^{l}\dot{\xi}_G^5}{\sum_{i=1}^{l}\dot{\xi}_G^4}.$$

In the above, l points are sampled in an appropriate velocity range which is selected as $\dot{\xi}_G \in [-c/2, 0]$ based on some preliminary calculations. By requesting the maximum ship speed to be equal to the wave celerity, Spyrou obtained the exact analytical formula of surf-riding threshold as follows:

$$f = [R(c) - T(c,n)]\sqrt{\frac{k^2(m+m_x)^2}{4\gamma^2} + 1} \tag{11}$$

This formula can be also deduced by requesting the heteroclinic bifurcation (See Appendix).

CONTINIUOUS PIECEWISE LINEAR APPROXIMATION FOR NON-LINEAR SURGE EQUATION

As shown in the above section, Spyrou converted nonlinear damping to quadratic one. Equation (9) has two nonlinear elements; one is nonlinear damping and the other is a restoring term represented by a sinusoidal function.

A quadratic approximation, however, is not necessarily reasonable because the shape of the original damping is almost linear. As an alternative, the authors approximate with the following equivalent linear expression:

$$(m+m_x)\ddot{\xi}_G+\beta(n)\dot{\xi}_G+f\sin(k\xi_G) = T(c,n)-R(c) \qquad (12)$$

where

$$\beta(n)=\frac{A_1(c\,;n)\sum_{i=1}^{l}\dot{\xi}_G^3+A_2(c)\sum_{i=1}^{l}\dot{\xi}_G^4+A_3\sum_{i=1}^{l}\dot{\xi}_G^5}{\sum_{i=1}^{l}\dot{\xi}_G^3}$$

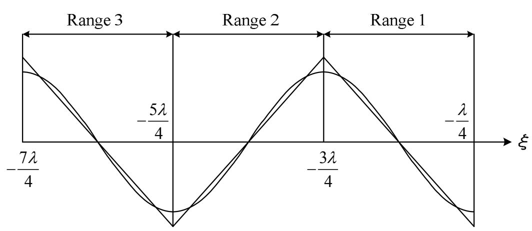

Fig. 4: Schematic view of continuous piecewise linearization for sinusoidal function.

Since this equation cannot be exactly solved, the authors attempt to approximate the nonlinear restoring term with continuous piece-wise linear (CPL) curves as shown in Fig. 4. Then, the following differential equation is obtained:

$$\ddot{\xi}_G+\alpha_1\dot{\xi}_G+\alpha_2\sin(k\xi_G)=\alpha_3 \qquad (13)$$

where

$$\alpha_1\equiv\frac{\beta(n)}{(m+m_x)}$$

$$\alpha_2\equiv\frac{f}{(m+m_x)}$$

$$\alpha_3\equiv\frac{T(c,n)-R(c)}{(m+m_x)}$$

$$\sin(k\xi_G)\approx\begin{cases}-\frac{4}{\lambda}\left(\xi_G+\frac{1}{2}\lambda\right) & \left(-\frac{3}{4}\lambda<\xi_G<-\frac{1}{4}\lambda\right)\\ \frac{4}{\lambda}(\xi_G+\lambda) & \left(-\frac{5}{4}\lambda<\xi_G<-\frac{3}{4}\lambda\right)\\ -\frac{4}{\lambda}\left(\xi_G+\frac{3}{2}\lambda\right) & \left(-\frac{7}{4}\lambda<\xi_G<-\frac{5}{4}\lambda\right)\end{cases}$$

This equation can be easily solved for each range. To determine the surf-riding threshold as a heteroclinic bifurcation point, it is necessary to find a trajectory connecting two saddles. Thus we should find the trajectories to and from the saddles first. This requires that the solution of Range 1 and that of Range 3 do not tend to infinity when the time tend to infinity. Taking account of the connection conditions at the borders, the following equation can be reduced.

$$-\alpha=2e^{\lambda_R\tau}\left[c_R\cos\lambda_I\tau-c_I\sin\lambda_I\tau\right] \qquad (14)$$

where

$$\tau=\frac{1}{\lambda_R}\ln\left\|\frac{\alpha\sqrt{\Lambda_1{}^2+\Lambda_2{}^2}}{2\lambda_I\left(c_R{}^2+c_I{}^2\right)}\right\|$$

$$\Lambda_1\equiv c_R\lambda_I+c_I\lambda_R+c_I\lambda_2$$

$$\Lambda_2\equiv c_R\lambda_R-c_I\lambda_I+c_R\lambda_2$$

$$\alpha\equiv\frac{1}{4}\lambda+\frac{\lambda\alpha_3}{4\alpha_2}$$

$$\lambda_{1,2}\equiv\frac{-\alpha_1\pm\sqrt{\alpha_1^2+16\alpha_2/\lambda}}{2}$$

$$\lambda_{3,4}\equiv\frac{-\alpha_1\pm\sqrt{\alpha_1^2-16\alpha_2/\lambda}}{2}$$

$$c_{1,2}=\left[\pm Z_1\pm\frac{1}{4}\lambda\lambda_{2,1}\mp\frac{\lambda\alpha_3}{4\alpha_2}\lambda_{2,1}\right]/\left(\lambda_1-\lambda_2\right)$$

$$c_{3,4}=\left[\mp\frac{1}{4}\lambda\pm\frac{\lambda\alpha_3}{4\alpha_2}\right]\cdot\frac{\left(\lambda_1+\lambda_{4,3}\right)}{\left(\lambda_3-\lambda_4\right)}$$

Here $c_R=\mathrm{Re}[c_3]$, $c_I=\mathrm{Im}[c_3]$, $\lambda_R=\mathrm{Re}[\lambda_3]$ and $\lambda_I=\mathrm{Im}[\lambda_3]$. If this equation is satisfied, a heteroclinic bifurcation point is determined. It can be solved using simple Newton iterations with respect to a single variable, such as propeller rate, *n*, when the bifurcation point is required as a function of the propeller rate. Although the Newton method is a numerical calculation procedure, it does not require to discretize Equation (28). This can be regarded as an analytical approach.

NUMERICAL RESULTS

In order to validate those analytical approaches, calculated results were obtained using the analytical formulae proposed above for a 135 gross tonnage-type purse seiner known as the ITTC Ship A-2 are compared with those obtained by the numerical bifurcation analysis.

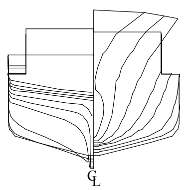

Fig. 5: Body plan of the ITTC Ship A-2.

Its body plan is shown in Fig.5. Its detailed data are available in Umeda and Hashimoto (2002).

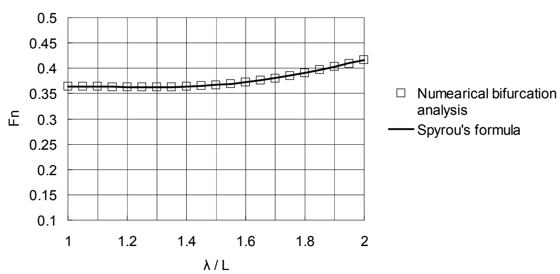

Fig. 6: Comparison of the surf-riding thresholds predicted using the numerical bifurcation analysis with quadratic damping, and that predicted using the Spyrou's solution, for $H/\lambda = 0.06$.

Fig.6 indicates the comparison between the surf-riding threshold predicted using the numerical bifurcation analysis (Maki et al., 2007) of equivalent quadratic damping, and that predicted by using the Spyrou's analytical solution, Equation (10). In this figure the abscissa is the wavelength to ship length ratio, while the ordinate indicates the nominal Froude number, defined as the ship velocity in calm water under the same propeller revolutions. Here the wave steepness, H/λ, is 0.06 and the wavelength to ship length ratio, λ/L, ranges from 1.0 to 2.0. Since there is no visible difference between the two, it can be concluded that the Spyrou's formula is equivalent to the numerical bifurcation analysis for predicting the surf-riding threshold with Equation (10).

Next, the threshold using the present formula, i.e. Equation (14), is compared with that obtained from the numerical bifurcation by using the CPL approximated wave-induced surge force, as shown in Fig.7.

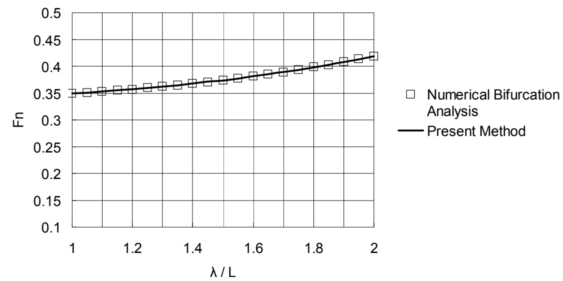

Fig. 7: Comparison of the surf-riding thresholds predicted using the numerical solution, and that predicted using the analytical solution with CPL approximation, for $H/\lambda = 0.06$.

Since there is also no visible difference between the two, it can be concluded that the present formula is equivalent to the numerical bifurcation analysis for predicting the surf-riding threshold with Equation (13). Therefore, these analytical methods are reliable enough.

Then, the comparisons are extended to cover the numerical bifurcation analysis for the original surge equation, i.e. Equation (1), as shown in Figs.8-9. When the wave steepness is small, the three methods show similar trends. Compared to the numerical bifurcation analysis, Spyrou's formula slightly overestimates the value of the threshold for lower wave steepness, but underestimates it for longer waves with higher wave steepness. On the other hand, the present method provides reasonable agreements with the numerical bifurcation analysis. The reason for the difference between the two could be the approximation errors for the damping term. The estimation of surf-riding threshold could depend on the errors in damping, which could be more significant for large ship motion in higher wave steepness and longer wavelength.

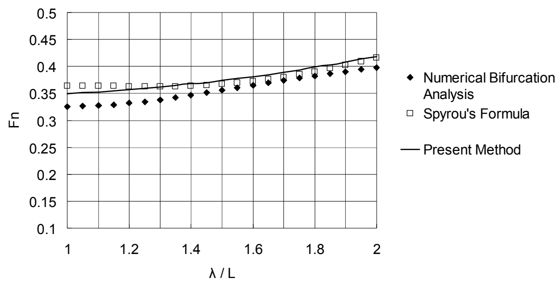

Fig. 8: Comparison of the surf-riding thresholds predicted using several methods, for $H/\lambda = 0.06$.

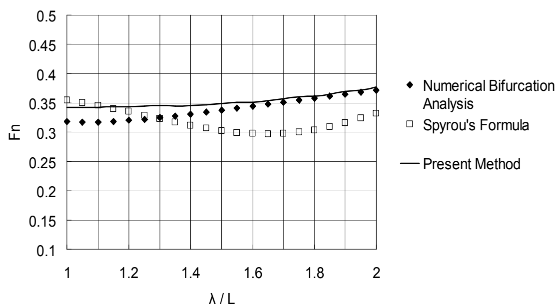

Fig. 9: Comparison of the surf-riding thresholds predicted using several methods, for $H/\lambda = 0.08$.

VALIDATION AGAINST FREE-RUNNING MODEL EXPERIMENTS

In order to validate the present formula, particularly its applicability to unconventional ships to which the IMO vulnerability criterion is expected to apply, predictions were compared with results obtained from a free-running model experiment carried out in the seakeeping and manoeuvring basin of NRIFE (National Research Institution of Fishing Engineering) with the ONR tumblehome vessel. The vessel's body plan is shown in Fig.10. This is a good example of an unconventional vessel, and has all the required information in the public domain. Her above-water hull has tumblehome and a wave-piercing bow. The ship is equipped with twin screws and twin rudders. The details of the experiments will be reported by Umeda et al. (2008).

Fig.11 shows a comparison of the predicted surf-riding thresholds, using both the present formula and the numerical bifurcation analysis, with the experimental results. In this the autopilot course was set to -5 degrees, because it was known that the effect of such small autopilot course on the surf-riding threshold is negligibly small (Umeda et al., 2006) and an autopilot course of 0 degrees could cause a collision with the tank wall at the beginning of the model run. Here the ship model initially drifted near the wave maker and then the propellers and the autopilot control were activated. The propeller revolutions were set to attempt to control the specified nominal Froude numbers during the model runs and a proportional autopilot was used.

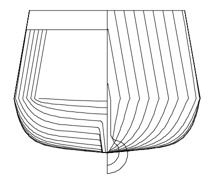

Fig. 10: Body plan of the ONR tumblehome vessel.

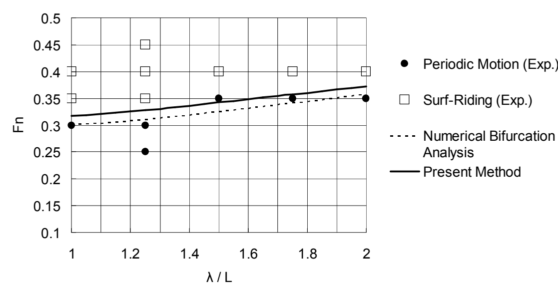

Fig. 11: Comparison of the predicted surf-riding thresholds for the two methods with the experimental results for $H/\lambda = 0.05$.

The predicted surf-riding thresholds obtained both from the numerical bifurcation analysis and the present formula agree well with the results from the experiments, where the surf-riding threshold is between the runs showing periodic motion, and those showing surf-riding (as shown in Fig.11). The slight underestimate in the Froude number for the predicted threshold could be caused by the nonlinearity in the wave-induced surge force, which was discussed by Hashimoto et al. (2004). For practical purpose, however, the present formula seems to provide sufficiently accurate prediction even for this unconventional vessel.

CONCLUDING REMARKS

The main conclusions from this work are summarized as follows:

(1) Applicability of the analytical solution proposed by Spyrou was examined. Numerical simulation results using Spyrou's formula show good agreements with numerical bifurcation analysis of its assumed model.
(2) Based on continuous piecewise linear approximation, analytical formulae to estimate the deterministic surf-riding threshold are presented and are reduced to a simple formula for practical use.
(3) These formulae were validated by computing results with those obtained using a numerical bifurcation analysis. While Spyrou's formula could underestimate the Froude number for the surf-riding threshold for longer and steeper waves, the present formula agrees with the numerical bifurcation analysis fairly well.
(4) The present formula was well validated with free-running model experiments using the ONR tumblehome vessel as an example of unconventional vessels.

ACKNOWLEDMENTS

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REFERENCES

Hashimoto, H., Umeda, N. and Matsuda, A. 2004, "Importance of Several Nonlinear Factors on Broaching Prediction", Journal of Marine Science and Technology, Vol.9,pp.80-93.

Japan, the Netherlands and the United States, 2007, "Framework for the Development of New Generation Criteria for Intact Stability", SLF50/4/4, IMO.

Maki, A., Umeda, N. and Hori, M., 2007, "Prediction of Global Bifurcation Points as Surf-Riding Threshold in Following Seas, Journal of the Japan Society of Naval Architects and Ocean Engineers, Vol., 5, p. 205-215, (in Japanese).

Spyrou, K. J., 2001, "Exact Analytical Solutions for Asymmetric Surging and Surf-Riding", Proceeding of 5th International Workshop on Stability and Operational Safety of Ships, University of Trieste (Trieste), pp.4.5.1-4.5.3.

Umeda, N. and Hashimoto, 2002, "Qualitative Aspects of Nonlinear Ship Motions in Following and Quartering Seas with High Forward Velocity", Journal of Marine Science and Technology, Vol., 6, pp. 111-121.

Umeda, N., Hori, M. and Hashimoto, H., 2006, "Theoretical Prediction of Broaching in the Light of Local and Global Bifurcation Analysis", Proceeding the 9th International Conference on Stability of Ships and Ocean Vehicles, Federal University of Rio de Janeiro (Rio de Janeiro), Vol.1, pp.353-362.

Umeda, N., Yamamura, S., Matsuda, A., Maki, A., and Hashimoto, H. 2008, "Extreme Motion of a Tumblehome Hull in Following and Quartering Waves", Proceeding of the 6th Osaka Colloquium on Seakeeping and Stability of Ships, Osaka Prefecture University (Osaka), in press.

APPENDIX

The authors attempt to derive Spyrou's formula by a different way. Unstable equilibrium point is represented as follows:

$$\tilde{\xi}_G = \left(\nu - \frac{1}{2}\right)\lambda - \frac{\lambda}{2\pi}\sin^{-1}\frac{T(c,n) - R(c)}{f} \qquad \text{(A.1)}$$

Here ν is arbitrary integer. And a trajectory in phase plane can be represented as follows:

$$\dot{\xi}_G = -\frac{1}{k}\sqrt{c_u q e^{2pk\xi_G} + \frac{2q(\cos k\xi_G + 2p\sin k\xi_G)}{1+4p^2} - \frac{r}{p}} \qquad \text{(A.2)}$$

where $p = \frac{\gamma}{k(m + m_x)}$ and $q = \frac{fk}{m + m_x}$.

Here c_u is arbitrary constant to be determined by an initial condition.

A heteroclinic bifurcation requires that the trajectory from the saddle type equilibrium point reach the another saddle, taking infinite time. Put the positions of the two saddles in the phase-plane as follow:

$$(\xi_G, \dot{\xi}_G) = (\tilde{\xi}_G, 0) \quad \text{(A.3a)}$$

$$(\xi_G, \dot{\xi}_G) = (\tilde{\xi}_G - \lambda, 0) \quad \text{(A.3b)}$$

Substituting Equation (A.3a) into Equation (A.2), we can determine the arbitrary constant c_u as follows:

$$c_u = \exp\left(2pk\tilde{\xi}_G\right)\frac{-r \pm 2pq\sqrt{1-\left(r/q\right)^2}}{pq\left(1+4p^2\right)} \quad \text{(A.4)}$$

Then Substituting Equation (A.3b) into Equation (A.2) yields following relation:

$$\left[\exp\left(-2pk\lambda\right) - 1\right]\left[r \pm 2pq\sqrt{1-\left(r/q\right)^2}\right] = 0 \quad \text{(A.5)}$$

Setting the second term of Equation (A.5) to zero yields Spyrou's formula. Furthermore, setting the first term to zero gives the following condition:

$$\lambda = 0 \quad \text{(A.6)}$$

This can be regarded as a kind of calm water condition.